Mathematics

A Textbook for

Grade VII

Free from Government
NOT FOR SALE

Khyber Pakhtunkhwa Textbook Board Peshawar

Table of contents

Unit No	Topic Name	Page No
1	Sets	01
2	Rational Numbers	20
3	Decimals	44
4	Exponents	57
5	Square Root of Positive Number	77
6	Direct and Inverse Variation	98
7	Financial Arithmetic	111
8	Algebraic Expressions	126
9	Linear Equations	148
10	Fundamentals of Geometry	165
11	Practical Geometry	185
12	Circumference, Area and Volume	199
13	Information Handling	210
	Answers	222

Unit 1 Sets

What You'll Learn

- Express a set in descriptive form, set builder form, tabular form.
- Define union, intersection and difference of two sets.
- Find union of two or more sets, intersection of two or more sets, difference of two sets.
- Define and identify disjoint and overlapping sets.
- Define a universal set and complement of a set.
- Verify different properties involving union of sets, intersection of sets, difference of sets and complement of a set, e.g., $A \cap A' = \phi$.
- Represent sets through Venn diagram.
- Perform operations of union, intersection, difference and complement on two sets A and B when A is subset of B, B is subset of A, A and B are disjoint sets, A and B are overlapping sets, through Venn diagram.

Why It's Important

The purpose of sets is to make a collection of related objects. They are as important for building more mathematical structures as hammer, saw, and nails are important for building a table or chair. They are important everywhere in mathematics because every field of mathematics uses or refers to sets in some way.

1.1 Sets

We know that a set is "a collection of well-defined distinct objects".
For example,

- The collection of natural numbers from 1 to 10.
- The collection of months in a calendar year.

1.1.1 Methods of expressing a set

Any set can be expressed by following three methods.

(i). Descriptive form (ii). Tabular form (iii). Set builder form

(a) Descriptive form

In this form, instead of writing all the members of the set, such a sentence is written which makes inclusion of each and every member of the set clear.

Example 1 Give two examples of a set in Descriptive form

Solution

(i). Set of first ten odd integers.
(ii). Set of vowels of English alphabets.

(b) Tabular Form

In this form, members of a set are written within braces and are separated by commas. Here is an example of Tabular form:

$\{3, 6, 91, ...\}$

element element element three dots means goes on forever (infinite)

"element" or "member" mean the same thing

Example 2 Use Tabular Form to express the sets given in Example1.

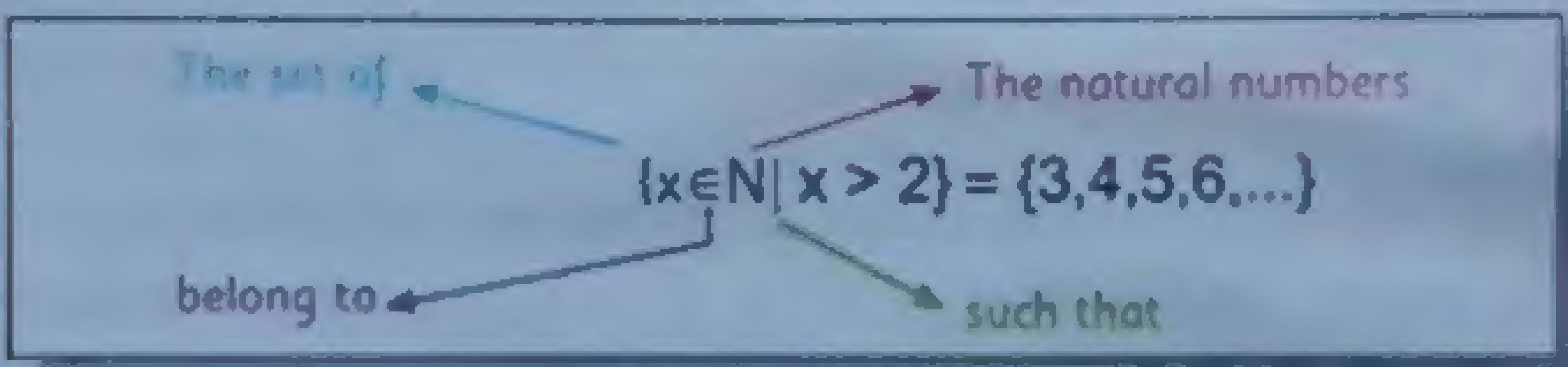
(i). $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

(ii). $B = \{a, e, i, o, u\}$

(c) Set Builder Form

In this form of a set, a general element 'x' and its characteristic property is mentioned which is common to all the elements of the set.

Here is an example of set-builder form:



It is read as " the set of all x' s in natural numbers, such that x is greater than 2".

Example 3 Use Set builder form to express the sets given in Example1.

(i). $A = \{x \mid x$ is odd number and is less than 20}.

(ii). $B = \{x \mid x$ is a vowel of English alphabets}.

Remember the symbol "|" stands for "such that".

Guided Practice

Write the following sets in

(i). Tabular form (ii). Set builder form

(iii). Set of prime numbers less than 20

(iv). Set of days of a week.

Exercise 1.1

1. Write the following sets in descriptive form.

   (i). A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

   (ii). B = {a, b, c, d, e, f}

   (iii). C = {2, 4, 6, 8, 10}

   (iv). D = {2, 3, 5, 7, 11, 13, 17, 19}

2. Write the following sets in tabular form.

   (i). The set of first five multiples of 5.

   (ii). The set of natural numbers between 10 and 20.

   (iii). The set of the names of the days in a week.

   (iv). The set of the positive even numbers less than 10.

3. Write the following sets in set builder form.

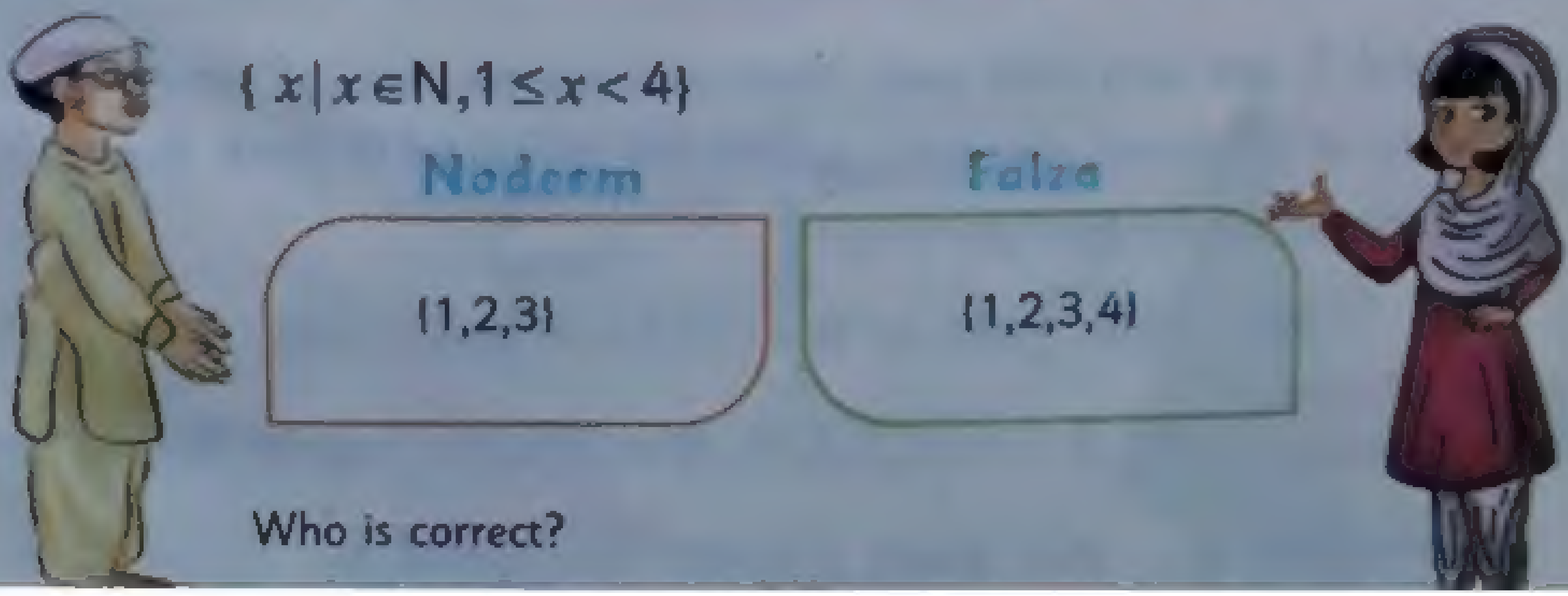
   (i). A = {1, 2, 3, ... 20}

   (ii). B = {a, e, i, o, u}

   (iii). C = {Peshawar, Lahore, Karachi, Quetta}

   (iv). D is the set of the odd numbers.

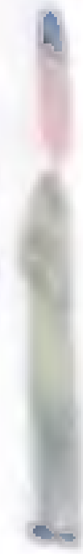
Find the error. Nadeem and Faiza are writing the set in tabular form.



Who is correct?

1.2 Operations on sets

Sets can be combined in a number of different ways to produce some other sets. Here three basic operations are introduced and their properties are discussed.



(a) Union of two or more sets

If A and B are any two sets, then the union of set A and set B consists of all elements in set A or in set B and is denoted by $A \cup B$,

In set builder form. $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Example 4 If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ find $A \cup B$ and $B \cup A$

Solution

$A \cup B = \{1,2,3\} \cup \{3,4,5\} = \{1,2,3,4,5\}$

$B \cup A = \{3,4,5\} \cup \{1,2,3\} = \{1,2,3,4,5\}$

Example 5 Let $A = \{a, b, c\}$, $B = \{b, c, d, e\}$ and $C = \{c, d, e, f, g\}$ then find $A \cup (B \cup C)$

Solution

$$A \cup (B \cup C) = \{a, b, c\} \cup (\{b, c, d, e\} \cup \{c, d, e, f, g\})$$

$$= \{a, b, c\} \cup \{b, c, d, e, f, g\}$$

$$= \{a, b, c, d, e, f, g\}$$

(b) Intersection of two or more sets

If A and B are any two sets, then the intersection of set A and set B and consists of all those elements which are common to both A and B, and it is denoted by $A \cap B$.

In set builder form $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Example 6 If $A = \{1,2,3,4\}$, $B = \{3,4,5\}$, find $A \cap B$ and $B \cap A$

Solution

$A \cap B = \{1,2,3,4\} \cap \{3,4,5\} = \{3,4\}$

$B \cap A = \{3,4,5\} \cap \{1,2,3,4\} = \{3,4\}$

Example 7 Let $A = \{a, b, c\}$, $B = \{b, c, d, e\}$ and $C = \{c, d, e, f, g\}$ then find $A \cap (B \cap C)$.

Solution $A \cap (B \cap C) = \{a, b, c\} \cap [\{b, c, d, e\} \cap \{c, d, e, f, g\}]$

$= \{a, b, c\} \cap \{c, d, e\}$

$= \{c\}$

Guided Practice

If $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 3, 5, 7\}$, i. Find $A \cup B$? ii. Find $A \cap B$?

(c) Difference of two sets

If A and B are two sets then their difference consists of all those elements of set A which are not in set B and it is denoted by $A-B$ or $A \backslash B$.

In set builder form.

$$A \backslash B = \{x \mid x \in A \text{ and } x \notin B\}$$

Example 8 If $A = \{2,3,4,5,6\}$ and $B = \{5,6,7,8\}$, then

$A \backslash B = \{2,3,4,5,6\} - \{5,6,7,8\}$

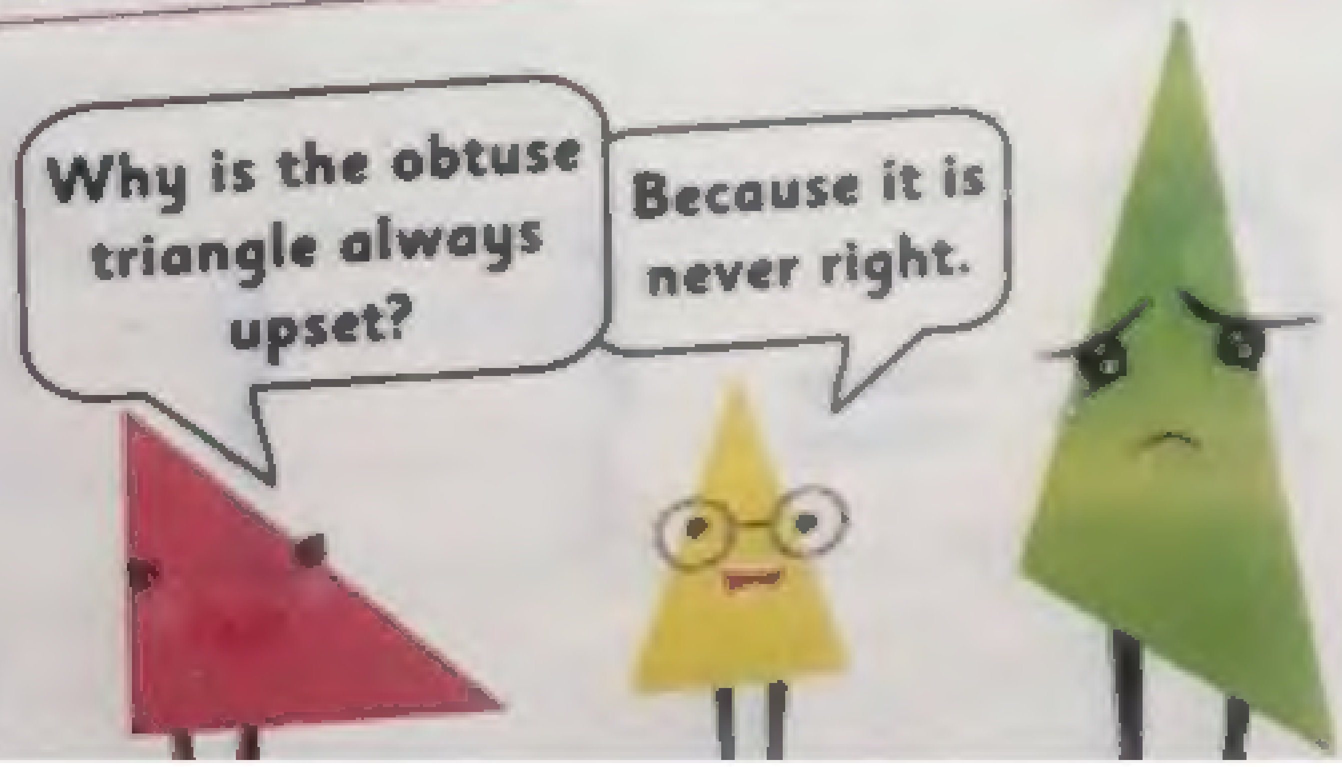
$= \{2,3,4\}$

$B \backslash A = \{5,6,7,8\} - \{2,3,4,5,6\}$

$= \{7,8\}$

Guided Practice

If $X = \{a, e, i, o, u\}$ and $Y = \{a, b, c, d, e\}$, then what is $Y - X$?



Exercise 1.2

1. Find union and intersection of the following.

   (i). $A = \{1,2,3,4\}$, $B = \{3,4,5\}$

   (ii). $A = \{-1,-2,-3\}$, $B = \{-2,-3,-4,-5\}$

   (iii). $A = \{1, 2, 3, ... 10\}$, $B = \{1, 3, 5, 7\}$

   (iv). $A = \{1,3,5,7,11,13\}$, $B = \{5,6,7,8,9,10,11\}$

   (v). A = Set of first 10 natural numbers
   B = set of first 5 positive even numbers

2. If $A=\{0,1,2,3,4,5\}$
   $B=\{1,3,5\}$ and
   $C=\{3,4,5\}$, then find

   (i). $(A\cup B)\cup C$ (ii). $(A\cap B)\cap C$

   (iii). $A\cap(B\cup C)$ (iv). $A\cup(B\cap C)$

   (v). $(A\cup B)\cap(A\cup C)$

3. If $A=\{1,2,3,4,5,6,7,8,9,10\}$ and $B=\{2,4,6,8,10\}$, then find $A\backslash B$ and $B\backslash A$

4. If $C=\{a, b, c..., x, y, z\}$ and $D=\{a, e, i, o, u\}$, then find $C\backslash D$ and $D\backslash C$

Did you know?

$$\emptyset = \{\ \}$$

1.2.1 Disjoint and overlapping sets

(a) Disjoint sets

Two sets A and B are disjoint sets, if they do not have any common element i.e. $A \cap B = \varphi$

Example 9

If $A = \{3,4,5\}$ and $B = \{6,7,8\}$,

Solution then show that A and B are disjoint sets.

$$A \cap B = \{3,4,5\} \cap \{6,7,8\}$$

$$A \cap B = \{\ \} = \varphi$$

This shows that A and B are disjoint sets.

(b) Overlapping sets

Two sets A and B are called overlapping sets if none of them is a subset of the other and there is at least one element which is common to both the sets.

For example,

If $A = \{1,2,3,4,5\}$ and $B = \{4,6,8,10\}$

then set A and set B are overlapping sets because 4 is a common element of both the sets A and B, also $A \not\subseteq B$ and $B \not\subseteq A$.

Guided Practice

Find whether the sets P and Q are overlapping sets or disjoint sets:

i. $P = \{10, 20, 30, 40\}$; $Q = \{15, 25, 35, 45\}$

ii. $P = \{2, 4, 6, 8, 10\}$; $Q = \{1, 3, 5, 7, 8\}$

iii. $P = \{S, U, N\}$; $Q = \{S, T, A, R\}$

1.2.2 Universal set and complement of a set

(a) Universal set

A set which consists of all the elements under consideration for a particular problem is called universal set and is denoted by U. For example, if the set of positive integers is under consideration, then a set of all integers will be universal set i.e

$U = \{0, \pm1, \pm2, \pm3, \ldots\}$

$W = \{0, 1, 2, 3, \ldots\}$

(b) Complement of a set

If U is a universal set and A is any subset of U, then the complement of set A is the set of all elements of U which are not in set A, and it is denoted by A' or A

In set builder form,

$A' = U \setminus A = \{x : x \in U \text{ and } x \notin A\}$

Remember

Union of a set and its complement is the universal set i.e $A \cup A' = U$

Intersection of a set and its complement is the null set i.e $A \cap A' = \phi$

10 If $U = \{1,2,3,4,5,6\}$

$A = \{1,2,3\}$

$B = \{3,4,5,6\}$

Find A' and B'.

Solution $A' = U \setminus A = \{1,2,3,4,5,6\} \setminus \{1,2,3\} = \{4,5,6\}$

and $B' = U \setminus B = \{1,2,3,4,5,6\} \setminus \{3,4,5,6\} = \{1,2\}$

1.2.3 Different properties involving union, int[illegible] difference and complement of sets

[illegible] 11

If $U=\{1,2,3,4,5,6,7,8,9,10,12\}$, $A=\{2,4,6,8,10,12\}$ [illegible]

then verify the following the properties

(i) $A\cup A'=U$ (ii) $A\cap A'=\phi$ (iii) [illegible]

(iv). $\phi'=U\backslash\phi=U$ (v) $(A')'=A$

Solution $A'=U\backslash A=\{1,2,3,\ldots,10,12\}\backslash\{2,4,6,8,10,12\}=$ [illegible]

(i) $A\cup A'=\{2,4,6,8,10,12\}\cup\{1,3,5,7,9\}=\{1,2,3,5,6,$ [illegible]

Hence $A\cup A'=U$

(ii) $A\cap A'=\{2,4,6,8,10,12\}\cap\{1,3,5,7,9\}=\phi$

Hence $A\cap A'=\phi$

(iii) $U'=U\backslash U=\{1,2,3,\ldots,10,12\}\backslash\{1,2,3,\ldots,10,12\}=\phi$

Hence $U'=\phi$

(iv) $\phi'=U\backslash\phi$

$=\{1,2,3,5,6,7,8,9,10,12\}\backslash\phi=\{1,2,3,5,6,7,8,9,10,12\}=U$

Hence $\phi'=U$

(v) $(A')'=U\backslash A'$

$=\{1,2,3,5,6,7,8,9,10,12\}\backslash\{1,3,5,7,9\}=\{2,4,6,8,10,12\}=A$

Hence $(A')'=A$

Exercise 1.3

1. If $A=\{1,2,3,4,6,12\}$

   $B=\{3,6,9,12,18\}$

   then, find $A \backslash B$ and $B \backslash A$ and check whether $A \backslash B = B \backslash A$?

2. If $U=\{1,2,3,...20\}$ then, find complements of the following sets.

   (i) $A=\{2,4,6,...,20\}$ (ii) $B=\{1,3,5,...,19\}$

   (iii) $C=\{3,6,9,12,15,18\}$ (iv) $D=\{4,8,16,20\}$

3. If $U=\{4,8,12,16,20\}$ then find U' and φ'

4. If $U=\{a, b, c, d, e, f\}$

   $A=\{a, b, c\}$

   then find $A \cup A'$ and $A \cap A'$ and check whether $A \cup A' = U$ and $A \cap A' = \varphi$?

5. If $U=\{1,2,3,...,10\}$

   $A=\{1,3,5,7,9\}$

   $B=\{2,4,6,8,10\}$

   then find.

   (i) $(A \cup B)'$ (ii) $(A \cap B)'$ (iii) $A' \cap B'$

   (iv) $A' \cup B'$ (v) $A \cap B'$ (vi) $B \cap A'$

6. Find whether the sets P and Q are overlapping sets or disjoint sets:

   (i) A = {Natural numbers between 35 and 60} and

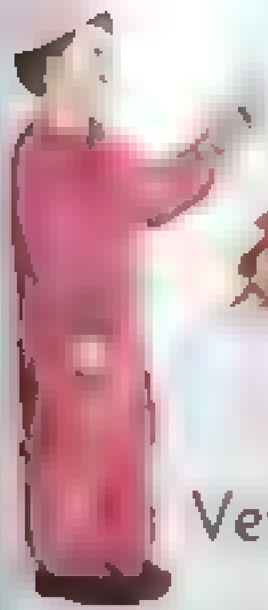
   B = {Natural numbers between 50 and 80}

   (ii) A = {Letters in the word 'MOON'} and

   B = {Letters in the word 'STAR'}

1.3 Venn diagrams

Now we shall illustrate the concept of sets with the help of Venn diagrams Union, intersection, difference and complements of sets can be explained eas y with the help of Venn diagrams In these diagrams a set is usually represented by a circle and the universal set is represented by a rectangle



1.3.1 Venn diagrams of union, intersection, difference and complements of two sets

Example 12

If $U = \{1,2,3,4,5,6\}$ and A and B are any subsets of U, the draw Venn diagrams for $A \cup B$, $A \cap B$, $A \setminus B$ and $B \setminus A$, when

(i) A is a subset of B (ii) B is a subset of A

(iii) A and B are disjoint sets. (iv) A and B are overlapping sets

Also find A' and B'.

Solution

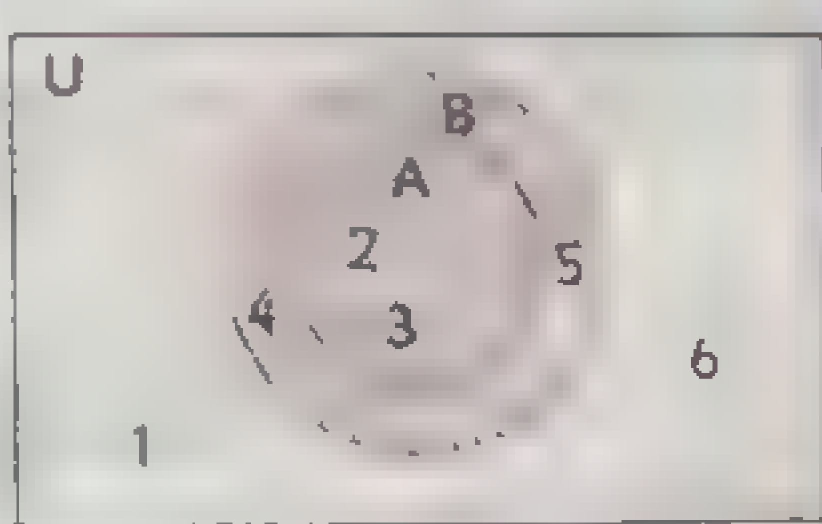
(i). When A is a subset of B.

Let $A = \{2,3\}$

$B = \{2,3,4,5\}$, then

$A \cup B = \{2,3\} \cup \{2,3,4,5\}$

$= \{2,3,4,5\}$



$A \cup B$
(Shaded area represents $A \cup B$)

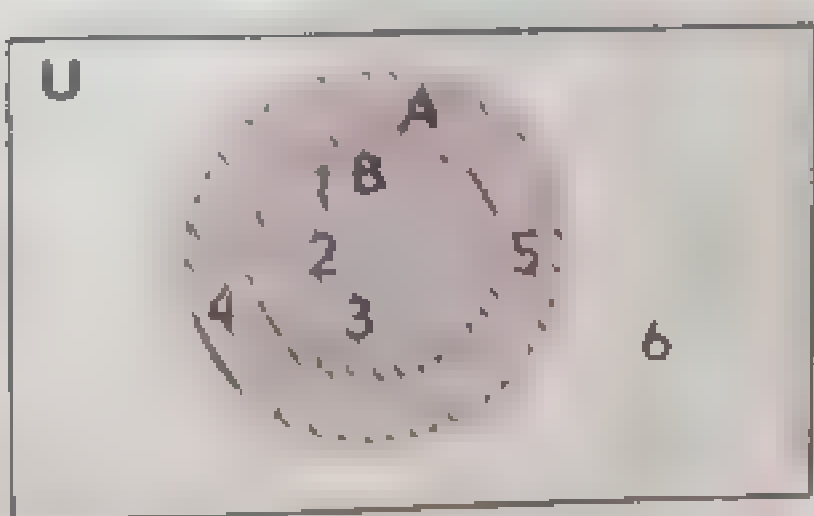
(ii). When B is a subset of A.

Let $A = \{1,2,3,4,5\}$

$B = \{1,2,3\}$, then

$A \cup B = \{1,2,3,4,5\} \cup \{1,2,3\}$

$= \{1,2,3,4,5\}$



(Shaded area represents $A \cup B$)

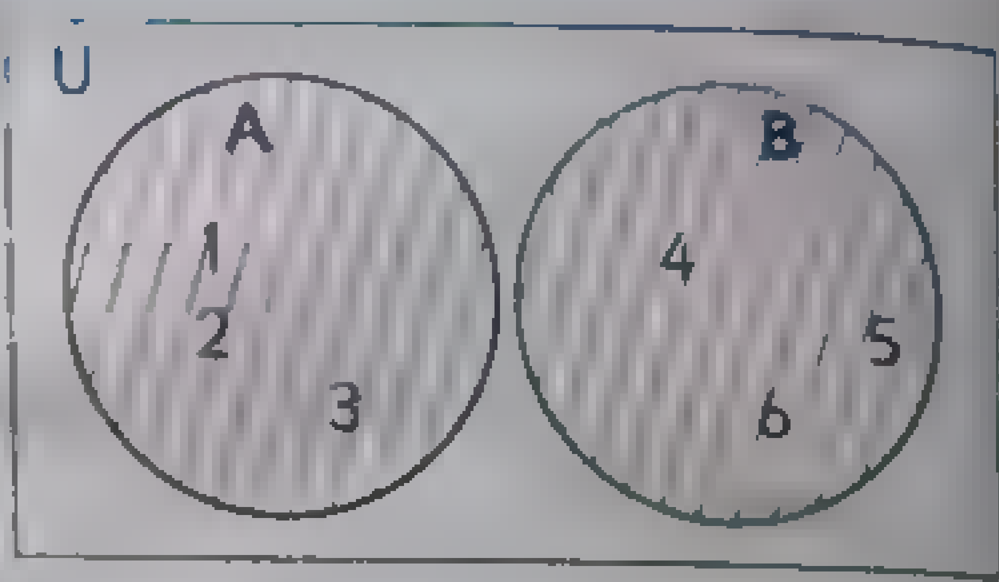
(ii) When A and B are disjoint sets.

Let $A = \{1,2,3\}$

$B = \{4,5,6\}$, then

$A \cup B = \{1,2,3\} \cup \{4,5,6\}$

$= \{1,2,3,4,5,6\}$



Shaded area represents $A \cup B$

When A and B are overlapping sets.

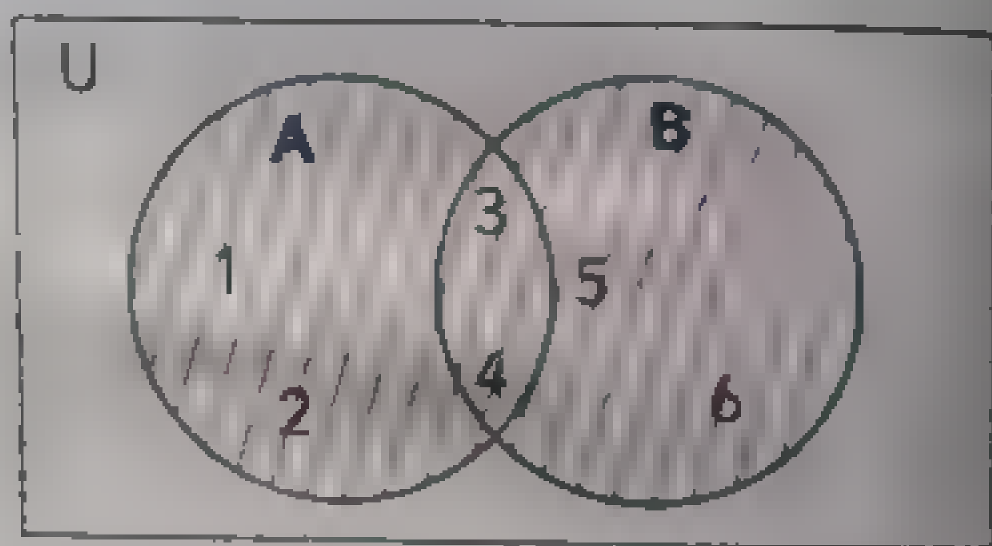
Let $A = \{1,2,3,4\}$

$B = \{3,4,5,6\}$

then,

$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6\}$

$= \{1,2,3,4,5,6\}$



Shaded area represents $A \cup B$

Venn diagrams of intersection

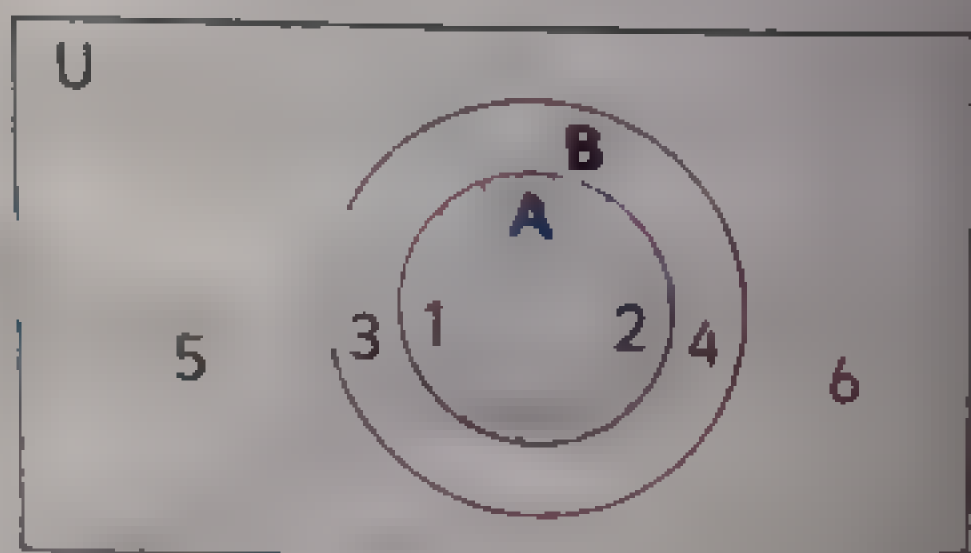
(i). When A is a subset of B.

Let $A = \{1,2\}$

$B = \{1,2,3,4\}$, then

$A \cap B = \{1,2\} \cap \{1,2,3,4\}$

$= \{1,2\}$



Shaded area represents $A \cap B$

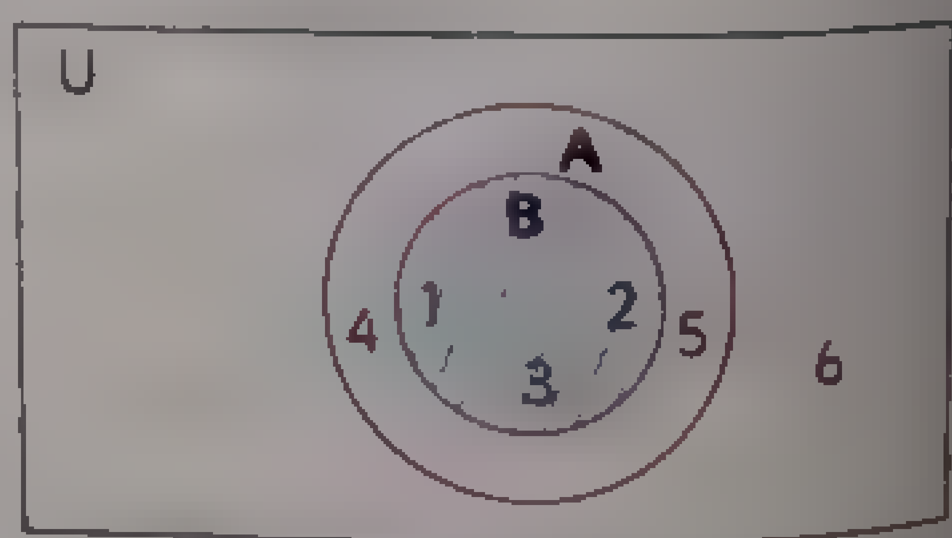
(ii) When B is a subset of A.

Let $A = \{1,2,3,4,5\}$

$B = \{1,2,3\}$, then

$A \cap B = \{1,2,3,4,5\} \cap \{1,2,3\}$

$= \{1,2,3\}$



Shaded area represents $A \cap B$

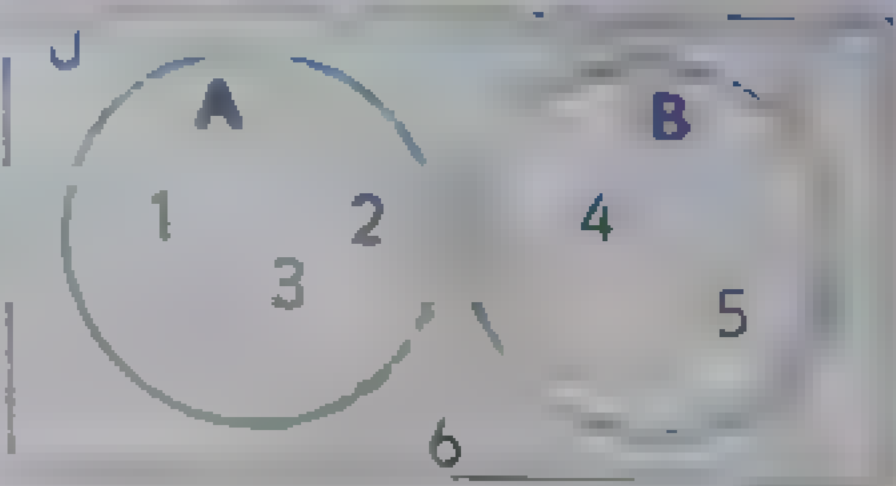
When A and B are disjoint sets

Let $A = \{1,2,3\}$

$B = \{4,5\}$, then

$A \cap B = \{1,2,3\} \cap \{4,5\}$

$= \{\ \}$



Shaded are a $A \cap B$

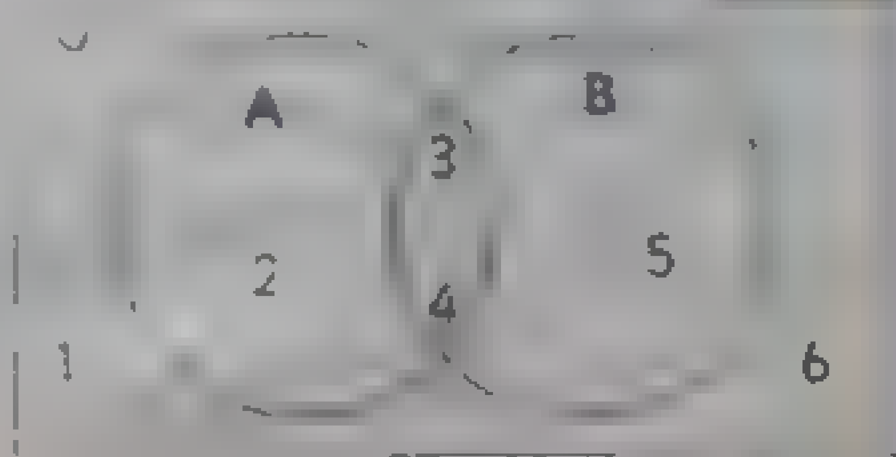
When A and B are overlapping sets

Let $A = \{2,3,4\}$

$B = \{3,4,5\}$, then

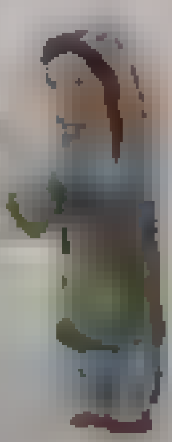
$A \cap B = \{2,3,4\} \cap \{3,4,5\}$

$= \{3,4\}$



Shaded area $A \cap B$

diagrams of A\B and B\A



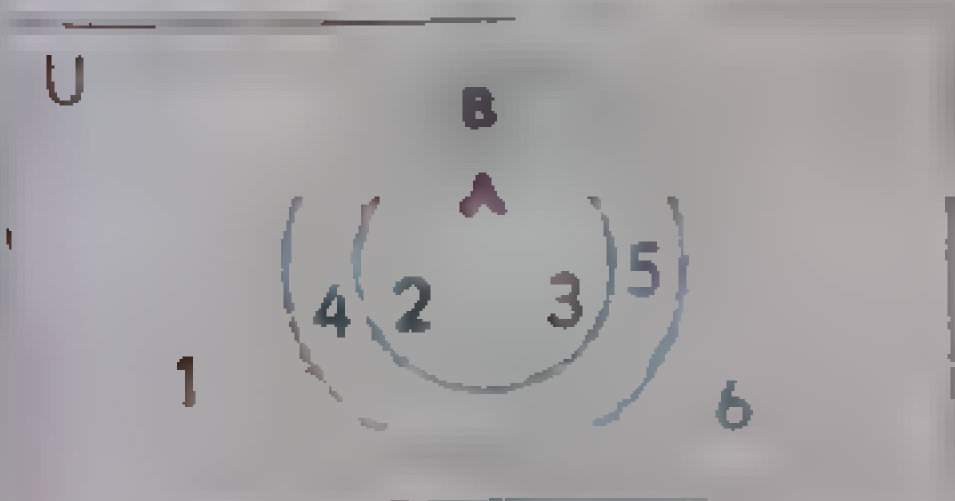
When A is a subset of B

Let $A = \{2,3\}$

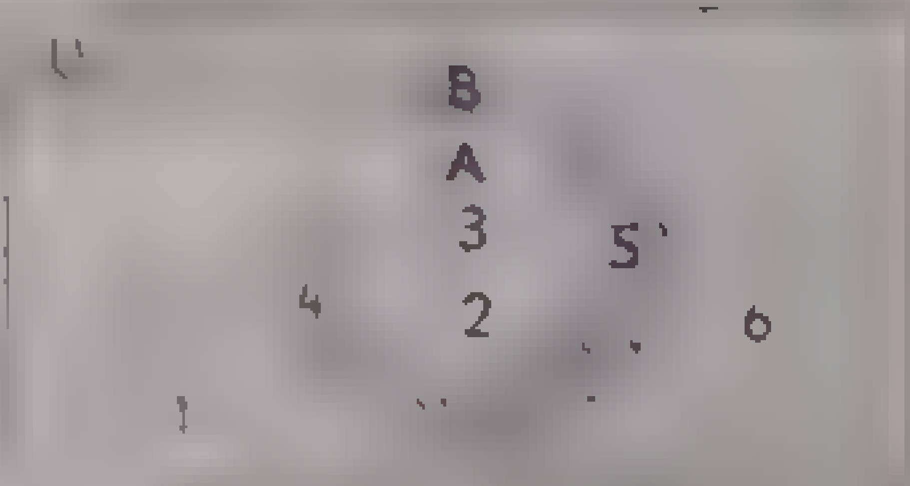
$B = \{2,3,4,5\}$, then

$A \setminus B = \{2,3\} \setminus \{2,3,4,5\} = \varphi$

$B \setminus A = \{2,3,4,5\} \setminus \{2,3\} = \{4,5\}$



Shaded area A\B



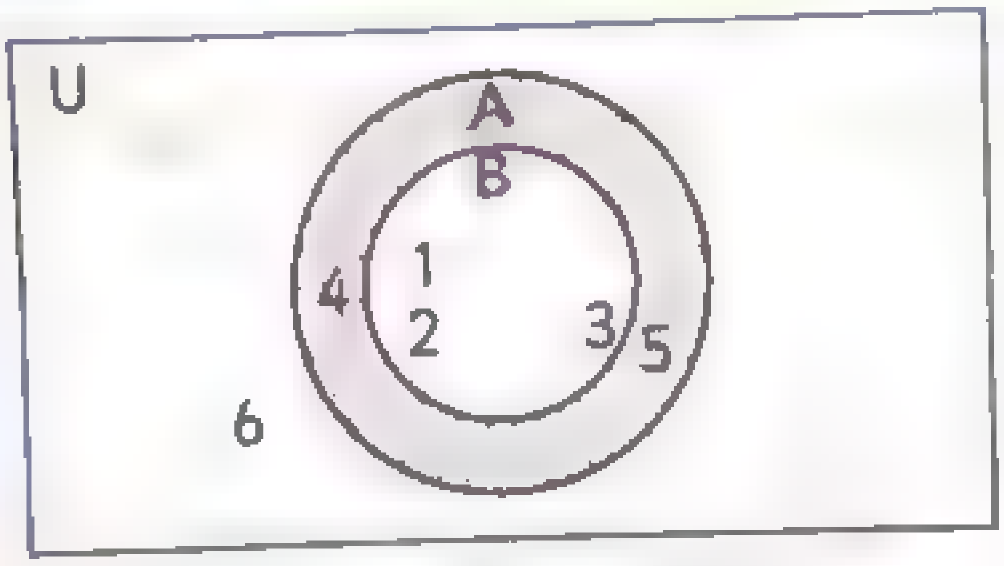
Shaded area B\A

(ii). When B is a subset of A.

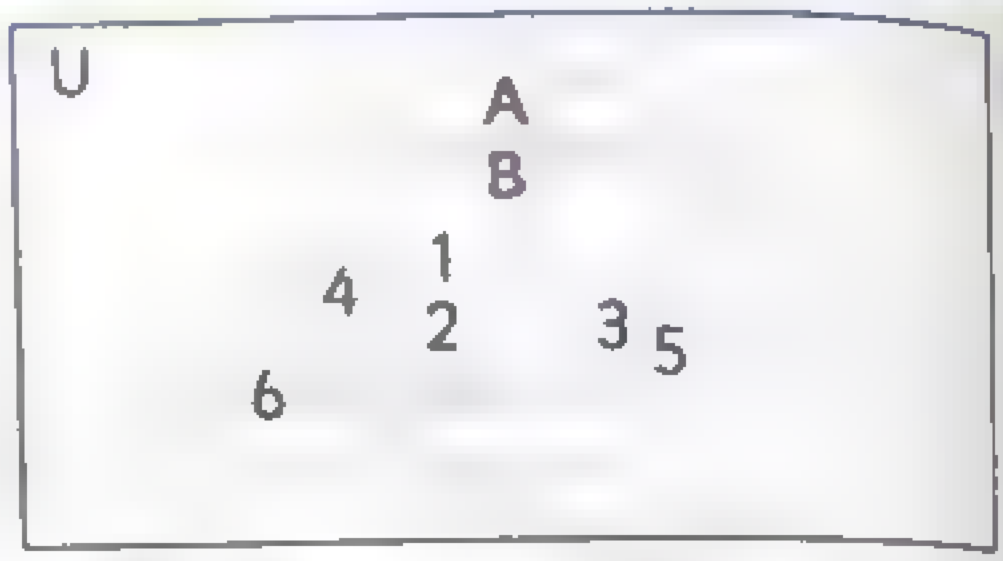
Let $A = \{1,2,3,4,5\}$, $B = \{1,2,3\}$, then

$A \setminus B = \{1,2,3,4,5\} \setminus \{1,2,3\} = \{4,5\}$

$B \setminus A = \{1,2,3\} \setminus \{1,2,3,4,5\} = \{\ \}$



Shaded area A\B

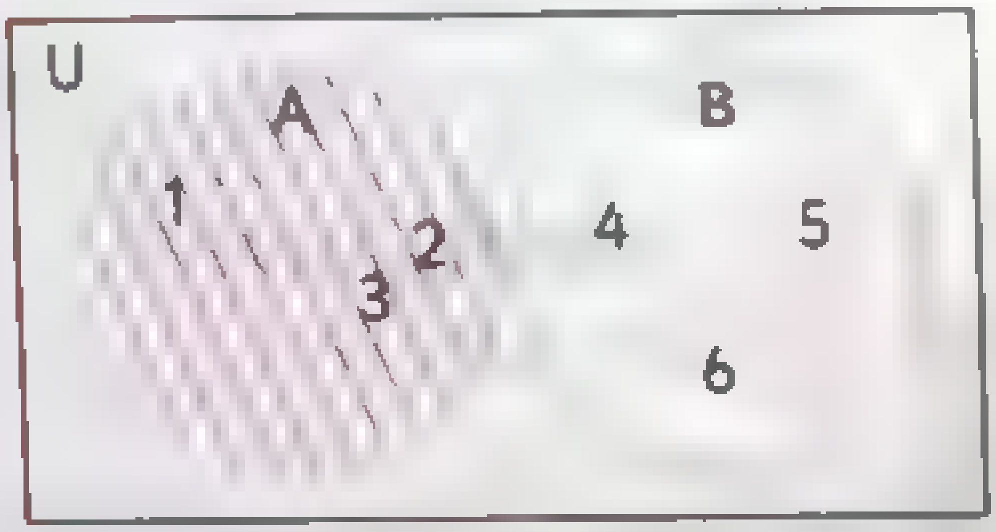


Shaded area B\A

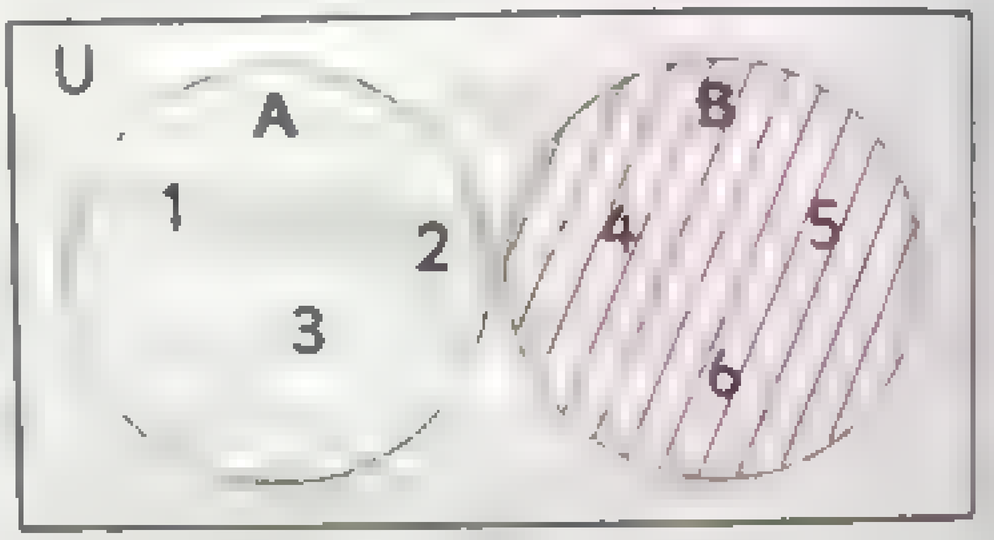
(iii) When A and B are disjoint sets.

Let A = {1,2,3}, B = {4,5,6}, then

A\B = {1,2,3}\{4,5,6} = {1,2,3}, B\A = {4,5,6}\{1,2,3} = {4,5,6}



Shaded area A\B

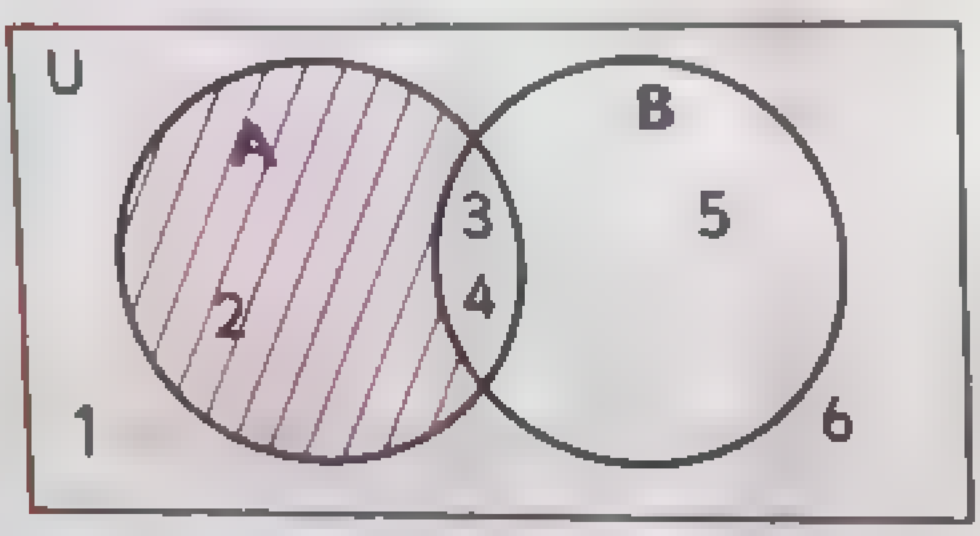


Shaded area B\A

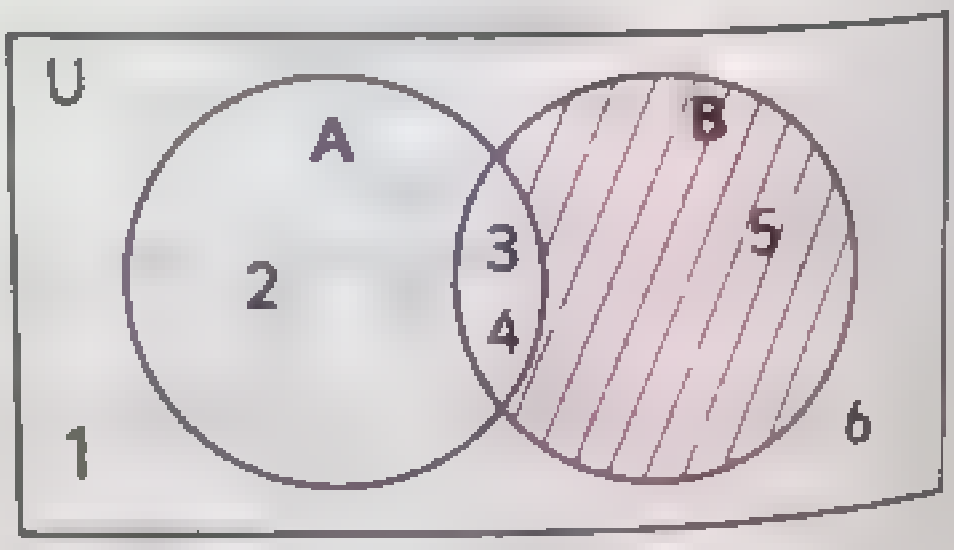
(iv) When A and B are overlapping sets.

Let A = {2,3,4}, B = {3,4,5}, then A\B = {2,3,4}\{3,4,5} = {2}

B\A = {3,4,5}\{2,3,4} = {5}



Shaded area A\B



Shaded area B\A

Venn diagrams of A′ and B′

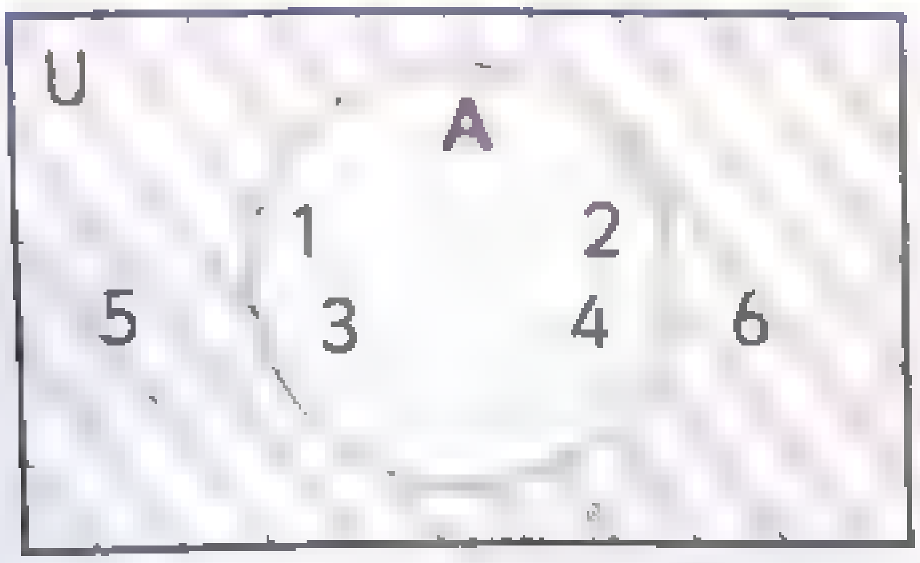
Let A = {1,2,3,4}and B = {3,4,5}

Then

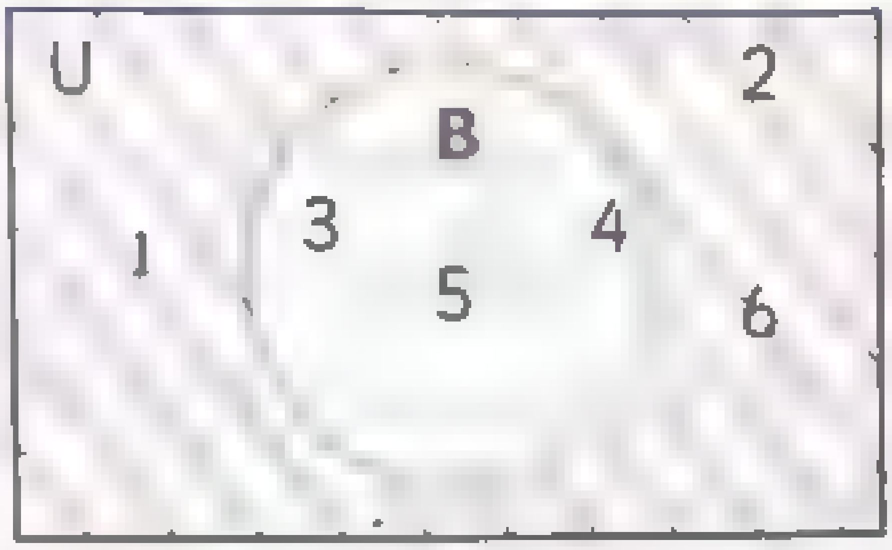
A′ = U\A = {1,2,3,4,5,6}\{1,2,3,4} = {5,6}

and

B′ = U\B = {1,2,3,4,5,6}\{3,4,5} = {1,2,6}



Shaded area A′={5,6}



Shaded area B′={1,2,6}

Exercise 1.4

1 If U={1,2,3,4,.....,10}, then find A∪B and A∩B in each of the following cases and draw their Venn diagrams

(i). A = {2,3,4}, B = {1,2,3,4,5}

(ii) A = {2,4,6,8,10}, B = {6,8}

(iii) A = {1,3,5,7,9}, B = {2,3,5,7}

(iv). A = {1,5,6}, B = {2,4,7}

2. If U = {a,b,c,d,e,f}, then find A′, B′, A\B and B\A in the following questions, also draw their Venn diagrams.

(i) A = {a,b}, B = {a,b,c} (ii) A = {d,e,f}, B = {e,f}

(iii). A = {b,c,d}, B = {c,e} (iv) A = {a,b}, B = {c,d,e}

Review Exercise 1

1. Read the following statements carefully and write 'T' in front of true statement and 'F' in front of false statement.

(i) A collection of well-defined and distinct objects is called a set. ☐

(ii) In descriptive form members of a set are written within braces. ☐

(iii) If A∩B = φ then set A and set B are called overlapping sets. ☐

(iv) Difference of two sets A–B consists of all those elements of set A which are not in B. ☐

(v) U–A is called complement of A. ☐

2. Fill in the blanks.

(i) $A \cup A' =$ ________

(ii) φ' $=$ ________

(iii) $A \cap A' =$ ________

(iv) $(A')'$ $=$ ________

(v) If A and B are disjoint sets then $A \cap B =$ ________

3. Colour the correct answer.

(i) $A \cup B =$

(a) $\{x|x \in A \text{ and } x \in B\}$ (b) $\{x|x \in A \text{ or } x \in B\}$

(c) $\{x|x \in A \text{ and } x \notin B\}$ (d) $\{x|x \notin A \text{ and } x \in B\}$

(ii) $A - B =$

(a) $\{x|x \in A \text{ and } x \in B\}$ (b) $\{x|x \in A \text{ or } x \in B\}$

(c) $\{x|x \in A \text{ and } x \notin B\}$ (d) $\{x|x \notin A \text{ and } x \in B\}$

(iii) $B - A =$

(a) $\{x|x \in A \text{ and } x \in B\}$ (b) $\{x|x \in A \text{ or } x \in B\}$

(c) $\{x|x \in A \text{ and } x \notin B\}$ (d) $\{x|x \in B \text{ and } x \notin A\}$

(iv) $(A')' =$

(a) U (b) φ (c) A (d) A'

(v) $A \cup A' =$

(a) U (b) φ (c) A (d) A'

(vi) $A \cap A' =$

(a) U (b) φ (c) A (d) A'

(vii) $U' =$

(a) U (b) φ (c) U–A (d) U-B

(viii) $\varphi' =$

(a) U (b) φ (c) U' (d) U-A

(ix) If $A \cap B = \varphi$ then A and B are

(a) Overlapping sets (b) Disjoint sets

(c) Equal sets (d) None of these

(x) If $A = \{1,2,3\}, B = \{3,4,5\}$ then $A \cap B =$

(a) $\{2,3\}$ (b) $\{1,3\}$ (c) $\{3\}$ (d) $\{1,2,3,4,5\}$

4. If $A = \{1,2,4,6\}$, $B = \{1,2,3,\ldots,10\}$, then find

(i) $A \cup B$ (ii) $A \cap B$

5. If $A = \{0,1,2,3,4\}$, $B = \{2,4,6\}$, then find

(i) $A \backslash B$ (ii) $B \backslash A$

6. If $U = \{a,b,c,d,e,f\}$, $A = \{a,c,e\}$ and $B = \{b,d,f\}$, then find

(i) A' (ii) B' (iii) $(A \cup B)'$

(iv) $(A \cap B)'$ (v) $A' \cup B'$ (vi) $A' \cap B'$

7. If $U = \{1,2,3,4,5,6\}$, $A = \{2,3,4\}$ then show that $A \cup A' = U$ and $A \cap A' = \varphi$.

8. If A is the set of factors of 15,
B is the set of prime numbers less than 10,
C is the set of even numbers less than 9,
then what is $(A \cup B) \cup C$?

9. If $U = \{a,b,c,d,e,f\}$, $A = \{a,b,c\}$ and $B = \{c,d,e\}$ then draw Venn diagrams of

(i) A' (ii) B' (iii) $A \cup B$

(iv) $A \cap B$ (v) $A \backslash B$ (vi) $B \backslash A$

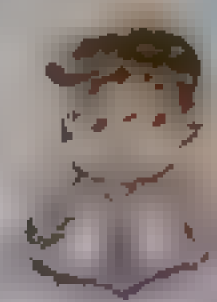
10. Find whether the sets P and Q are overlapping sets or disjoint sets:

(i) $P = \{x : x \text{ is a factor of } 24\}$;
$Q = \{x : x \text{ is a factor of } 33\}$;

(ii) $P = \{x : x \text{ is a multiple of } 7 \text{ between } 1 \text{ and } 50\}$;
$Q = \{x : x \text{ is a multiple of } 11 \text{ between } 1 \text{ and } 50\}$;

Glossary

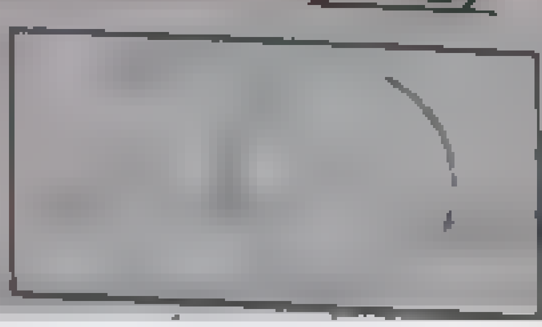
- **Set** "A Collection of well-defined distinct objects" is called a set.
- **Descriptive form** In this form instead of writing all the members of the set, a sentence is written which makes each and every member of the set clear.
- **Set builder form** In this form a general element is denoted by a variable 'x' and a characteristic property is mentioned which is common to all the elements of the set
- **Tabular form** In this form we list all the members of a set.
- **Union of two sets** If A and B are any two sets then their union consists of all the elements in set A or in set B, and it is denoted by $A \cup B$.
- **Intersection of two sets** If A and B are any two sets then their intersection consists of all those elements which are common to both A and B, and it is denoted by $A \cap B$.
- **Difference of two sets** If A and B are any two sets, then their difference consists of all those elements of set A which are not in set B, and it is denoted by $A \setminus B$ and vice versa.
- **Disjoint sets** Two sets A and B are said to be disjoint sets, if their intersection is a null set i.e. $A \cap B = \varphi$.
- **Overlapping set** Two sets A and B are said to be overlapping sets if none, of them is the subset of the other and there is at-least one element which is common to both set A and set B.
- **Universal set** A set which consists of all the elements under consideration in a particular problem is called the universal set, and it is denoted by capital U.
- **Complement of a set** The complement of a set A, denoted by A' is the set of all elements of the universal set U that are not elements of A. i.e. $A' = U - A$.
- **Venn diagrams** In these diagrams the sets under consideration are usually represented by a circle and the universal set is represented by a rectangle.



Shade the Venn diagrams

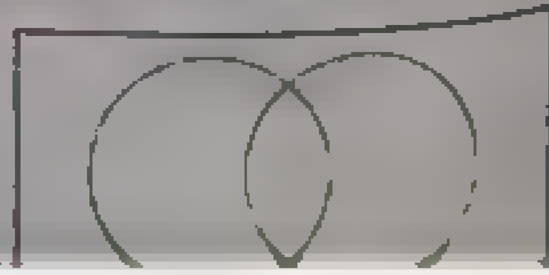
i. Shade $A' - B'$

ii. Shade $(A \cap B)'$



iii. Shade $(A - B)'$

iv. Shade $A' \cup B'$



Unit 2 Rational Numbers

What You'll Learn

- Definition of rational numbers.
- Representation of rational numbers on a number line.
- Addition of two or more rational numbers.
- Subtraction of rational numbers from another.
- Additive inverse of a rational number
- Multiplication of two or more rational numbers.
- Division of a rational number by a non-zero rational number.
- Multiplicative inverse of a rational number.
- Reciprocal of a rational number.
- Verification of commutative property of rational numbers with respect to addition and multiplication.
- Verification of associative property of rational numbers with respect to addition and multiplication.
- Verification of distributive property of rational numbers with respect to multiplication over addition/subtraction.
- Comparison of two rational numbers.
- Arrangement of rational numbers in ascending or descending order.

Why It's Important

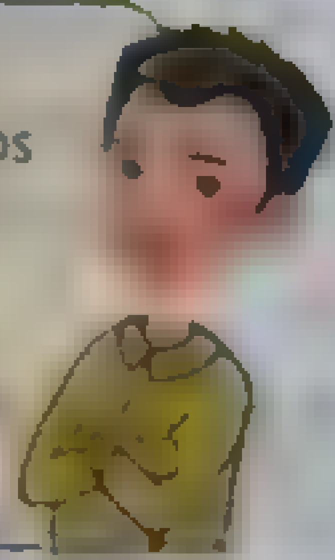
Take a thermometer that can measure room temperature and start your AC or heater in a room. Record the readings in a chart after every minute in your room. You will see definitely some readings that will not be integers but will lie in between the integers. What are these numbers?

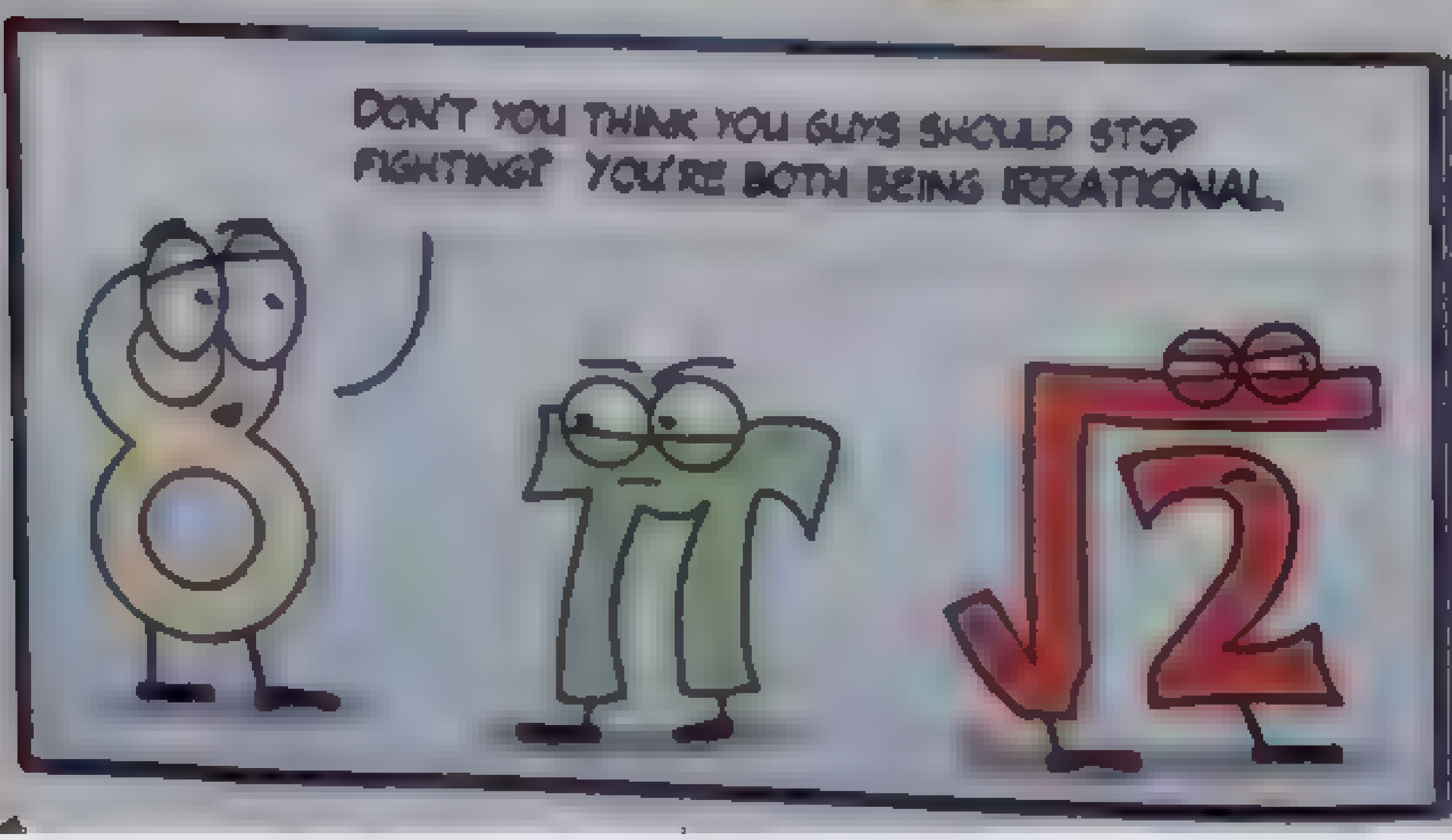
In our daily life we frequently use some quantities which are not whole numbers. For example, a half liter of milk, a quarter of an hour etc etc. Therefore, there is a need of some other numbers like rational numbers.

Did you know?

Beautiful Number Relationships

$$135 = 1^1 + 3^2 + 5^3$$
$$175 = 1^1 + 7^2 + 5^3$$
$$518 = 5^1 + 1^2 + 8^3$$
$$598 = 5^1 + 9^2 + 8^3$$





2.1 Rational Numbers

2.2.1 Definition Rational Numbers

The set of all numbers of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called the set of rational numbers.

For example, $9, 0, \frac{-3}{7}, \frac{-6}{4}, 2\frac{1}{5}$ are rational numbers

How are rational numbers related to other sets of numbers?

The set of natural numbers

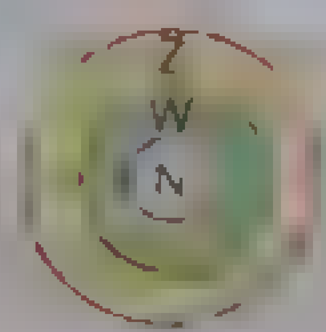
$N = \{1, 2, 3, ...\}$. ⟶

The set of whole numbers

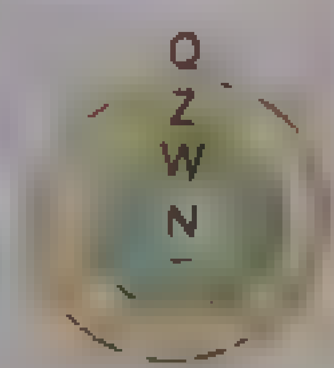
$W = \{0, 1, 2, 3, ...\}$ ⟶

The set of integers

$Z = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$ ⟶

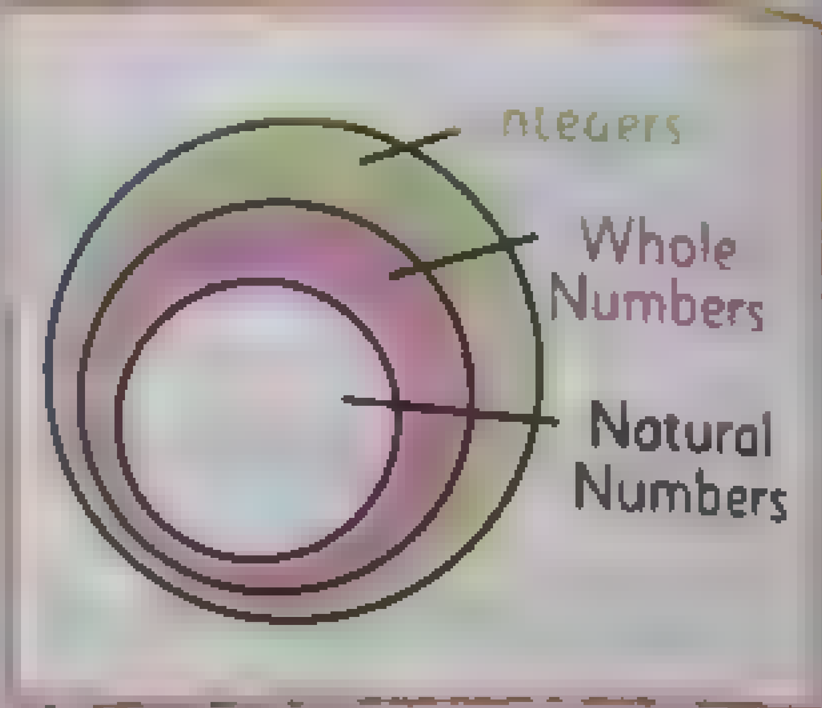


The set of rational numbers Q ⟶



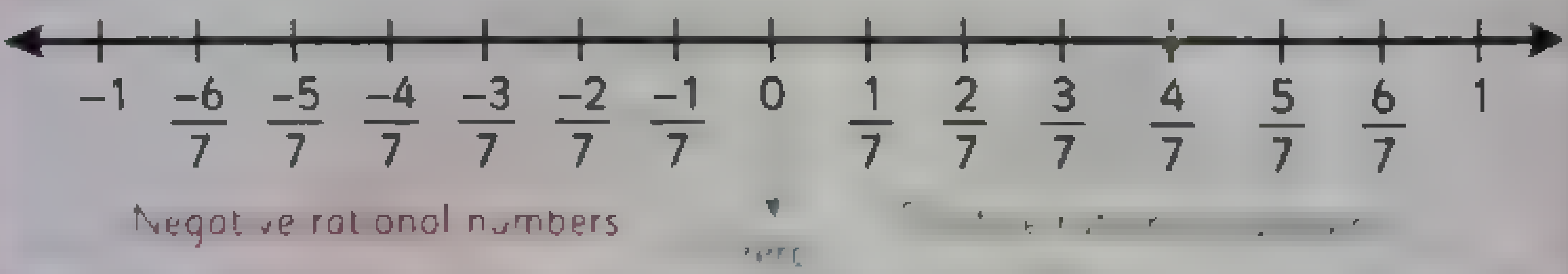
Rational numbers include fractions and decimals as well as natural numbers, whole numbers, and integers

Natural Numbers	{1, 2, 3,....
Whole Numbers	{0, 1, 2, 3,...}
Integers	{..., –2, –1, 0, 1, 2,...}
Rational Numbers	numbers expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.



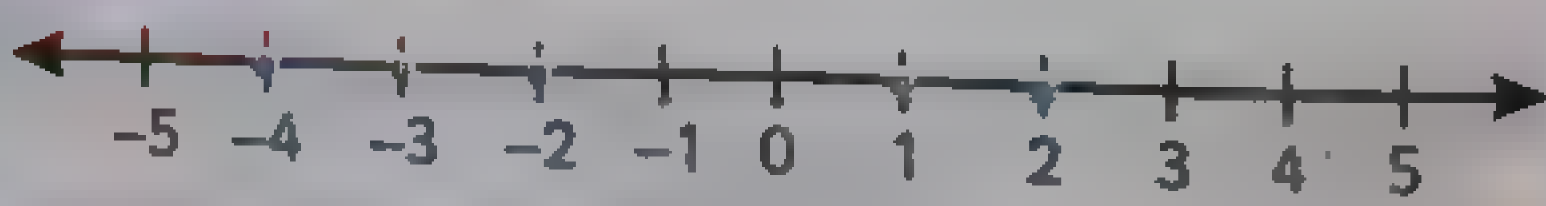
2.1.2 Representation of rational numbers on a number line

Likewise integers we can also represent rational numbers on number line. The number line consists of negative numbers on its left, zero in the middle, and positive numbers on its right. The following figure shows some rational numbers on the number line.



To graph a set of numbers means to draw, or plot, the points named by those numbers on a number line. The number that corresponds to a point on a number line is called the coordinate of that point.

1 Name the coordinates of the points graphed on the number line.



The dots indicate each point on the graph.

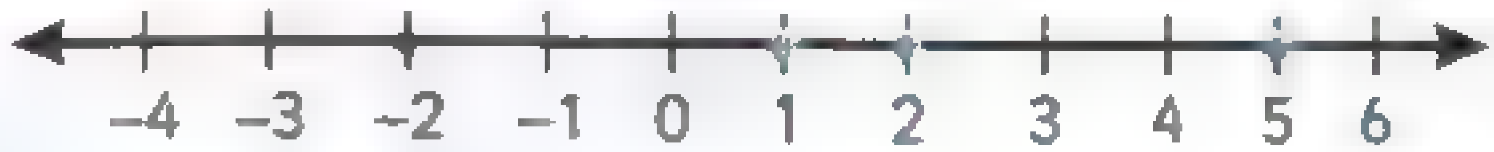
The coordinates are {–4, 3, –2, 1, 2}.

The bold arrows mean that the graph continues indefinitely in that direction.

Guided Practice

Name the coordinates of the points graphed on each number line.

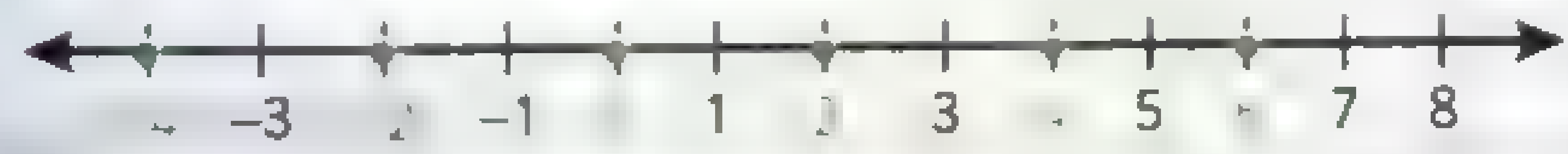
i



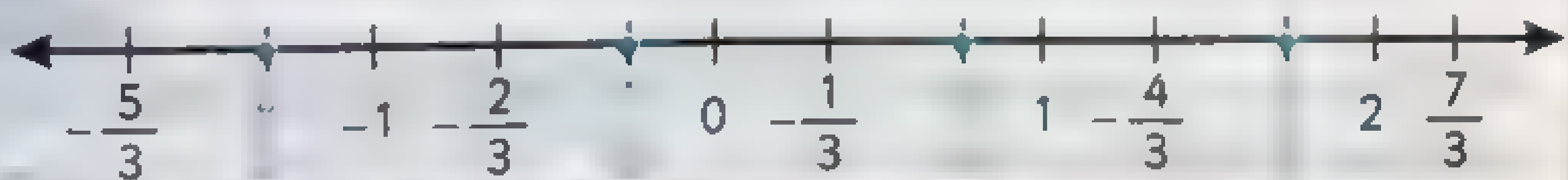
ii

Example 2 Graph each set of numbers.

(i) {..., –4, –2, 0, 2, 4, 6}



(ii) $\left\{-\frac{4}{3}, -\frac{1}{3}, \frac{2}{3}, \frac{5}{3}\right\}$



(iii) {integers less than –3 or greater than or equal to 5}

Guided Practice

Graph each set of numbers.

i. {–4, –2, 1, 5, 7}

ii {–2.8, –1.5, 0.2, 3.4,}

iii $\left\{-\frac{1}{2}, 0, \frac{1}{4}, \frac{2}{5}, \frac{5}{3}\right\}$

iv {integers less than or equal to –4}

Exercise 2.1

1. Read the following statements carefully and write 'T' in front of true statement and 'F' in front of false statement.

(i) Any integer can be expressed in the form of $\frac{p}{q}$ where $q \neq 0$. ☐

(ii) Zero is not a rational number ☐

(iii) All integers are rational numbers ☐

(iv) Rational numbers may be positive or negative. ☐

(v) In any rational number $\frac{p}{q}$, q may be zero ☐

2. Show the following rational numbers on a number line.

(i) $-2\frac{1}{2}, -\frac{3}{2}, -5\frac{1}{2}, \frac{5}{2}, 3\frac{1}{2}$

(ii) $-\frac{7}{2}, -\frac{5}{2}, 2, \frac{3}{2}, 2\frac{3}{4}$

2.2 Operations on Rational Numbers

We have learnt the addition, subtractions and multiplication of two or more integers and fractions in the previous classes. On similar pattern we proceed for rational numbers.

2.2.1 Addition of two or more than two rational numbers

Example 3 Find each sum.

(i) $-11 + (-7)$

$$-11 + (-7) = -(|-11| + |-7|)$$
$$= -(11 + 7)$$
$$= -18$$

$$\frac{7}{16} + \left(-\frac{3}{8}\right)$$

$$\frac{7}{16} + \left(-\frac{3}{8}\right) = \frac{7}{16} + \left(-\frac{6}{16}\right)$$

$$= + \left(\left|\frac{7}{16}\right| - \left|-\frac{6}{16}\right| \right)$$

$$= + \left(\frac{7}{16} - \frac{6}{16}\right)$$

absolute value is [illegible], the sum is positive

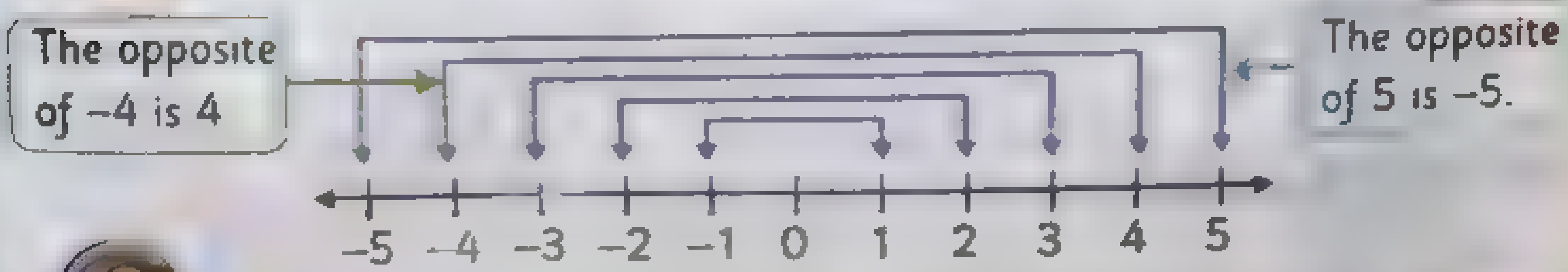
$$= \frac{1}{16}$$

Guided Practice

Add. $\frac{3}{10} + \frac{7}{10}$ $\frac{1}{12} + \left(-\frac{7}{12}\right)$ $2\frac{5}{12} + \left(2 - \frac{7}{12}\right)$

2.2.2 Subtraction of two rational numbers

Every positive rational number can be paired with a negative rational number. These pairs are called opposites.



A number and its opposite are additive inverses of each other. When we add two opposites, the sum is always 0.

2.2.3 Additive Inverse Property

Key Concept — Additive Inverse Property

Words	The sum of a number and its additive inverse is 0
Symbols	For every number a, $a + (-a) = 0$

Guided Practice

What is the additive inverse of [illegible] -7 [illegible] $\frac{2}{7}$ [illegible] $\frac{-5}{8}$ iv. $2\frac{1}{2}$

Example 4

Evaluate $a - b$ if $a = 9\frac{1}{6}$ and $b = 5\frac{2}{6}$.

$a - b = 9\frac{1}{6} - 5\frac{2}{6}$ [illegible] $9\frac{1}{6}$ [illegible] $5\frac{2}{6}$.

$= \frac{55}{6} - \frac{32}{6}$ [illegible]

$= \frac{23}{6}$ Subtract the numerators.

$= 3\frac{5}{6}$ Simplified

Guided Practice

Solve.

i. $\frac{10}{11} - \frac{8}{11}$ ii. $9 - \left(-\frac{7}{20}\right)$ iii. $2\frac{3}{8} - 1\frac{5}{8}$

Exercise 2.2

1. Solve.

(i) $\frac{1}{2}+\frac{5}{8}$ (ii) $3\frac{1}{4}+2$ (iii) $\frac{3}{7}+\left(-\frac{5}{2}\right)$ (iv) $\frac{5}{6}-\left(-\frac{3}{8}\right)$

(v) Find the additive inverse of $\frac{1}{7}$.

(v.) Find the additive inverse of -10.

(vi.) Find the sum of $4\frac{1}{8}$ and $1\frac{1}{2}$

(viii) Evaluate $x+y$ if $x=2$ and $y=8\frac{4}{9}$.

(ix) Nadeem was $62\frac{1}{8}$ inches tall at the end of school in June. He was $63\frac{7}{8}$ inches tall in September. How much did he grow during the summer?

2. Najam and Nazia are subtracting fractions.

[illegible]	[illegible]
$\left(-\frac{4}{9}\right)-\left(-\frac{2}{3}\right)$	$\left(-\frac{4}{9}\right)-\left(-\frac{2}{3}\right)$
$=\left(-\frac{4}{9}\right)-\left(-\frac{6}{9}\right)$	$=\left(-\frac{4}{9}\right)-\left(-\frac{6}{9}\right)$
$=\left(-\frac{4}{9}\right)+\left(-\frac{6}{9}\right)$	$=\left(-\frac{4}{9}\right)+\left(-\frac{6}{9}\right)$
$=\left(\frac{6}{9}-\frac{4}{9}\right)$	$=\left(-\frac{6}{9}+\frac{4}{9}\right)$
$=\frac{2}{9}$	$=\frac{10}{9}$

2.2.4 Multiplication of two or more than two rational numbers

Key Concept — Multiplication of rational numbers

To multiply rational numbers, multiply the numerators and multiply the denominators

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}, \text{ where } b, d \neq 0$$

5 Find $\frac{4}{7} \times \frac{1}{6}$. Write the product in simplest form.

$$\frac{4}{7} \times \frac{1}{6} = \frac{\overset{2}{\cancel{4}}}{7} \times \frac{1}{\underset{3}{\cancel{6}}}$$ [illegible] HCF, 2

$$= \frac{2 \times 1}{7 \times 3}$$ [illegible]

$$= \frac{2}{21}$$ Simplified

6 Find $1\frac{2}{5} \times 2\frac{1}{2}$. Write the product in simplest form.

$$1\frac{2}{5} \times 2\frac{1}{2} = \frac{7}{5} \times \frac{5}{2}$$ Rename $1\frac{2}{5}$ as $\frac{7}{5}$ and rename $2\frac{1}{2}$ as $\frac{5}{2}$

$$= \frac{7}{\cancel{5}} \times \frac{\cancel{5}}{2}$$ Cancel 5 with 5

$$= \frac{7 \times 1}{1 \times 2}$$ Multiply

$$= \frac{7}{2} \text{ or } 3\frac{1}{2}$$ [illegible]

Guided Practice

Multiply.

i. $\left(\frac{5}{3}\right)\left(-\frac{2}{7}\right)$

ii. $\left(\frac{4}{9}\right)\left(\frac{7}{15}\right)$

iii. $\left(-3\frac{1}{5}\right)\left(-7\frac{1}{2}\right)$

2.2.5 Division of a rational number by a non-zero rational number

Since multiplication and division are inverse operations, the rule for finding the sign of the quotient of two integers is similar to the rule for finding the sign of a product of two integers.

2.2.6 Reciprocal of a rational number

For any rational number $\frac{p}{q}$, $p, q \neq 0$, $\frac{q}{p}$ is its reciprocal,

e.g. reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

Remember

(i) Reciprocal of a rational number is its multiplicative inverse. The product of rational number and its reciprocal is "1".

(ii) Rational number of the type $\frac{0}{p}$ has no reciprocal, because $\frac{p}{0}$ is undefined (do not exist).

Example 7 Find the multiplicative inverse of each number.

(i). $-\frac{3}{8}$

$-\frac{3}{8}\left(-\frac{8}{3}\right) = 1$

The product is 1.

The multiplicative inverse or reciprocal of $-\frac{3}{8}$ is $-\frac{8}{3}$.

(ii). $2\frac{1}{5}$

$2\frac{1}{5} = \frac{11}{5}$ Write as an improper fraction

$\frac{11}{5} \times \frac{5}{11} = 1$ The product is 1

The multiplicative inverse or reciprocal of $2\frac{1}{5}$ is $\frac{5}{11}$

Key Concept **Division of rational numbers**

Words.	To divide by a fraction, multiply by its multiplicative inverse.
Symbols	$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$, where $b, c, d \neq 0$

Example 8 Find $\frac{1}{3} \div \frac{5}{9}$. Write the quotient in simplest form.

$$\frac{1}{3} \div \frac{5}{9} = \frac{1}{3} \times \frac{9}{5}$$

$$= \frac{1}{\cancel{3}} \times \frac{\cancel{9}^{3}}{5}$$

$$= \frac{3}{5}$$

Guided Practice

Find each quotient. Write in simplest form.

$\frac{1}{6} \div \frac{3}{4}$ $\quad -\frac{5}{8} \div \frac{1}{3}$ $\quad \frac{2}{5} \div 1\frac{1}{2}$

Exercise 2.3

1. Find the additive and multiplicative inverses of the following.

(i) -5 (ii) $-2\frac{1}{11}$ (iii) $\frac{4}{15}$

(iv) $\frac{105}{200}$ (v) $\frac{6}{7}$

2. Solve the following.

$\frac{2}{3} \times \frac{7}{8}$ $\quad \left(\frac{-2}{3}\right) \times (-7)$ $\quad 2\frac{5}{8} \times 3\frac{4}{5}$

$(-3) \times \left(\frac{3}{17}\right)$ $\quad \frac{-2}{3} \div \left(\frac{-2}{3}\right)$ $\quad \left(\frac{-1}{2}\right) \div \frac{3}{16}$

$(-5) \div \left(\frac{10}{9}\right)$ $\quad 1\frac{3}{5} \div \frac{1}{10}$ $\quad \frac{30}{7} \times \frac{14}{6}$

2.2.7 Verification of commutative property of rational numbers

Key Concept — Commutative Property

The order in which you add or multiply rational numbers does not change their sum or product.

For any two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$,

- $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$ It is called the commutative property of rational addition.
- $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$ It is called the commutative property of rational numbers w.r.t. multiplication.

9 Verify $\frac{3}{5} + \frac{4}{7} = \frac{4}{7} + \frac{3}{5}$.

Solution

$$\text{L.H.S} = \frac{3}{5} + \frac{4}{7} = \frac{3 \times 7 + 4 \times 5}{35} = \frac{21 + 20}{35} = \frac{41}{35}$$

$$\text{R.H.S} = \frac{4}{7} + \frac{3}{5} = \frac{4 \times 5 + 3 \times 7}{35} = \frac{21 + 20}{35} = \frac{41}{35}$$

As L.H.S = R.H.S

Hence, $\frac{3}{5} + \frac{4}{7} \quad \frac{4}{7} + \frac{3}{5}$.

Rational numbers satisfy commutative property w r t addition

Guided Practice

Verify. $\frac{4}{5} + \frac{7}{10} = \frac{7}{10} + \frac{4}{5}$

10 Verify $\frac{2}{3} \times \frac{-4}{5} = \frac{-4}{5} \times \frac{2}{3}$

Solution

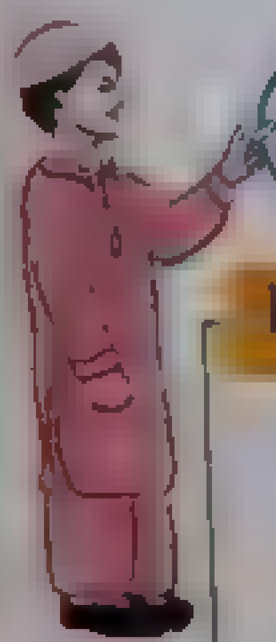
$$\text{L.H.S} = \frac{2}{3} \times \frac{-4}{5}$$
$$= -\frac{2 \times 4}{3 \times 5}$$
$$= \frac{-8}{15}$$

Rational numbers satisfy commutative property w.r.t multiplication.

$$\text{R.H.S} = \frac{-4}{5} \times \frac{2}{3}$$
$$= \frac{-4 \times 2}{5 \times 3}$$
$$= \frac{-8}{15}$$

As L.H.S = R.H.S

Hence, $\frac{2}{3} \times \frac{-4}{5} = \frac{-4}{5} \times \frac{2}{3}$



2.2.8 Verification of associative property of rational numbers

Key Concept — **Associative Property**

The way you group three or more rational numbers when adding or multiplying does not change their sum or product.

For any two rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$.

$\frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f}$ It is called the associative property of rational numbers w.r.t. addition.

$\frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) = \left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f}$. It is called the associative property of rational numbers w.r.t. multiplication.

Example 11 Verify $\frac{1}{2}+\left(\frac{3}{4}+\frac{4}{5}\right)=\left(\frac{1}{2}+\frac{3}{4}\right)+\frac{4}{5}$

Solution

$$\frac{1}{2}+\left(\frac{3}{4}+\frac{4}{5}\right)=\left(\frac{1}{2}+\frac{3}{4}\right)+\frac{4}{5}$$

$$\frac{1}{2}+\left(\frac{3\times 5+4\times 4}{20}\right)=\left(\frac{1\times 4+2\times 3}{8}\right)+\frac{4}{5}$$

$$\frac{1}{2}+\left(\frac{15+16}{20}\right)=\left(\frac{4+6}{8}\right)+\frac{4}{5}$$

$$\frac{1}{2}+\frac{31}{20}=\frac{10}{8}+\frac{4}{5}$$

$$\frac{1\times 10+31\times 1}{20}=\frac{10\times 5+4\times 8}{40}$$

$$\left(\frac{10+31}{20}\right)=\left(\frac{50+32}{40}\right)$$

$$\frac{41}{20}=\frac{82}{40}$$

$$\frac{41}{20}=\frac{41}{20}$$

Rational numbers satisfy associative property w.r.t addition

As L.H.S = R.H.S

Hence, $\frac{1}{2}+\left(\frac{3}{4}+\frac{4}{5}\right)=\left(\frac{1}{2}+\frac{3}{4}\right)+\frac{4}{5}$

Example 12 Verify $\frac{1}{3}\times\left(\frac{4}{5}\times\frac{2}{7}\right)=\left(\frac{1}{3}\times\frac{4}{5}\right)\times\frac{2}{7}$

Solution

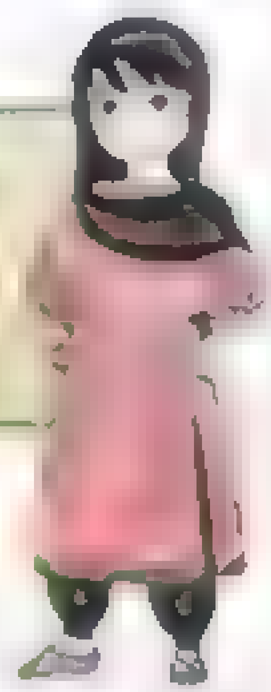
$$\frac{1}{3}\times\left(\frac{4}{5}\times\frac{2}{7}\right)=\left(\frac{1}{3}\times\frac{4}{5}\right)\times\frac{2}{7}$$

$$\frac{1}{3}\times\frac{4\times2}{5\times7}=\frac{1\times4}{3\times5}\times\frac{2}{7}$$

$$\frac{1}{3}\times\frac{8}{35}=\frac{4}{15}\times\frac{2}{7}$$

$$\frac{1\times8}{3\times35}=\frac{4\times2}{15\times7}$$

$$\frac{8}{105}=\frac{8}{105}$$ As L H S = R H S



Hence, $\frac{1}{3}\times\left(\frac{4}{5}\times\frac{2}{7}\right)=\left(\frac{1}{3}\times\frac{4}{5}\right)\times\frac{2}{7}$

2.2.9 Verification of distributive property of rational numbers

Key Concept — **Distributive Property**

For any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ the following hold

$$\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\frac{a}{b}\times\frac{e}{f}$$

This is called the distributive property of rational numbers of multiplication over addition.

$$\frac{a}{b}\times\left(\frac{c}{d}-\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}-\frac{a}{b}\times\frac{e}{f}$$

This is the distributive property of rational numbers of multiplication over subtraction.

13 Verify $\frac{1}{5} \times \left(\frac{2}{3} + \frac{5}{6}\right) = \frac{1}{5} \times \frac{2}{3} + \frac{1}{5} \times \frac{5}{6}$.

Solution

$$\frac{1}{5} \times \left(\frac{2}{3} + \frac{5}{6}\right) = \frac{1}{5} \times \frac{2}{3} + \frac{1}{5} \times \frac{5}{6}.$$

$$\frac{1}{5} \times \frac{2 \times 2 + 5 \times 1}{6} = \frac{1 \times 2}{5 \times 3} + \frac{1 \times 5}{5 \times 6}$$

$$\frac{1}{5} \times \frac{4 + 5}{6} = \frac{2}{15} + \frac{1}{6}$$

$$\frac{1}{5} \times \frac{9}{6} = \frac{2}{15} + \frac{1}{6}$$

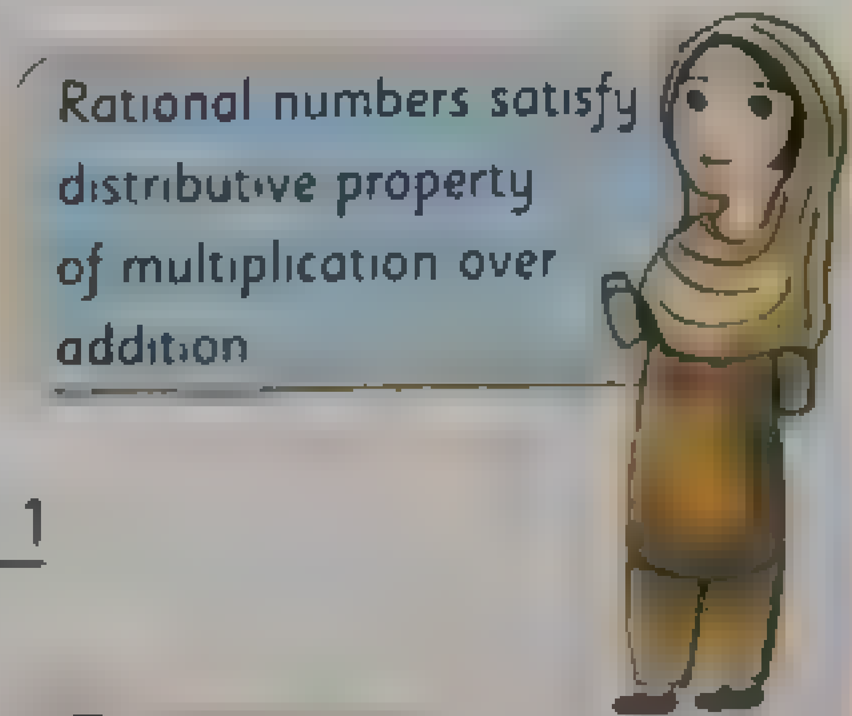
$$\frac{1}{5} \times \frac{3}{2} = \frac{2}{15} + \frac{1}{6}$$

$$\frac{1 \times 3}{10} = \frac{2 \times 2 + 5 \times 1}{30}$$

$$\frac{3}{10} = \frac{4 + 5}{30}$$

$$\frac{3}{10} = \frac{9}{30}$$

$$\frac{3}{10} = \frac{3}{10}$$



As L.H.S = R.H.S

Hence, $\frac{1}{5} \times \left(\frac{2}{3} + \frac{5}{6}\right) = \frac{1}{5} \times \frac{2}{3} + \frac{1}{5} \times \frac{5}{6}$.

Guided Practice

Verify. $\frac{2}{3} \times \left(\frac{5}{9} + \frac{4}{17}\right) = \frac{2}{3} \times \frac{5}{9} + \frac{2}{3} \times \frac{4}{11}$.

Inam and Zohra are checking the following problem.

$$\frac{1}{2}\times\left(\frac{7}{8}-\frac{3}{4}\right)=\frac{1}{2}\times\frac{7}{8}-\frac{1}{2}\times\frac{3}{4}$$

$$\frac{1}{2}\times\left(\frac{7}{8}-\frac{3}{4}\right)=\frac{1}{2}\times\frac{7}{8}-\frac{1}{2}\times\frac{3}{4}$$

$$\frac{1}{2}\times\left(\frac{7}{8}-\frac{3}{4}\right)=\frac{1}{2}\times\frac{7}{8}-\frac{1}{2}\times\frac{3}{4}$$

$$\frac{1}{2}\times\frac{7\times1-2\times3}{8}=\frac{1}{2}\times\frac{7}{8}-\frac{1}{2}\times\frac{3}{4}$$

$$\frac{1}{2}\times\frac{7-6}{8}=\frac{7}{16}-\frac{3}{8}$$

$$\frac{1}{2}\times\frac{1}{8}=\frac{7\times1-3\times2}{16}$$

$$\frac{1\times1}{2\times8}=\frac{7-6}{16}$$

$$\frac{1}{16}=\frac{1}{16}$$

As L.H.S = R.H.S

Hence, $\frac{1}{2}\times\left(\frac{7}{8}-\frac{3}{4}\right)=\frac{1}{2}\times\frac{7}{8}-\frac{1}{2}\times\frac{3}{4}$

$$\frac{1}{2}\times\left(\frac{7}{8}-\frac{3}{4}\right)=\frac{1}{2}\times\frac{7}{8}-\frac{1}{2}\times\frac{3}{4}$$

$$\frac{1}{2}\times\frac{7\times2-3\times1}{4}=\frac{1\times7}{2\times8}-\frac{1\times3}{2\times4}$$

$$\frac{1}{2}\times\frac{14-3}{4}=\frac{7}{16}-\frac{3}{8}$$

$$\frac{1}{2}\times\frac{11}{4}=\frac{7\times1-3\times2}{8}$$

$$\frac{1\times11}{2\times4}=\frac{7-6}{8}$$

$$\frac{11}{8}=\frac{1}{8}$$

As L.H.S $\neq$ R.H.S

Hence,

$$\frac{1}{2}\times\left(\frac{7}{8}-\frac{3}{4}\right)\neq\frac{1}{2}\times\frac{7}{8}-\frac{1}{2}\times\frac{7}{8}$$

Who is correct?

ACTIVITY

Buy 12 bananas, give $\frac{1}{3}$ of it to your sister and then $\frac{1}{2}$ of the remaining to your youngest brother. How many bananas are left with you?

The Distributive Property can be used to simplify mental calculations.

Example 14 Use the Distributive Property to find each product.

(i). 15×99

15×99	$= 15(100 - 1)$	Think: $99 = 100 - 1$
	$= 15(100) - 15(1)$	Distributive Property
	$= 1500 - 15$	Multiply
	$= 1485$	Subtract

(ii). $35\left(2\frac{1}{5}\right)$

$$35\left(2\frac{1}{5}\right) = 35\left(2 + \frac{1}{5}\right)$$

$$= 35(2) + 35\left(\frac{1}{5}\right)$$

$$= 70 + 7 \quad \text{Multiply}$$

$$= 77 \quad \text{Add}$$

Exercise 2.4

1. Name the property used in each of the following and also verify them

(i) $\frac{1}{4} + 5 = 5 + \frac{1}{4}$

(ii) $\frac{-2}{5} + \left(\frac{3}{4} + \frac{1}{5}\right) = \left(\frac{-2}{5} + \frac{3}{4}\right) + \frac{1}{5}$

(iii) $\frac{3}{6} \times \frac{4}{7} = \frac{4}{7} \times \frac{3}{6}$

(iv) $\frac{2}{7} \times \left(\frac{4}{5} - \frac{3}{7}\right) = \frac{2}{7} \times \frac{4}{5} - \frac{2}{7} \times \frac{3}{7}$

(v) $\frac{1}{3} \times \left(2 \times \frac{3}{8}\right) = \left(\frac{1}{3} \times 2\right) \times \frac{3}{8}$

(vi) $\frac{3}{8} \times \left(\frac{1}{2} + \frac{3}{5}\right) = \frac{3}{8} \times \frac{1}{2} + \frac{3}{8} \times \frac{3}{5}$

2. Find each sum or difference. Write in simplest form.

(i) $\frac{x}{8} + \frac{4x}{8}$ (ii) $-2\frac{1}{6}y + 8\frac{5}{6}y$ (iii) $\frac{12}{m} - \frac{9}{m}, m \neq 0$

2.2.10 Comparison of two rational numbers

For comparing any two rational numbers, we use the symbols "=" (equal to) ">" (greater than) "<" (less than)

Rules for comparison of two rational numbers

To compare any two rational numbers the following rules must be followed

- Make the denominators of both the given rational numbers the same by taking L.C.M.
- Then compare according to numerators, the greater the numerator the greater will be the rational number and vice versa.

15 Compare $\frac{3}{5}$ and $\frac{5}{6}$

Solution Since L.C.M of 5 and 6 is 30. Therefore we can write

$$\frac{3}{5} \text{ and } \frac{5}{6} \text{ as } \frac{3 \times 6}{5 \times 6} \text{ and } \frac{5 \times 5}{6 \times 5}$$

$$\frac{18}{30} \text{ and } \frac{25}{30}$$

Since $18 < 25$

$$\frac{18}{30} < \frac{25}{30} \text{ or } \frac{3}{5} < \frac{5}{6}$$

16

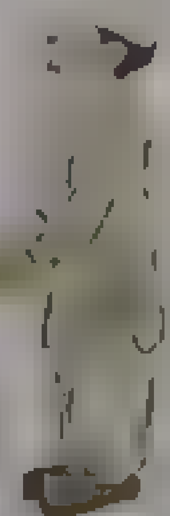
Which one is greater of the two rational numbers $\frac{-4}{9}$ and $\frac{-5}{6}$.

Solution L C M of 9 and 6 is 18 we can write

$$\frac{-4}{9} \text{ and } \frac{-5}{6} \text{ as } \frac{-4 \times 2}{9 \times 2} \text{ and } \frac{-5 \times 3}{6 \times 3}$$

$$\frac{-8}{18} \text{ and } \frac{-15}{18}$$

Since $-8 > -15$



Therefore $\frac{-8}{18} > \frac{-15}{18}$ or $\frac{-4}{9} > \frac{-5}{6}$

2.2.11 Arrangement of rational numbers in ascending or descending order

Rational numbers can be arranged in ascending or descending order by making the denominators the same as illustrated by the following examples

17

Arrange the following rational numbers in ascending order

$\frac{3}{5}, \frac{2}{15}, -3\frac{1}{2}, -2\frac{7}{10}, 7$

Solution $\frac{3}{5}, \frac{2}{15}, -3\frac{1}{2}, -2\frac{7}{10}, 7$

$= \frac{3}{5}, \frac{2}{15}, -3\frac{1}{2}, -2\frac{7}{10}, 7$

$= \frac{3}{5}, \frac{2}{15}, \frac{-7}{2}, \frac{-27}{10}, 7$

L C M of 5, 15, 2 and 10 is 30

Therefore,

$$\frac{3}{5}, \frac{2}{15}, \frac{-7}{2}, \frac{27}{10}, 7 = \frac{3 \times 6}{5 \times 6}, \frac{2 \times 2}{15 \times 2}, \frac{7 \times 15}{2 \times 15}, \frac{-27 \times 3}{10 \times 3}, \frac{7 \times 30}{1 \times 30}$$

$$= \frac{18}{30}, \frac{4}{30}, \frac{-105}{30}, \frac{-81}{30}, \frac{210}{30}$$

Writing the numerators in the ascending order

$$\frac{-105}{30}, \frac{-81}{30}, \frac{4}{30}, \frac{18}{30}, \frac{210}{30}$$

$\therefore \frac{-7}{2}, \frac{-27}{10}, \frac{2}{15}, \frac{3}{5}, 7$ is the required ascending order

18 Arrange the following rational numbers in descending order. $\frac{19}{30}, \frac{8}{15}, \frac{-7}{10}, \frac{2}{5}$

Solution $\frac{19}{30}, \frac{8}{15}, \frac{-7}{10}, \frac{2}{5}$

L.C.M of 30, 15, 10, 5 is 30

$$\frac{19}{30}, \frac{8}{15}, \frac{-7}{10}, \frac{2}{5} = \frac{19}{30}, \frac{8 \times 2}{15 \times 2}, \frac{-7 \times 3}{10 \times 3}, \frac{2 \times 6}{5 \times 6} \text{ or } \frac{19}{30}, \frac{16}{30}, \frac{-21}{30}, \frac{12}{30}$$

Arrange the numerators in descending order: $\frac{19}{30}, \frac{16}{30}, \frac{12}{30}, \frac{-21}{30}$

$\frac{19}{30}, \frac{8}{15}, \frac{2}{5}, \frac{-7}{10}$ is the required descending order.

Arrange the numerators in descending order: $\frac{19}{30}, \frac{8}{15}, \frac{2}{5}, \frac{-7}{10}$

2.5

1. Compare the following rational numbers by using <, > or =

   (i) $\frac{3}{7}, \frac{5}{21}$ (ii) $\frac{-2}{5}, \frac{3}{5}$ (iii) $\frac{7}{11}, \frac{-3}{5}$

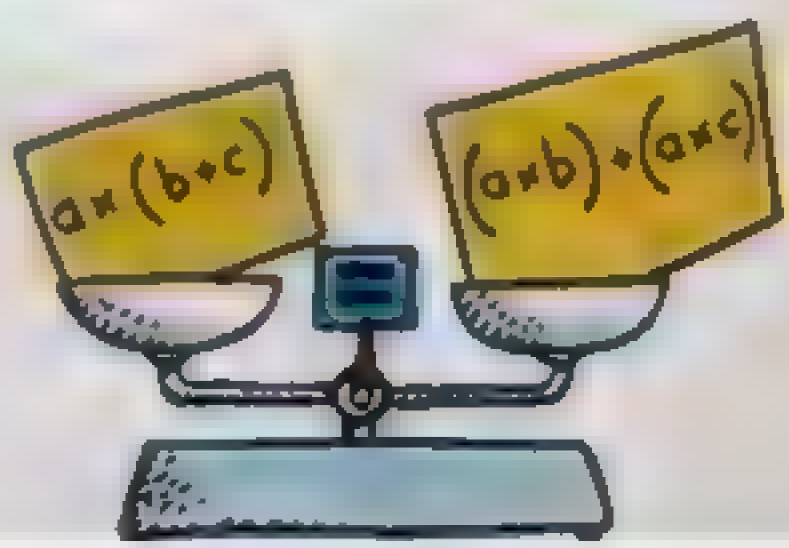
   (iv) $\frac{5}{6}, \frac{10}{12}$ (v) $\frac{6}{7}, \frac{8}{15}$

2. Arrange the following rational numbers in descending order

   $1\frac{1}{3}, \frac{3}{5}, -5\frac{7}{6}, 4\frac{2}{5}$

3. Arrange the following rational numbers in ascending order

   $3\frac{7}{8}, 3\frac{7}{25}, -5\frac{5}{3}, -5\frac{7}{12}$



REVIEW EXERCISE 2

1. Fill in the blanks

(i) The additive inverse of $\frac{-1}{2}$ is ________.

(ii) All integers are __________ numbers.

(iii) 0 has __________ reciprocal.

(iv) ________ is the reciprocal of itself.

2. Choose the correct answer.

(i) What is $\frac{3}{10}$ divided by $1\frac{4}{5}$?

a $\frac{1}{2}$ b $\frac{3}{8}$ c $\frac{1}{6}$ d $\frac{27}{50}$

(ii) The multiplicative inverse of $\frac{1}{4}$ is:

a 4 b -4 c $-\frac{1}{4}$ d 0

(iii) Find $\frac{13}{20}-\frac{7}{20}$. Write it in simplest form.

a $\frac{6}{10}$ b $\frac{3}{5}$ c $\frac{6}{20}$ d $\frac{3}{10}$

(iv) For any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ we have $\frac{a}{b}\left(\frac{c}{d}-\frac{e}{f}\right)=\frac{ac}{bd}-\frac{ae}{bf}$. This shows which property?

a Associative property w.r.t. multiplication

b Distributive property of multiplication over subtraction.

c Distributive property of addition over multiplication.

d Associative property w.r.t. addition.

3. Solve the following.

(i) $\frac{4}{5}+\frac{3}{7}$ (ii) $1\frac{3}{5}-\frac{6}{11}$ (iii) $4\frac{1}{8}\times\frac{6}{11}$ (iv) $\frac{-1}{2}\div\frac{3}{18}$

Arrange the following rational numbers in the ascending and descending order

$$\frac{-1}{5}, \frac{6}{7}, \frac{-3}{10}, \frac{4}{7}$$

Evaluate each expression if $x = \frac{8}{15}$, $y = 2\frac{1}{15}$ and $z = \frac{11}{15}$

$x + y$ (ii) $z + y$

$z - x$ (iv) $y - x$

Glossary

A number of the form $\frac{p}{q}$, $q \neq 0$ where p and q are integers is called rational number.

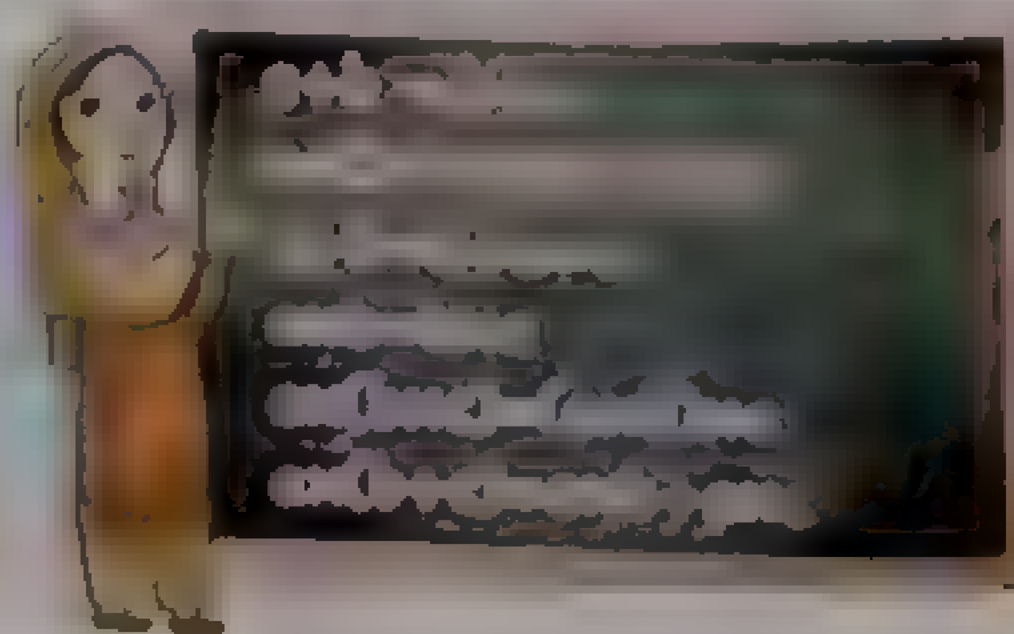
If a is any rational number such that $a + (-a) = (-a) + a = 0$ then $-a$ is called the additive inverse of a.

If a is any rational number such that $a \times \frac{1}{a} = \frac{1}{a} \times a = 1$, then $\frac{1}{a}$ is called the multiplicative inverse of a

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$ is called the commutative property of rational numbers w.r.t. addition.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$ is called the commutative property of rational numbers w.r.t. multiplication.

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$ is called the distributive property of rational numbers.



Unit 3 DECIMALS

You'll Learn

- Convert decimals to rational numbers
- Define terminating decimals as decimals having a finite number of digits after the decimal point
- Define recurring decimals as non-terminating decimals in which a single digit or a block of digits repeats itself infinite number of times after the decimal point

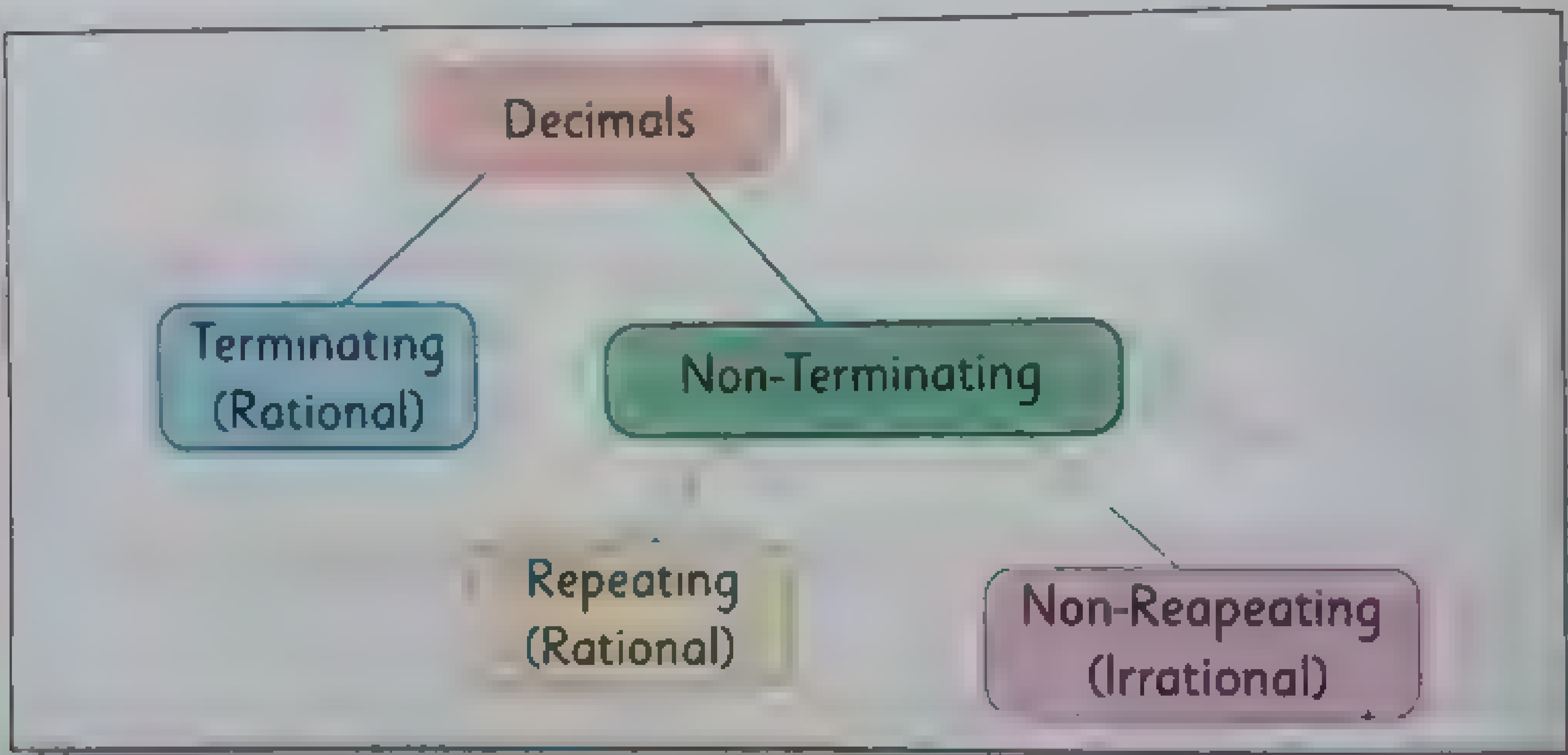
  $\left(\text{e.g. } \frac{2}{7} = 0.285714285714285714\right)$
- Use the following rule to find whether a given rational number is terminating or not.
  - Rule: If the denominator of a rational number in the standard form has no prime factor other than 2, 5 or 2 and 5, then and only then the rational number is a terminating decimal.
- Express a given rational number as a decimal and indicate whether it is terminating or recurring.
- Get an approximate value of a number, called rounding off, to a desired number of decimal places.

Why It's Important

Decimals are important because people use them every day in different situations, such as counting money, looking at price tags, reading an odometer, comparing run-rate in cricket and reviewing scores.

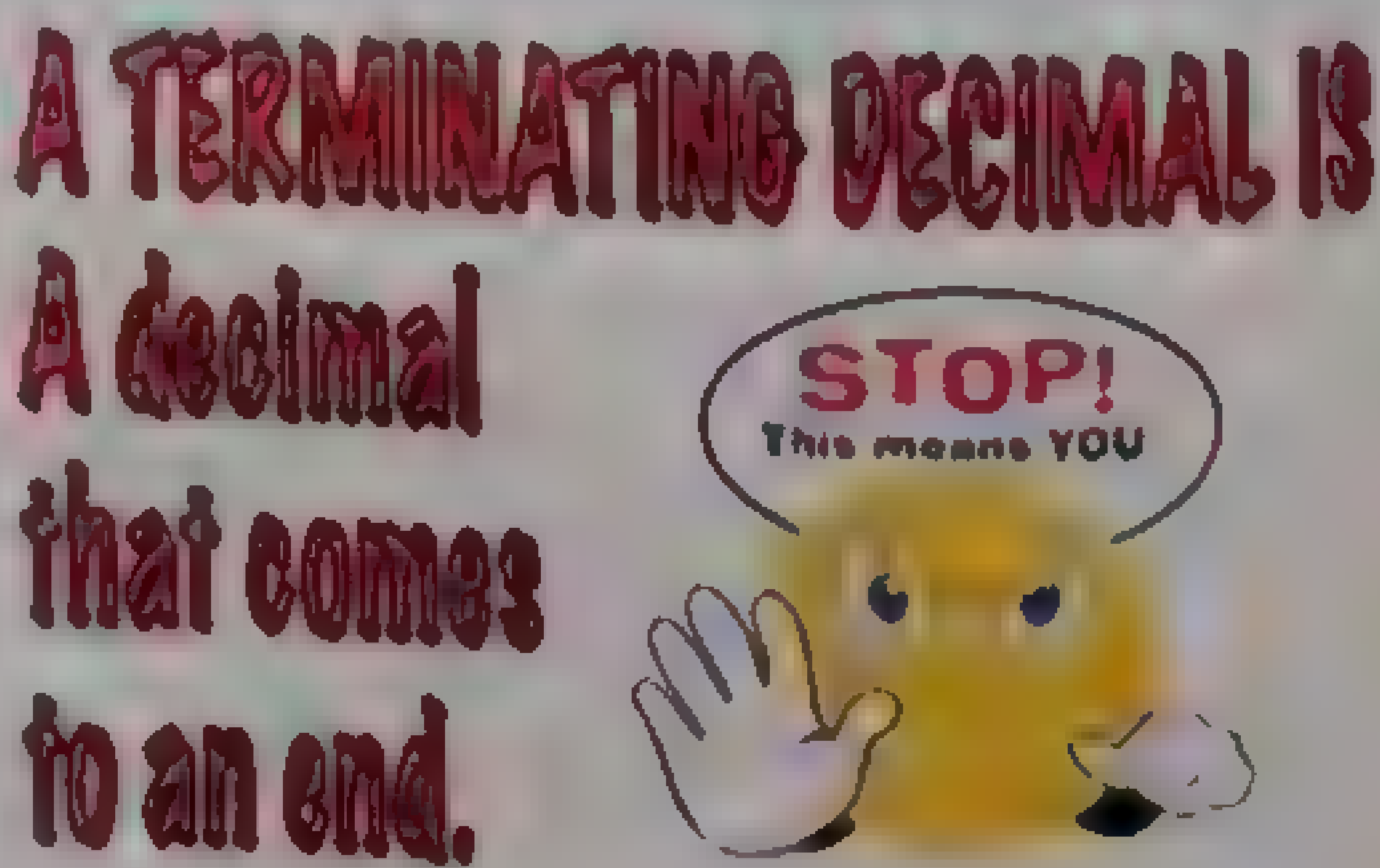
How Decimals are classified

When Numbers are expressed in decimal form, the division process will either be terminating or will be non-terminating. The non-terminating decimals further have two kinds, one is recurring or repeating decimals and the other is non-repeating or non-recurring decimals. The numbers of the two kinds that are terminating and recurring are called rational numbers..



3.1 Terminating and non Terminating Decimals

Terminating decimals are decimals having a finite number of digits after the decimals. e g 7.5, the last digit is 5. Otherwise the decimals are called non-terminating. Any fraction where $b \pm 0$, can be written as a decimal by dividing the numerator by the denominator. The division ends, or terminates, when the remainder is zero, the decimal is a terminating decimal.

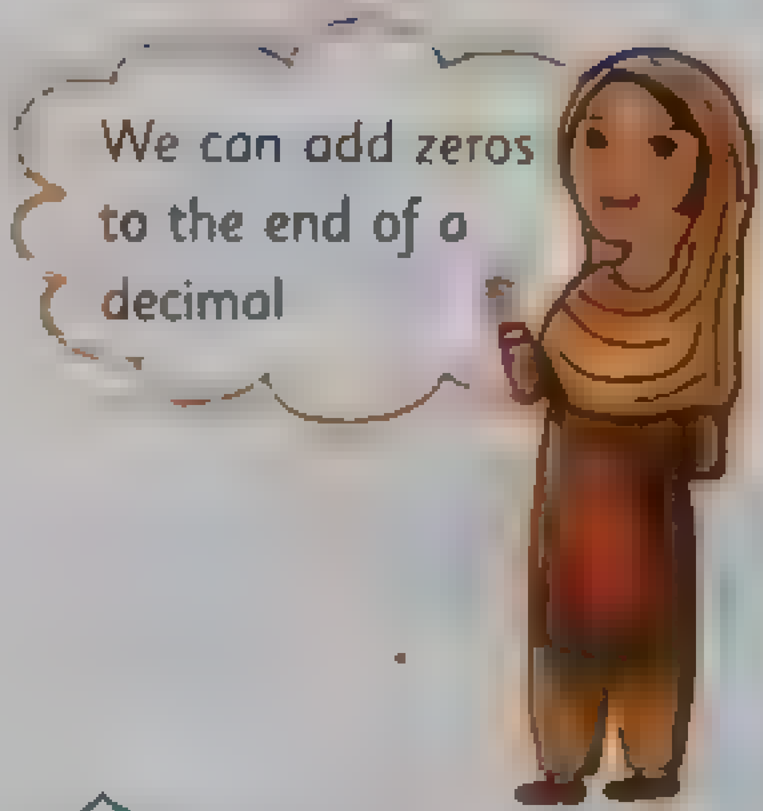


Example 1 Write $\frac{3}{8}$ as decimals

Solution

$$
\begin{array}{r}
0.375 \\
8\overline{)\,3.000} \\
-24 \\
\hline
60 \\
56 \\
\hline
40 \\
-40 \\
\hline
0
\end{array}
$$

Division ends when the remainder 0.



0.375 is a terminating decimal.

Tidbit
$\frac{3}{8}$ is a proper fraction.

Example 2 Write $\frac{16}{7}$ as decimals.

Solution

$$
\begin{array}{r}
2.285... \\
7\overline{)\,16.000} \\
14 \\
\hline
20 \\
14 \\
\hline
60 \\
56 \\
\hline
40 \\
35 \\
\hline
5
\end{array}
$$

Tidbit
$\frac{16}{7}$ is an improper fraction.

As the division goes on without getting zero as remainder, therefore 2.285... is a non-terminating decimals.

Tidbit
Improper fraction can be written as mixed numbers

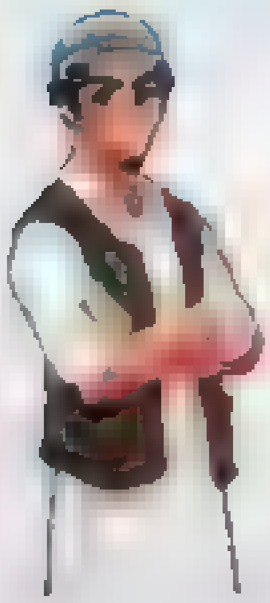
Recurring decimals

Recurring decimals are non-terminating decimals in which a single digit or a block of digits repeats itself infinite number of times after the decimal point

e.g. $\frac{2}{7} = 0.285714285714285714..$

Example 3 Convert the $\frac{5}{3}$ in the recurring decimals.

Solution $\frac{5}{3}$



$$
\begin{array}{r|l}
 & 1.666 \\
\hline
3 & 5 \\
 & -3 \\
\hline
 & 20 \\
 & -18 \\
\hline
 & 20 \\
 & -18 \\
\hline
 & 20 \\
 & -18 \\
\hline
 & 2
\end{array}
$$

The number 6 repeats

The remainder after each step is 2

So, $\frac{5}{3} = 1.666666666..$

This decimal is a repeating or recurring decimal.

We can use bar notation to indicate that the 6 repeats forever.

The digit 6 repeats, so place a bar over the 6

Thus $1.6666666666 = \overline{1.6}$.

The period of a repeating decimal is the digit or digits that repeat.

So, the period of 1.6 is 6.

4 Convert the $\frac{3}{7}$ in decimal fraction

Solution

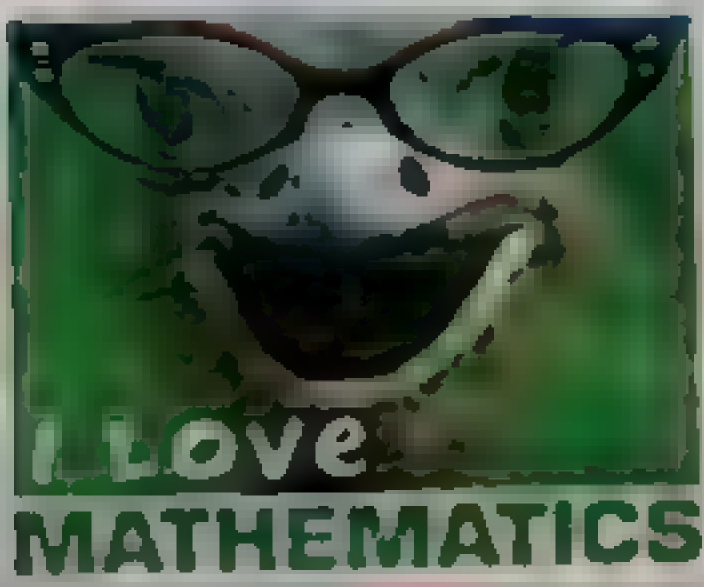
```
       0.428571
  7 ) 3.0000
      28
      ---
       20
      -14
      ---
        60
      - 56
      ----
        40
        35
      ----
         50
         49
        ----
         10
        - 7
        ---
          3   → Repeating starts
```

Tidbit

Be careful, not to add or remove zeros from a written number

$7.01 \neq 7.1$

Math fun



428571 will remain repeating again and again

Hence it is a non-terminating recurring decimal

Guided Practice

Convert the following rational numbers into decimals and separate recurring and non-recurring decimals

i. $\frac{54}{11}$ ii. $\frac{69}{17}$ iii. $\frac{67}{23}$

5 Write the periods of the following numbers

(i). 0.13131313....

(ii). 16.855555....

(iii). 19.1724724....

Decimal	Bar Notation	Period
0.13131313	$0.1\bar{3}$	13
6.855555	6.85	5
19.1724724	$19.1\overline{724}$	724

Factorize the number in the denominator and in the numerator separately. If in the denominator (after simplification) only 2, 5, or 2 and 5 are left the decimal will be terminating.

6

Using the rule, without a long division separate the terminating and non-terminating decimals from the following.

(i) $\frac{39}{26}$ (ii) $\frac{35}{15}$ (iii). $\frac{63}{8}$ (iv). $\frac{30}{18}$

Solutions (i). $\frac{39}{26}$

$$\frac{\cancel{13} \times 3}{\cancel{13} \times 2} = \frac{3}{2}$$

As in the denominator we have only 2 (after cancellation), therefore it is a terminating decimal.

(ii). $\frac{35}{15}$

$$\frac{7 \times 5}{3 \times 5} = \frac{7}{3}$$

As in the denominator we do not have only 2 or 5, therefore it is non-terminating decimal.

(iii). $\frac{63}{8}$

$$\frac{7 \times 3 \times 3}{2 \times 2 \times 2}$$

As in the denominator we have only 2 therefore it is a terminating decimal.

(iv). $\frac{30}{18}$

$$\frac{\cancel{3} \times 5 \times \cancel{2}}{\cancel{3} \times \cancel{2} \times 3} = \frac{5}{3}$$

As in the denominator we have 3 therefore it is non-terminating.

3.1

1. Convert the following decimals into fractions and also simplify where ever possible

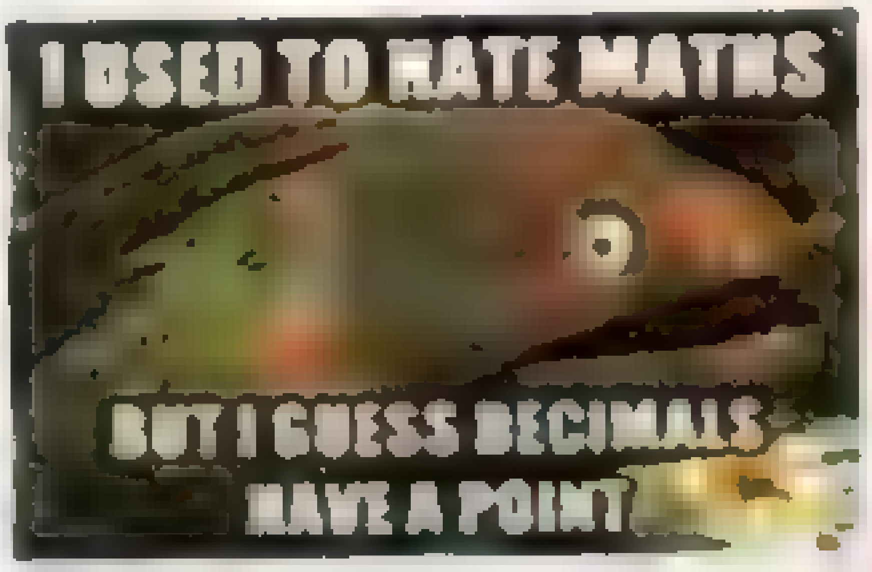
   (i) 0.45 (ii) 0.774 (iii) 7.2 (iv) 1.5771 (v) 192.14

2. Which of the following rational numbers are non-terminating and recurring decimals (Divide up to five decimal places)

   (i) $\frac{5}{3}$ (ii) $\frac{9}{7}$ (iii) $\frac{16}{6}$ (iv) $\frac{57}{13}$ (v) $\frac{342}{169}$

3. Which of the following are terminating/ non-terminating decimals (using division method up to 5 points)

   (i) $\frac{17}{3}$ (ii) $\frac{135}{72}$ (iii) $\frac{63}{11}$



4. Which of the following are terminating/ non-terminating decimals (without using division method).

   (i) $\frac{21}{6}$ (ii) $\frac{66}{16}$ (iii) $\frac{8}{26}$ (iv) $\frac{25}{10}$ (v) $\frac{6}{20}$

3.2 Conversion of decimals to Rotational Numbers

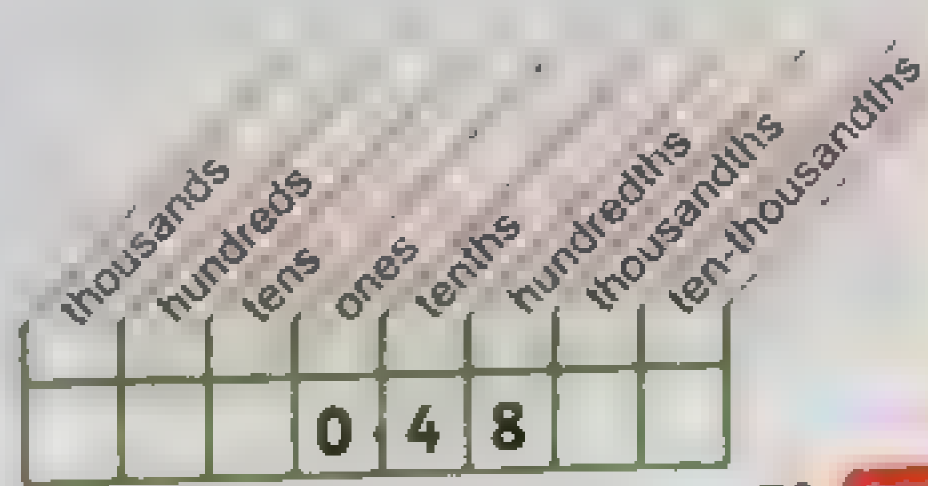
Terminating decimals are rational numbers because they can be written as a fraction with a denominator of 10,100,1000, and so on.

7 Write each decimal as a fraction or mixed number in simplest form.

(a). 0.48

$$0.48 = \frac{48}{100}$$

$$= \frac{12}{25}$$



8 Write 0.8 as fraction in simplest form.

$N = 0.888...$ Let N represent the number

$10N = 10\,(0.888...)$ Multiply each side by 10

$10N = 8.888...$

Subtract N from 10N to eliminate the repeating part, 0.888...

$$\begin{array}{rl} 10N &= 8.888... \\ -N &= .888 \\ \hline 9N &= 8 \end{array}$$

$$\frac{9N}{9} = \frac{8}{9}$$ Divide each side by 9

$$N = \frac{8}{9}$$

Therefore, $0.8 = \dfrac{8}{9}$

Guided Practice

Write each decimal as a fraction in simplest form.

i. $0.\overline{3}$ ii. $0.\overline{72}$

Rounding off

In daily life we experience that some values cannot be used exactly for example if the cost of 1 litre petrol is Rs. 92.14 we cannot pay 0.14 rupees. If the electricity bill is Rs. 918.72, 0.72 cannot be paid. There are thousands of other examples of such kind. To convert such values we round off them.

How to round off decimals

There are very simple rules of rounding to a desired number of decimal places. Following are the few guiding principles for rounding off.

Ones		Tenths	Hundredths

(i) Locate and underline the digit of decimal place which needs to be rounded off.

(ii) Consider the digit to the right of the underlined digit.

(iii) If this digit is 5 or more than 5 i.e. 6, 7, 8, 9 then increase the underlined digit by "1" for example 16.45 can be rounded up to 16.5.

(iv) If this digit is less than 5 i.e. 4, 3, 2 1 or 0 then keep the underlined digit unchanged as in 3.81. So it becomes 3.8.

9 Round off to the nearest tenth.

(i). 72.36 (ii). 72.84

Solution

(i). $72.\underline{3}6$

As the digit to the right of the underlined tenth digit is 6 (more than 5) therefore we add 1 to the underlined digit. So the number becomes 72.4.

(ii). $72.\underline{8}4$

As the digit to the right of the underlined tenth digit is 4 (less than 5) therefore we add nothing to the underlined digit. So the number becomes 72.8.

10 Round off to the nearest hundredth.

(i). 714.542 (ii). 714.545

Solution

(i) $714.5\underline{4}2$

As the hundredth digit is 4 and to its right is 2 which is less than 5 so we add nothing to the underlined digit. After rounding off we have 714.54.

(ii). $714.5\underline{4}5$

As the digit to the right of the underlined hundredth digit is 5 therefore we add 1 to the underlined digit. So the number becomes 714.55.

Example 11 Round off to the nearest thousandth

(i). 8.2342 (ii). 8.2437

Solutions

(i). 8.23<u>4</u>2

As the digit to the right of the underlined thousandth digit is 2 (less than 5) therefore we add nothing to the underlined digit So the number becomes 8.234

(ii). 8.24<u>3</u>7

As the digit to the right of the underlined thousandth digit is 7 (greater than 5) therefore we add 1 to the underlined digit So the number becomes 8 244

Exercise 3.2

1. Round off the following numbers up to the decimal values mentioned for each question

(i) 5 277 (to the nearest hundredth)

(ii) 262 5332 (to the nearest thousandth)

(iii) 1.35 (to the nearest tenth).

(iv) 0.223 (to the nearest hundredth).

(v) 0 917 (to the nearest hundredth)

(vi) 72 1688 (to the nearest thousandth)

(vii) 6.66 (to the nearest tenth)

(viii) 53 64 (to the nearest tenth)

REVIEW EXERCISE 3

1. Colour the correct answer:

i The ratio between the circumference of a circle and its radius is a

a Terminating decimal
b Non-terminating recurring decimal
c Non-terminating decimal
d Terminating-recurring decimal

ii. 5.36, after rounding off to nearest tenth will become

a 5.37 **b** 5.46 **c** 5.4 **d** 5.47

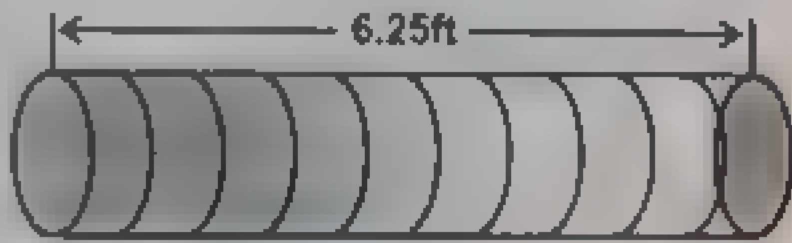
iii. 0.2727 when round off to thousandth becomes

a 0.2728 **b** 0.273 **c** 0.282 **d** 0.3000

iv. If the rod is cut as shown, how many inches long will each piece be?

a 0.625 ft. **b** 1.875 ft.

c 5.2 ft. **d** 7.5 ft



v. Which decimal is equivalent to $\frac{1}{100}$?

a 0.001 **b** 0.01 **c** 0.1 **d** –0.1

2. Convert the following decimals into rational numbers and simplify.

(i) 0.63 (ii) 4.26 (iii) 148.47

3. Using long division, separate the terminating and non-terminating decimals. (divide up to 5 decimal places)

(i) $\frac{25}{35}$ (ii) $\frac{155}{37}$ (iii) $\frac{185}{111}$ (iv) $\frac{69}{20}$ (v) $\frac{92}{21}$

4. Without division, using the rule separate the terminating and non-terminating decimals.

(i) $\frac{47}{40}$ (ii) $\frac{104}{36}$ (iii) $\frac{84}{105}$ (iv) $\frac{27}{93}$ (v) $\frac{37}{21}$

5. Divide the following fractions and determine the terminating and non-terminating recurring decimal. (carry out division up to 5 decimal places)

(i) $\frac{3}{7}$ (ii) $\frac{1}{27}$ (iii) $\frac{2}{26}$ (iv) $\frac{2}{7}$ (v) $\frac{1}{81}$

6. Round off the following to the desired decimal place as mentioned against each question.

(i) 5.72 (to the nearest tenth)

(ii) 0.092 (to the nearest hundredth)

(iii) 4.79 (to the nearest tenth)

(iv) 13.9345 (to the nearest thousandth)

Project

(i) Complete the given chart.

(ii) Complete the given chart for the following fractions.

2/5, 3/5, 4/5, 5/5, 6/5, 7/5, 8/5, 9/5, 11/5,12/5, 13/5, 14/5, 15/5, 16/5

Fraction	Decimal Represention	Is 2 a factor of denominator?	Is 5 a factor of denominator?	Terminating Decimal
1/3	0.333333333	No	No	No
1/4	0.25	Yes	No	No
1/5				
1/6				
1/7				
1/8				
1/9				
1/10				
1/11				
1/12				
1/13				
1/14				
1/15				

Glossary

- Rational number A decimal which can be converted to the form $\frac{p}{q}$ such that p, q, ∈ a c and $q \neq 0$.
- Rounding off A process to get an approximate value to the desired level.
- Terminating decimal When a numerator is divided by some denominator and the quotient has finite digits after decimal, it is called a terminating decimal.
- Non terminating decimal A numerator when divided by a denominator and the quotient has infinite digits the decimal is called non-terminating decimal.
- Non-terminating recurring decimal A non-terminating decimal is a decimal in which a digit or set of digits keeps on repeating.

ACTIVITY

Complete the table

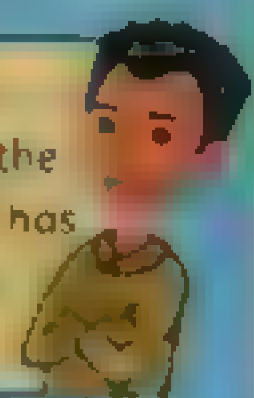
Q.No	Decimal number	Round to the nearest tenth	Round to the nearest hundredth	Round to the nearest thousandth
1)	54.285			
2)	7.69			
3)	19.711			
4)	9.003			
5)	4.6			
6)	81.644			
7)	2.529			
8)	57.407			
9)	3.192			
10)	67.038			

Unit 4 Exponents

What You'll Learn

- Identify base, exponent and value
- Use rational numbers to deduce laws of exponents
- Product law
    - When bases are same but exponents are different
      $$a^m \times a^n = a^{m+n}$$
    - When bases are different but exponents are same
      $$a^m \times a^n = (ab)^n$$
- Quotient law:
    - When bases are same but exponents are different
      $$a^m \div a^n = a^{m-n}$$
    - When bases are different but exponents are same
      $$a^n \div b^n = \left[\frac{a}{b}\right]^n$$
- Power law. $(a^m)^n = a^{mn}$
- For zero exponent: $a^0 = 1$
- For exponent as negative integer $a^{-m} = \frac{1}{a^m}$
- Demonstrate the concept of power of integer that is $(-a)^n$ when n is even or odd integer.
- Apply laws of exponents to evaluate expressions

Did you know?
The member 4 is the only number that has the same number of letters in it



Exponents

$$(5)(5)(5) = 5^3$$

Why It's Important

Very large quantities like planetary masses and very small distances like atomic size are very difficult to understand and compare without the use of exponents.

Mass of the sun: 1.989×10^{30} kg

How powers can be used in showing population increase?

Population Increase

If we have one person and they have 4 children, and then each of these children have 4 children, and so on, we get the following Exponential Population Growth.

Generation	0	1	2	3	4
Children	1	4	16	64	216
Powers	$(2^0)^2$	$(2^1)^2$	$(2^2)^2$	$(2^3)^2$	$(2^4)^2$

Rule $(2^{\text{Generation}})^2$

Population Tree

Guided Practice

i. How many children in generation 5?
ii. How many children in generation 8?
iii. How many children in generation n?

Note Exponents are also called powers or indices.

4.1 Exponents/indices

4.1.1 Base, exponent and value

When a number is repeatedly multiplied by itself, we get power of that number. For example, if 2 is multiplied by itself 5 times, then we write;

exponent

$$2^5 = \overbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}^{5\text{ factors}} \text{ or } 32$$

base

Example 1 How many children will be there in generation 5?

Solution $(2^5)^2 = 2^{5\cdot 2} = 2^{10} = 1024$

Example 2 Evaluate the powers

(i). 2^6 (ii). $(-2)^4$ (iii). -2^4

Solution

(i) $2^6 = 2\times2\times2\times2\times2\times2$

$= 64$ Multiply

(ii). $(-2)^4 = -2\times-2\times-2\times-2 = +16$

(iii). $-2^4 = -(2\times2\times2\times2) = -16$

$(-2)^4$ and -2^4 entirely have different values i e. +16 and −16

Remember

"ben" for Exponents $b^e = n$

exponent

$3^4 = 3.3.3.3 = 81$

base number

Guided Practice

Evaluate each expression.

i. 9^2 ii. 4^4 iii. 10^5 iv. $4(5)^2$ v. $2(-7)^2$

4.2 Laws of exponents/indices

4.2.1 Product Law

(i) Product of powers (ii) Power of a product

Product of powers

Look for a pattern in the examples below.

$$2^3 \times 2^5 = \underbrace{\overbrace{2\times2\times2}^{3\text{ factors}}\times\overbrace{2\times2\times2\times2\times2}^{5\text{ factors}}}_{3+5\text{ or }8\text{ factors}} = 2^8,\quad 3^2 \times 3^4 = \underbrace{\overbrace{3\times3}^{2\text{ factors}}\times\overbrace{3\times3\times3\times3}^{4\text{ factors}}}_{2+4\text{ or }6\text{ factors}} = 3^6$$

Key Concept — Product of powers

Words: To multiply two powers that have the same base, add the exponents.

Symbols: For any number a and all integers m and n, $a^m \times a^n = a^{m+n}$

Example 3 Evaluate.

(i) $3^5 \times 3^3$ (ii) $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$ (iii) $(-5)^2 \times (-5)^4$

Solution

(i) $3^5 \times 3^3 = 3^{5+3} = 3^8$

(ii) $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{3+2} = \left(\frac{1}{2}\right)^5$

(iii) $(-5)^2 \times (-5)^4 = (-5)^{2+4} = (-5)^6$

Guided Practice

Evaluate. i $4^3 \times 4^5$ ii $\left(\frac{1}{3}\right)^2 \times \left(\frac{1}{3}\right)^4$ iii $(-7)^2 \times (-7)^3$

Example 4 Simplify each expression

(i). $(5x^7)(x^6)$

$$(5x^7)(x^6) = (5x^7) \times (x^6)$$
$$= 5x^{13}$$

(ii) $(4ab^6)(-7a^2b^3)$

$$(4ab^6)(-7a^2b^3)$$
$$= (4)(-7)(a \times a^2)(b^6 \times b^3)$$
$$= -28(a^{1+2})(b^{6+3})$$
$$= -28a^3b^9$$

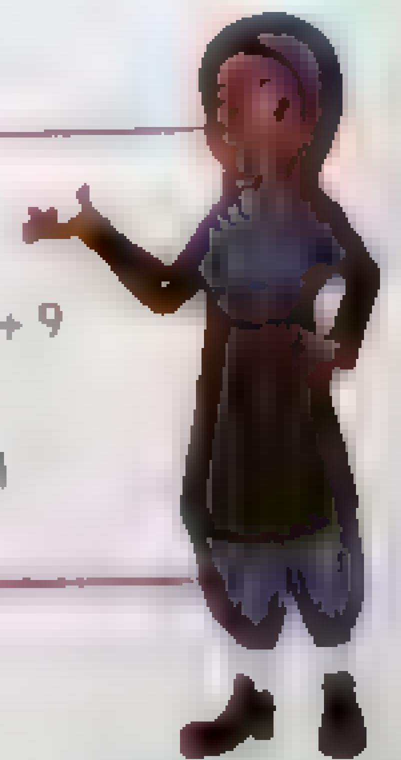
Tidbit

The only way to learn mathematics is to do mathematics

PAUL HALMAS

Find the error Salma and Awais are simplifying $(5^2)(5^9)$.

Salma	Awais
$(5^2)(5^9) = (5\times5)^{2+9}$	$(5^2)(5^9) = 5^{2+9}$
$= 25^{11}$	$= 5^{11}$



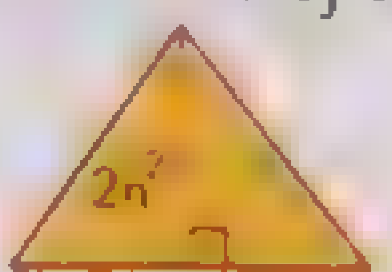
Who is correct?

Guided Practice

1. Simplify

i. $\left(\frac{2}{3}\right)^1 \times \left(\frac{2}{3}\right)^2$ ii. $x(x^4)(x^6)$ iii. $(4a^4b)(9a^2b^3)$

2 Calculate the area of each triangle

iv. 

v. 

Power of a product

Look for a pattern in the examples below

$$(xy)^4 = (xy)(xy)(xy)(xy)$$
$$= (x \cdot x \cdot x \cdot x)(y \cdot y \cdot y \cdot y)$$
$$= x^4 y^4$$

$$(6ab)^3 = (6ab)(6ab)(6ab)$$
$$= (6 \cdot 6 \cdot 6)(a \cdot a \cdot a)(b \cdot b \cdot b)$$
$$= 6^3 a^3 b^3 \text{ or } 216a^3b^3$$

Key Concept — **Power of a product**

Words	To find the power of a product, find the power of each factor and multiply.
Symbols	For all numbers a and b and any integer m, $(ab)^m = a^m b^m$.

Example 5 Find (i) $2^3 \times 3^3$ (ii) $5^3 \times 7^3$

Solution (i) $2^3 \times 3^3 = (2 \times 3)^3 = 6^3$

(ii) $5^3 \times 7^3 = (5 \times 7)^3 = (35)^3$

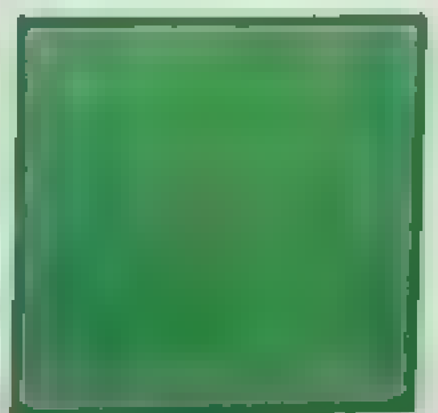
Example 6 Express the area of the square as a monomial.

$\text{Area} = s^2$ Formula for the area of a square

$= (4ab)^2$ $s = 4ab$

$= 4^2 a^2 b^2$ Power of a Product

$= 16a^2 b^2$

4ab

4ab

The area for the square is $16a^2 b^2$ square units.

Guided Practice

Find.

(i) $4^5 \times 5^5$ (ii) $(-2xy)^3$

Exercise 4.1

1. Write the base, and exponent in each of the following

(i) 2^5 (ii) $(-5)^7$ (iii) $\left(\frac{8}{5}\right)^{25}$

(iv) $(100)^{10}$ (v) $\left(\frac{125}{32}\right)^{12}$ (vi) $(-115)^{+20}$

2. Find the value in each of the following

(i) 2^4 (ii) $(-3)^5$ (iii) $(15)^2$

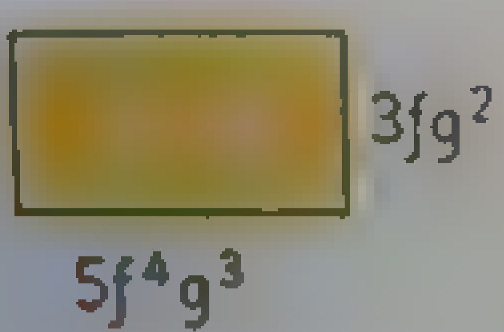
3. Simplify

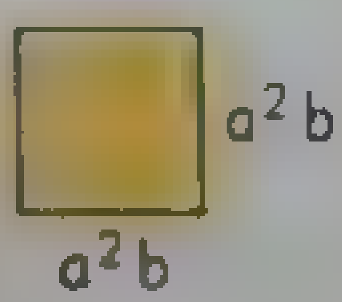
(i) $3^2 \times 3^2$ (ii) $5^4 \times 5^0$ (iii) $\left(\frac{2}{7}\right)^2 \times \left(\frac{2}{7}\right)^5 \times \left(\frac{2}{7}\right)^7$

(iv) $\left(\frac{2}{7}\right)^3 \times (3)^3$ (v) $\left(\frac{5}{8}\right)^4 \times \left(\frac{16}{5}\right)^4$ (vi) $3^2 \times 8^2$

(vii) $(-4)^3 \times (-5)^3$ (viii) $6^4 \times 6^7$

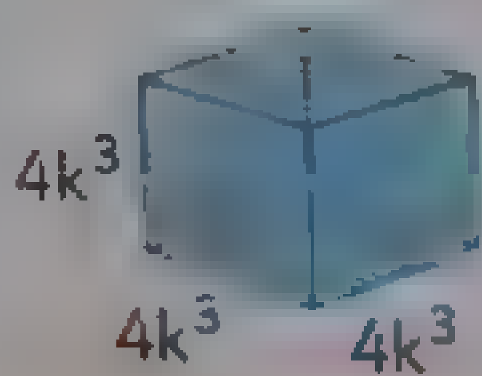
4. Express the area of each figure as a monomial

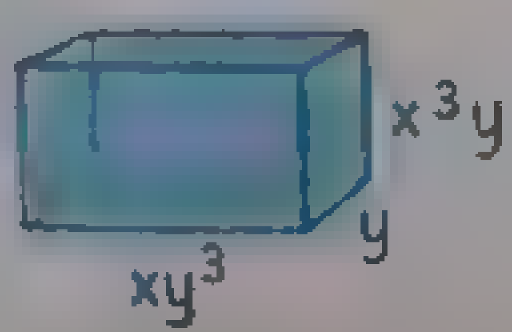
(i) 

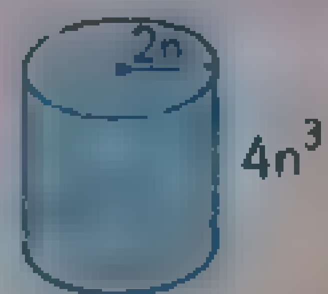
(ii) 

(iii) 

5. Express the volume of each solid as a monomial

(i) 

(ii) 

(iii) 

4.2.2 Quotient Law

Quotient of powers

Look for a pattern in the examples below

$$\frac{4^5}{4^3} = \frac{\overbrace{\cancel{4}.\cancel{4}.\cancel{4}.4.4}^{5\ factors}}{\underbrace{\cancel{4}.\cancel{4}.\cancel{4}}_{3\ factors}} = \underbrace{4.4}_{5-3\text{ or }2\ factors} \text{ or } 4^2 \qquad \frac{3^6}{3^2} = \frac{\overbrace{\cancel{3}.\cancel{3}.3.3.3.3}^{6\ factors}}{\underbrace{\cancel{3}.\cancel{3}}_{2\ factors}} = \underbrace{4.4}_{6-2\text{ or }4\ factors} \text{ or } 4^2$$

These and other similar examples suggest the following property for dividing powers having the same base.

Key Concept — **Quotient of power**

Words: To divide two powers that have the same base, subtract the exponents.

Symbols: For all integers m and n and any nonzero number a, $\frac{a^m}{a^n} = a^{m-n}$

Example 7 (i) $\frac{\left(\frac{2}{5}\right)^7}{\left(\frac{2}{5}\right)^3}$ (ii) $\frac{5^2}{5^5}$

(i) $\frac{\left(\frac{2}{5}\right)^7}{\left(\frac{2}{5}\right)^3} = \left(\frac{2}{5}\right)^{7-3} = \left(\frac{2}{5}\right)^4 = \frac{2^4}{5^4} = \frac{16}{625}$

Tidbit

Always write the answer in positive exponents.

(ii) $\frac{5^2}{5^5} = 5^{2-5} = 5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

Example 8

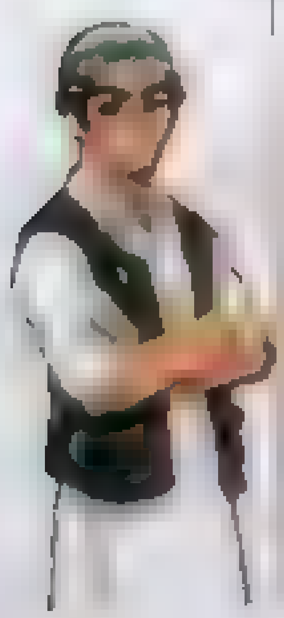
Simplify $\frac{a^5 b^8}{ab^3}$ Assume that a and b are not equal to zero

$$\frac{a^5 b^8}{ab^3} = \left(\frac{a^5}{a}\right)\left(\frac{b^8}{b^3}\right)$$

$$= (b^{5-1})(b^{8-3}) \quad \text{Quotient of Powers}$$

$$= a^4 b^5$$

Jamal and Tahira are simplifying $\frac{-4x^3}{x^5}$



Jamal

$$\frac{-4x^3}{x^5} = -4x^{3-5}$$
$$= -4x^{-2}$$
$$= \frac{-4}{x^2}$$

Tahira

$$\frac{-4x^3}{x^5} = \frac{x^{3-5}}{4}$$
$$= \frac{x^2}{4}$$
$$= \frac{1}{4x^2}$$

Who is correct?

Power of a quotient

Look for a pattern in the examples below

$$\left(\frac{2}{5}\right)^3 = \underbrace{\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)}_{3\text{ factors}} = \frac{\overbrace{2\,.\,2\,.\,2}^{3\text{ factors}}}{\underbrace{5\,.\,5\,.\,5}_{3\text{ factors}}} \text{ or } \frac{2^3}{5^3}$$

This and other similar examples suggest the following property

Key Concept — Power of a quotient

Words To find the power of a quotient, find the power of the numerator and the power of the denominator.

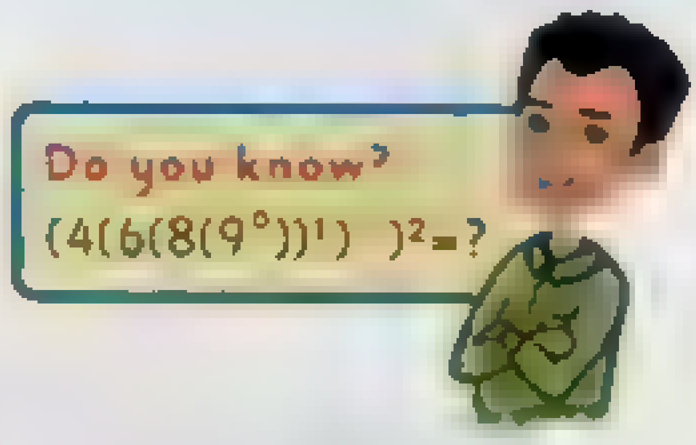
Symbols For any integer m and any real numbers a and

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Example 9 Evaluate.

(i) $\left(\frac{7}{4}\right)^2$ (ii) $\left(\frac{5}{3}\right)^3$ (iii) $\left(-\frac{2}{5}\right)^4$

Solution



(i) $\left(\frac{7}{4}\right)^2 = \frac{(7)^2}{(4)^2} = \frac{7 \times 7}{4 \times 4} = \frac{49}{16}$

(ii) $\left(\frac{5}{3}\right)^3 = \frac{(5)^3}{(3)^3} = \frac{5 \times 5 \times 5}{3 \times 3 \times 3} = \frac{125}{27}$

(iii) $\left(\frac{-2}{5}\right)^4 = \frac{(-2)(-2)(-2)(-2)}{(5)(5)(5)(5)} = \frac{16}{625}$

Guided Practice

Evaluate.

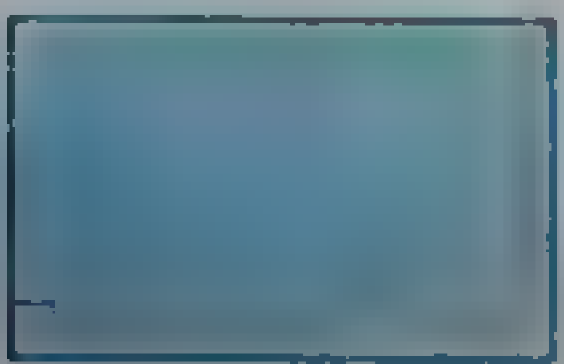
i. $\left(\frac{9}{4}\right)^2$ ii. $\left(\frac{3}{7}\right)^3$ iii. $\left(\frac{2}{5}\right)^4$ iv. $\left(\frac{-5}{9}\right)^2$

1. Simplify the following

(i) $\frac{3^8}{3^5}$ (ii) $\frac{4^7}{4^3}$ (iii) $\frac{(-7)^{10}}{(-7)^5}$ (iv) $\frac{8^{13}}{8^5}$ (v) $\frac{6^5}{3^5}$

(vi) $\frac{(-8)^3}{(-3)^3}$ (vii) $\frac{15^2}{7^2}$ (viii) $\frac{(-5)^{10}}{2^{10}}$

2. The area of the rectangle is $24x^5y^3$ square units. Find the length of the rectangle.



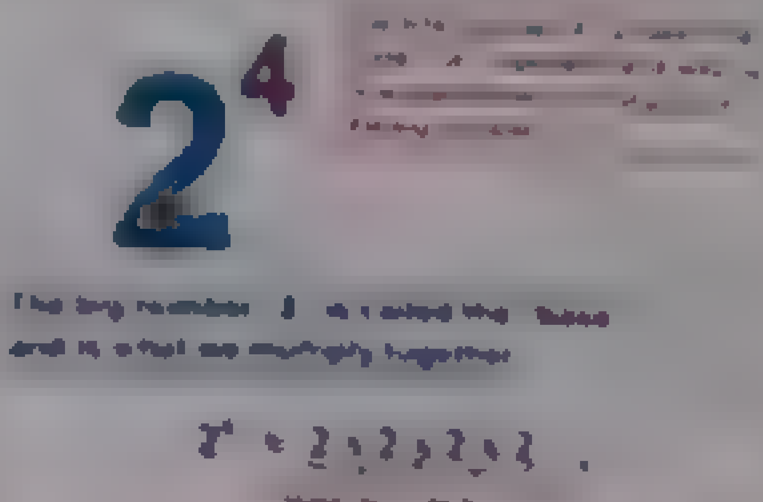
$8x^3y^2$

3. The area of the triangle is $100a^3b$ square units. Find the height of the triangle.

$20a^2$

Study Tip

The examples in the text are carefully chosen to prepare you for success with the exercise sets. Study the step-by-step solutions of the examples, noting that substitutions and explanations. The time you spend studying the examples will save you valuable time when you do your homework.



Exponents and Powers

4.2.3 Power of a Power

Look for a pattern in the examples below

$$(4^2)^5 = \overbrace{(4^2)(4^2)(4^2)(4^2)(4^2)}^{5\text{ factors}} \qquad (z^8)^3 = \overbrace{(z^8)(z^8)(z^8)}^{3\text{ factors}}$$

$$= 4^{2+2+2+2+2} \quad \xleftarrow{\text{Apply rule for Product of Powers}} \xrightarrow{\hspace{1em}} \quad = 4^{8+8+8}$$

$$= 4^{10} \qquad\qquad = 4^{24}$$

Therefore, $(4^2)^5 = 4^{10}$ and $(z^8)^3 = z^{24}$ These and other similar examples suggest the following property for finding the power of a power

Key Concept — **Power of a Power**

Words	To find the power of a power, multiply the exponents.
Symbols	For any number a and all integers m and n, $(a^m)^n = a^{m \cdot n}$

Example 10 Simplify $((3^2)^3)^2$

Solution

$((3^2)^3)^2 = (3^{2\times3})^2$ Power of a Power

$= (3^6)^2$ Simplify

$= 3^{6\times2}$ Power of a Power

$= 3^{12}$ or 531,441

Example 11 Simplify $\left(\frac{2p^2}{3}\right)^4$

Solution

$\left(\frac{2p^2}{3}\right)^4 = \frac{(2p^2)^4}{3^4}$ Power of a Quotient

$= \frac{2^4(p^2)^4}{3^4}$ Power of a Product

$= \frac{16p^8}{81}$ Power of a Power

Guided Practice

i. $[(4^2)^3]^2$ ii. $(9pq^7)^2$ iii. $(3y^5z)^2$

Key Concept — Zero Exponent

Words	Any nonzero number raised to the zero power is 1
Symbols	For any nonzero number a, $a^0 = 1$

Example 12 Simplify each expression. Assume that x and y are not equal to zero.

Solution

a. $\left(-\frac{3x^5y}{8xy^7}\right)^0$

$$\left(-\frac{3x^5y}{8xy^7}\right)^0 = 1$$

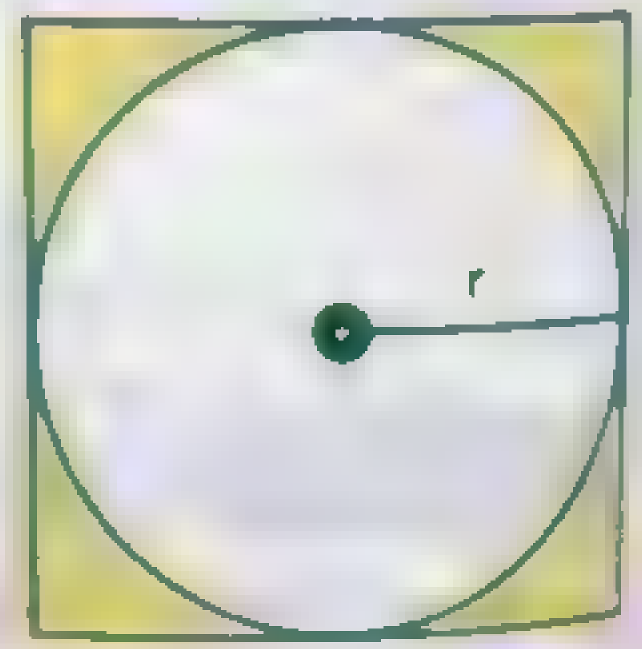
b. $\frac{t^3s^0}{t} = \frac{t^3(1)}{t}$

$= \frac{t^3}{t}$ Simplify

$= t^2$ Quotient of Powers

Example 13 Write the ratio of the area of the circle to the area of the square in simplest form.

Solution



area of circle $= \pi r^2$

length of square = diameter of circle or 2r

area of square $= (2r)^2$

$$\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{(2r)^2}$$

$= \frac{\pi}{4} r^{2-2}$ Quotient of Powers

$= \frac{\pi}{4} r^0$ or $\frac{\pi}{4}$

Guided Practice

Simplify

a. $\left(-\frac{5x^7 2^y}{10x^{2y}}\right)^0$

b. $2^7 \times 2^0$

Key Concept — Zero Exponent

Words For any nonzero number a and any integer n, a^{-n} is the reciprocal of a^n. similarly, the reciprocal of a^{-n} is a^n.

Symbols For any nonzero number a and any integer n,

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n .$$

Example 14 Simplify each expression. Assume that no denominator equal to zero.

Solution

(i). $\frac{b^{-3}c^2}{d^{-5}}$ (ii). $\frac{-3a^4b^7}{21a^2b^7c^{-5}}$

$$\frac{b^{3}c^{2}}{d^{-5}} = \left(\frac{b^{-3}}{1}\right)\left(\frac{c^2}{1}\right)\left(\frac{1}{d^{-5}}\right)$$

$$= \left(\frac{1}{b^3}\right)\left(\frac{c^2}{1}\right)\left(\frac{d^5}{1}\right)\left(a^{-n} = \frac{1}{a^n}\right)$$

$$= \frac{c^2d^5}{b^3}$$

$$\frac{-3a^4b^7}{21a^2b^7c^{-5}} = \left(\frac{-3}{21}\right)\left(\frac{a^{-5}}{a^2}\right)\left(\frac{b^7}{b^7}\right)\left(\frac{1}{c^{-5}}\right)$$

$$\frac{-1}{7}(a^{-4-2})(b^{7-7})(c^5)$$

$$= \frac{-1}{7}a^{-6}b^0c^5$$

$$= \frac{-1}{7}\left(\frac{1}{a^6}\right)(1)c^5 \quad = -\frac{c^5}{7a^6}$$

IT'S ALL FUN AND GAMES UNTIL SOMEONE DIVIDES BY ZERO

Guided Practice

Simplify i $-\frac{5x^5y^3}{15x^{-2}y^{-3}Z^4}$ ii $\frac{x^{-5}y^{-7}}{x^{-6}y^{4}}$

Exercise 4.3

1. Simplify the following.

(i) $(34)^{[illegible]}$ (ii) $51 \times (200)^{[illegible]}$ (iii) $\left(\frac{7}{2}\right)^{-3}$

2. Simplify the following.

(i) $(3^4)^2$ (ii) $[(5)^{-4}]^2$ (iii) $[(-7)^3]^5$ (iv) $[a^2b^3]^4$ (v) $\left[\left(\frac{6}{5}\right)^{[illegible]}\right]^3$

(vi) $\left[\left(-\frac{3}{8}\right)^2\right]^5$ (vii) $\left[\left(\frac{4}{7}\right)^5\right]^2$ (viii) $\left[\left(\frac{18}{5}\right)^2\right]^7$ (ix) $\left[\left(\frac{5}{11}\right)^3\right]^6$ (x) $\left[\left(\frac{p}{q}\right)^4\right]^{[illegible]}$

4.3 Concept of power of an integer

Consider the following examples

(i) $2^4 = 2 \times 2 \times 2 \times 2 = 16 > 0$ (positive)

(ii) $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32 > 0$ (positive)

(iii) $(-2)^4 = -2 \times -2 \times -2 \times -2 = +16 > 0$ (positive)

(iv) $(-2)^3 = -2 \times -2 \times -2 = -8 < 0$ (negative)

From the above examples we deduce that,

- If a is any positive rational number other than zero and its exponent is any positive integer (even or odd) then its value is positive as shown by Example (i) and (ii).
- If a is a negative rational number and its exponent is an even +ve integer then its value is a positive as shown in Example (iii).
- If a is a negative rational number and its exponent is an odd +ve integer then, its value is negative as shown by Example (iv)

Example 15

The value of $(-100)^{20}$ is positive because its exponent is an even integer (i.e. +20)

But the value of $(-100)^{19}$ is negative because its exponent is an odd integer (i.e. 19)

4.4 Simplify expressions

To simplify an expression, write it as such expression in which

- Each base appears exactly once.
- There are no powers of powers, and all fractions are in simplest form

Example 16 Simplify the following

$$\frac{2^3 \times 3^5 \times 12^3}{4^2 \times 6^3}$$

Solution

$$\frac{2^3 \times 3^5 \times 12^3}{4^2 \times 6^3} = \frac{2^3 \times 3^5 \times (3 \times 4)^3}{(2^2)^2 \times (2 \times 3)^3}$$

$$= \frac{2^3 \times 3^5 \times 3^3 \times 4^3}{2^{2\times2} \times 2^3 \times 3^3} = \frac{2^3 \times 3^5 \times 3^3 \times (2^2)^3}{2^4 \times 2^3 \times 3^3}$$

$$= \frac{2^3 \times 3^5 \times 3^3 \times 2^{2\times3}}{2^4 \times 2^3 \times 3^3} = \frac{2^3 \times 3^{5+3} \times 2^6}{2^{4+3} \times 3^3}$$

$$= \frac{2^{3+6} \times 3^8}{2^7 \times 3^3} = \frac{2^9 \times 3^8}{2^7 \times 3^3} = 2^{9-7} \times 3^{8-3}$$

$$= 2^2 \times 3^5 = 4 \times 243$$

$$= 972$$

Exercise 4.4

1. Simplify the following.

(i) $(-4)^4$ (ii) $(-3)^5$ (iii) $\dfrac{(-2)^2 \times 6^{-4}}{2^{-2} \times 4^{-3}}$ (iv) $\left(\dfrac{7}{2}\right)^{-3} \times 49$

(v) $\dfrac{4 \times 3^3}{9 \times (-8)^2}$ (vi) $\left(\dfrac{4}{5}\right)^{-6} \times \left(-\dfrac{4}{5}\right)^{0}$ (vii) $\dfrac{3^6 \times 7^4}{(-7)^3 \times (-3)^4}$ (viii) $\left(\dfrac{1}{4}\right)^{-6} \div (-2)^3$

(ix) $(-2)^5 \div \dfrac{1}{2}$ (x) $\dfrac{2^2 \times (-3)^5 \times 4^3 \times 5^2}{8 \times 9 \times 6^2 \times (-5)^4}$

REVIEW EXERCISE 4

1. Read the following statements carefully and write 'T' in front of true statement and 'F' in front of false statement.

(i) In 2^3, base is 3. ☐

(ii) In $(-5)^3$ exponent is 2. ☐

(iii) $2^3 \times 2^{-3} = 2^6$ ☐

(iv) $4^2 \div 4 = 4^3$ ☐

(v) $\dfrac{a^m}{a^m} = a^{2m}$ ☐

2. Fill in the blanks

(i) $(-50)^2$ has base ____________

(ii) $(a^m)^n$ = ____________.

(iii) a^0 = ____________

(iv) 2^{-3} = ____________

(v) $2^3 \times 3^2$ = ____________.

The only way to learn mathematics is to do mathematics.

PAUL HALMOS

3. Colour the correct answer:

(i) $[(-2)^2]^3 =$ ________.

a) 32   b) -32 c) 64   d) -64

(ii) $4^{-3} =$ ________.

a) 64   b) $\frac{1}{64}$ c) -12 d) $-\frac{1}{12}$

(iii) $2^0 =$ ________.

a) 1   b) 0   c) 2   d) $\frac{1}{2}$

(v) Write 4.4.4.c.c.c.c using exponents

a) $3^4 4^c$ b) $4^3 c^4$ c) $(4c)^7$ d) 4c

(v) $4^2 \times 4^5 = ?$

a) 16^7 b) 8^7 c) 4^{10} d) 4^7

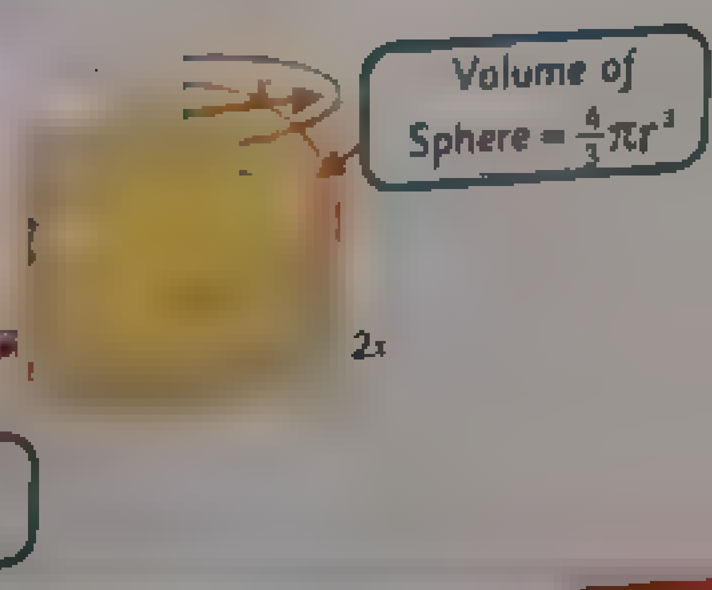
(vi) What is the value of $\frac{2^{\text{[illegible]}} \times 2^3}{2^{-2} \times 2^{-3}}$?

a) 2^{10} b) 2   c) 1^{10} d) $\frac{1}{2}$

(vii) Which of the following expression represents the volume of the cube?

a) $15x^3$ b) $25x^2$ c) $25x^3$ d) $125x^3$

(viii) Find the ratio of the volume of the cylinder to the volume of the sphere.



a) $\frac{1}{2}$ b) 1

c) $\frac{3}{2}$ d) $\frac{3\pi}{2}$

4. Write base, exponent and the value in each of the following questions

(i) 2^5 (ii) $(-3)^4$

5. Simplify

(i) $(-5)^2 \times (-5)^3$ (ii) $\left(\frac{1}{2}\right)^4 \div \left(\frac{1}{2}\right)^2$

(iii) $\left(\frac{3}{4}\right)^2 \times \left(\frac{4}{3}\right)^2$ (iv) $(1000)^0 \times 500$

(v) $(-6)^4 \div (-6)^2$ (vi) $[(-5)^4]^5$

(vii) $\dfrac{3^2 \times 5^3 \times 7^3}{15 \times 49}$ (viii) $(-2v^3w^4)^3(-3vw^3)^2$

6. Find the error. Umair and Sahiba are evaluating $3[4 + (27 \div 3)]^2$.

Umair	Sahiba
$3[4 + (27 \div 3)]^2 = 3(4 + 9^2)$	$3[4 + (27 \div 3)]^2 = 3(4 + 9)^2$
$= 3(4 + 81)$	$= 3(13)^2$
$= 3(85)$	$= 3(169)$
$= 255$	$= 507$

Who is correct?

GO DOWN DEEP ENOUGH INTO ANYTHING AND YOU WILL FIND MATHEMATICS ...

Glossary

Base, exponent and value

When a number is repeatedly multiplied by itself we get exponent of that numbers. In general $a^n = x$ where n is a +ve integer. Here a is base, n is exponent and x is the value of a.

Laws of exponents

i) If a and b are any rational number other than zero and, m, n are integers then,

$$a^m \times a^n = a^{m+n}$$

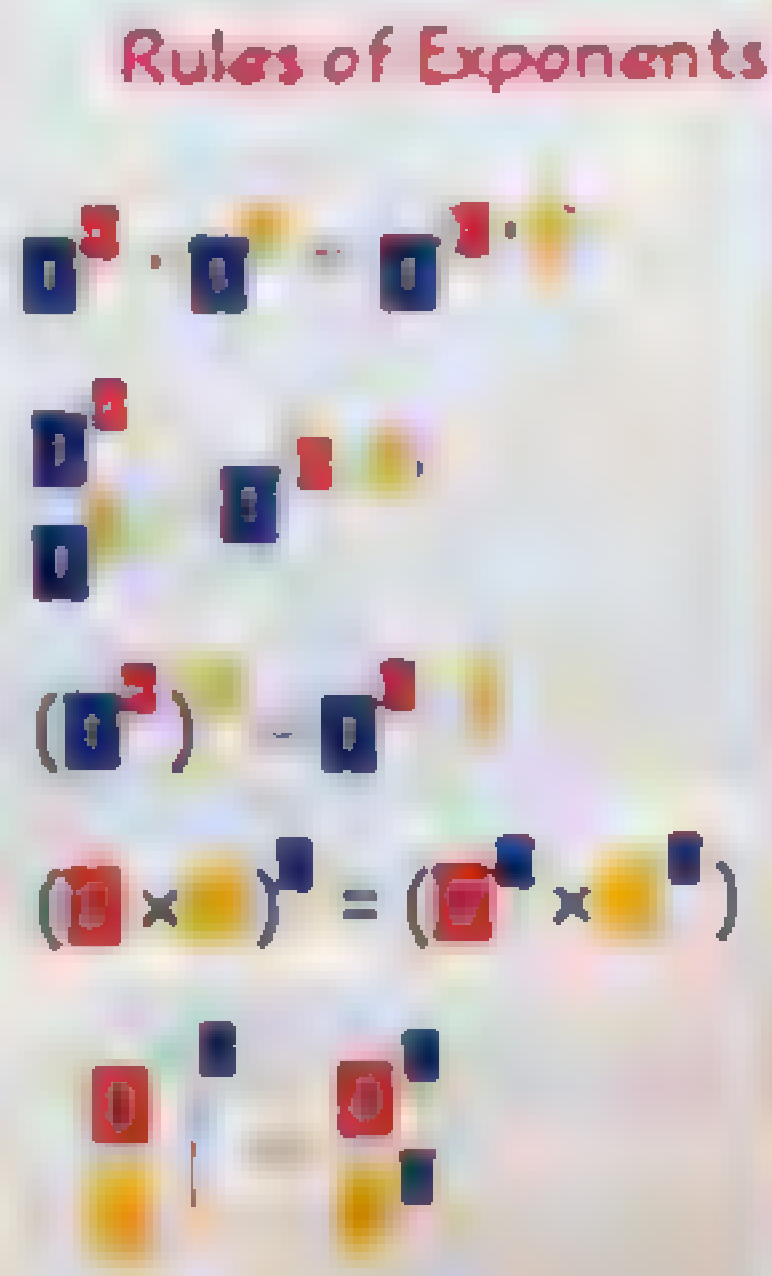
and $a^n \times b^n = (ab)^n$ (Product law)

$$\frac{a^m}{a^n} = a^{m-n}$$

and $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$ (Quotient law)

also $a^0 = 1$ (Power law)

and $a^{-m} = \frac{1}{a^m}$ (Negative exponent)



Remember

Division by zero is undefined

Tidbit

Note the very important difference between $(-2)^4$ and -2^4?

$(-2)^4 = 16$ while $-2^4 = -8$

Unit 5

Square Root of Positive Number

You'll Learn

- The concept of a perfect square
- Test whether a number is a perfect square or not
- Properties of perfect square of a number.
  - The square of an even number is even.
  - The square of an odd number is odd.
  - The square of a proper fraction is less than itself
  - The square of a decimal less than 1 is smaller than the decimal
- The concept of square root.
- Finding square root, by division method and factorization method, of a
  - natural number
  - fraction
  - decimal

  which are perfect squares.
- Solving real life problems involving the square roots

It's Important

Studying mathematics is like building a block wall or a building: you need the blocks on the lower part so you can build on them, and if you leave holes, you can't build on the hole.

The concept of a square root is a prerequisite to many other concepts in mathematics: For example,

square root ⟶ Pythagorean theorem ⟶ trigonometry

square root ⟶ irrational numbers ⟶ real numbers

Math fun

I'd tell you the joke about the roof
BUT IT'S OVER YOUR HEAD

5.1 Perfect Square

5.1.1 Definition

A number is called a perfect square if it is the square of a whole number.

For example, $4 = 2^2$

$9 = 3^2$

$16 = 4^2$

$25 = 5^2$

Here 4, 9, 16 and 25 are the perfect squares of 2, 3, 4, and 5

5.1.2 Testing whether a number is a Perfect Square or not

There are very interesting mathematical shortcuts by which we can test whether a number is a prefect square or not

Remember

All perfect squares end in 1, 4, 5, 6, 9 or 00 (i e Even number of zeros)
Therefore, a number that ends in 2, 3, 7 or 8 is not a perfect square

Example 1

Check whether the following numbers are perfect squares or not

(i) 540 (ii) 784 (iii) 364 (iv) 15628

Solution

Since 540 ends in a single zero so it is not a perfect square.

Since 784 ends in 4 it may or may not be a perfect square

By factorization, we see that

$$784 = (2 \times 2 \times 7)^2 = (28)^2$$

$$\begin{array}{r|l} 2 & 784 \\ \hline 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

This shows that 784 can be expressed as the square of 28.

Therefore, 784 is a perfect square

(iii) 364

Since the number ends in 4, it may or may not be a perfect square

By factorization we see that

$$364 = 2 \times 2 \times 7 \times 13$$

$$\begin{array}{r|l} 2 & 364 \\ \hline 2 & 182 \\ \hline 7 & 91 \\ \hline & 13 \end{array}$$

This shows that 364 cannot be expressed as the square of any number.

Therefore, 364 is not a perfect square

(iv) 15628

Since the number is ending in 8, it cannot be a perfect square

Guided Practice

All the following numbers are perfect squares or not

i. 500 ii. 127 iii. 3792 iv. 94538

5.1.3 Properties of a perfect square

Here are some important properties of perfect squares one by one.

1. The square of an even number is even e.g.

   (i) 2 is even $\Rightarrow 2^2 = 4$ which is also even.

   (ii) 4 is even $\Rightarrow 4^2 = 16$ which is also even.

   (iii) 6 is even $\Rightarrow 6^2 = 36$ which is also even

> **Tidbit**
> A proper fraction is always less than 1

2. The square of an odd number is odd e.g.

   (i) 3 is odd $\Rightarrow 3^2 = 9$ which is also odd

   (ii) 5 is odd $\Rightarrow 5^2 = 25$ which is also odd.

   (iii) 7 is odd $\Rightarrow 7^2 = 49$ which is also odd

3. The square of a proper fraction is less than itself e.g.

   (i) $\frac{2}{3}$ is a proper fraction $\Rightarrow \left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9} < \frac{2}{3}$

   (ii) $\frac{3}{4}$ is a proper fraction $\Rightarrow \left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16} < \frac{3}{4}$

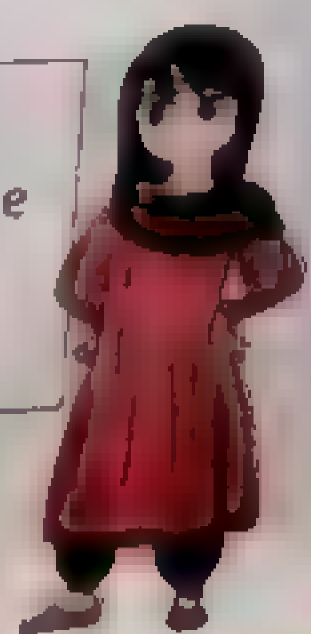
4. The square of a decimal less than 1 is smaller than the decimal e.g

   (i) $0.2 < 1 \Rightarrow (0.2) = 0.04 < 0.1$ (ii) $0.5 < 1 \Rightarrow (0.5) = 0.25 < 0.5$

Asfandyar and Farzana are of the opinion about 625 as

Asfandyar
Since the number ends in 5, surely it is a perfect square.

Farzana
It may or may not be a perfect square.



Who is correct?

Exercise 5.1

1. Check the following numbers whether they are perfect squares or not:

   16, 18, 25, 33, 200

2. Find square of the following numbers:

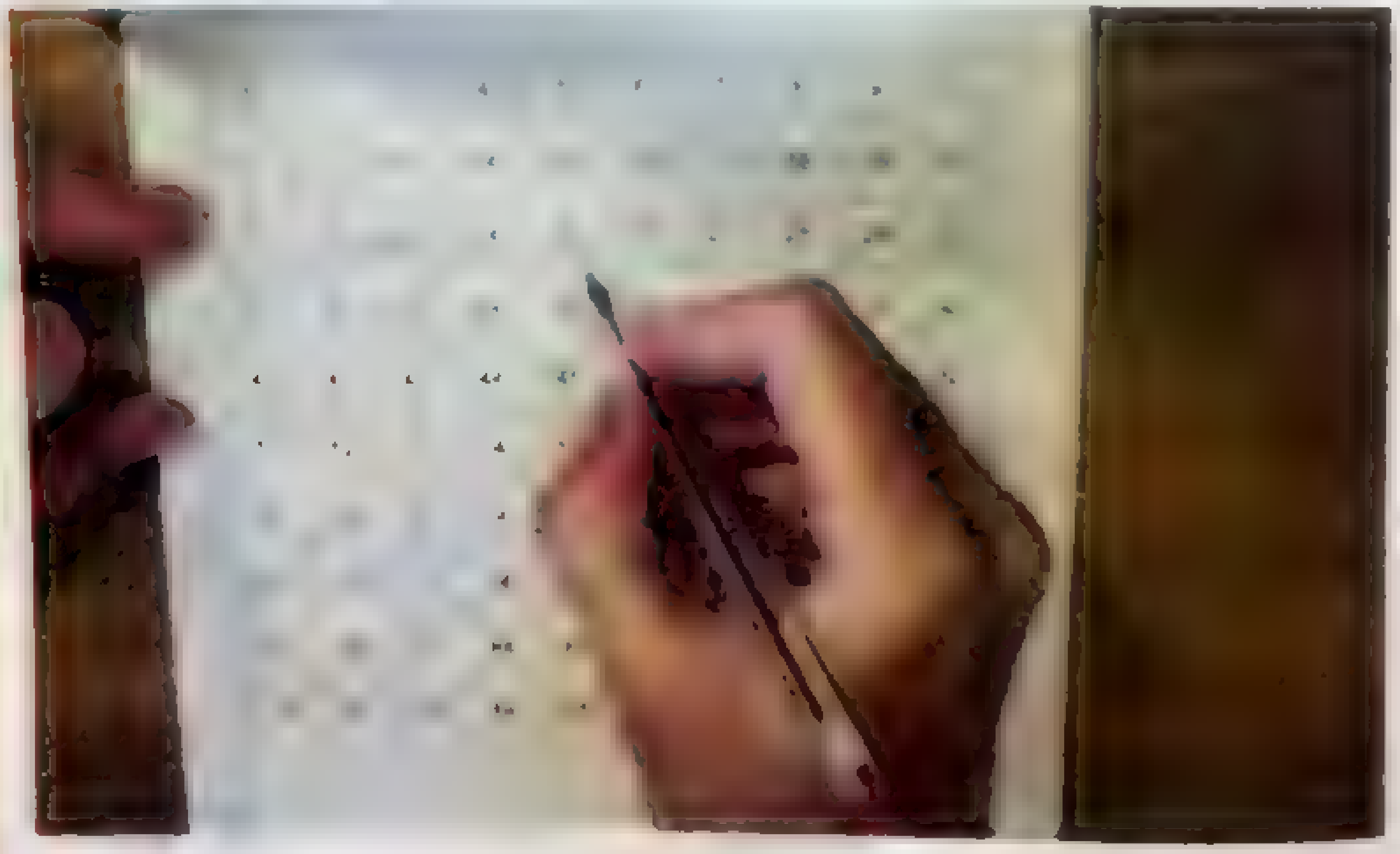
   (i) 35 (ii) 911 (iii) 2170 (iv) 125

3. Do not take square and tell whether the square of following numbers will be even or odd:

   (i) 34 (ii) 751 (iii) 1060 (iv) 32507

ACTIVITY

Make a table of first hundred number then encircle all perfect squares in that table.

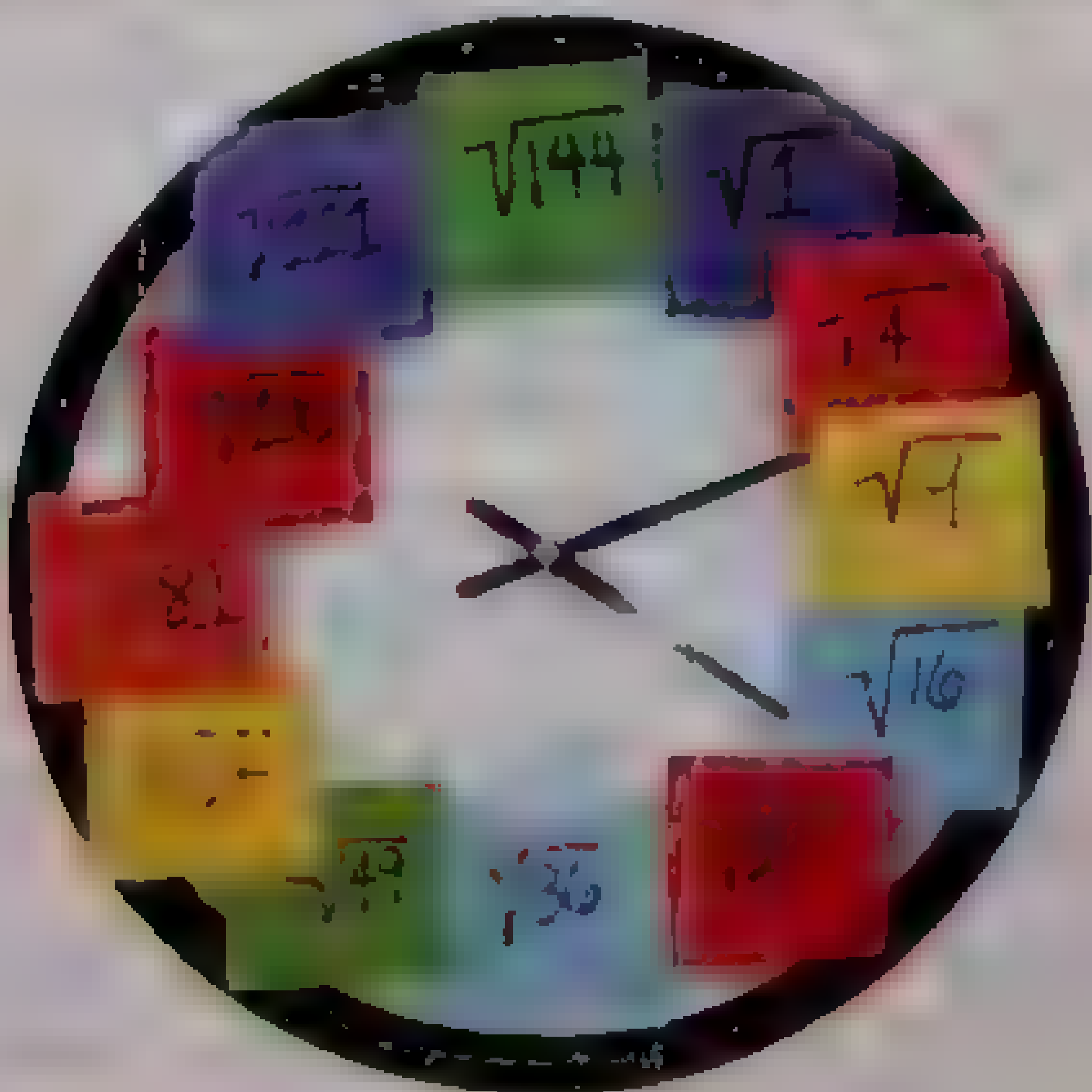


5.2 Square Root

5.2.1 Definition

The square root of a number is a number, whose square gives the same number For example, one square root of 64 is 8 since 8×8 or 8^2 is 64. Another square root of 64 is -8 since -8×-8 or $(-8)^2$ is also 64. A number like 64, whose square root is a rational number is called a perfect squ . The symbol '$\sqrt{}$' called a radical sign, is used to indicate a nonnegative or principal square root of the expression under the radical sign.

$\sqrt{64} = 8$	$\sqrt{64}$ indicates the principal square root of 64
$-\sqrt{64} = -8$	$-\sqrt{64}$ indicates the negative square root of 64
$\pm\sqrt{64} = \pm 8$	$\pm\sqrt{64}$ indicates the both square root of 64



[illegible] **2** Find square roots

> **Tidbit**
> Square roots are ALWAYS positive or zero

(i) $\sqrt{36}$ indicates positive square root of 36

Since $6^2 = 36$, $\sqrt{36} = 6$

(ii) $-\sqrt{81}$ indicates the negative square root of 81

Since $9^2 = 81$, $-\sqrt{81} = -9$

(iii) $\pm\sqrt{9}$ indicates both square root of 9

Since $3^2 = 9$, $\sqrt{9} = 3$, $-\sqrt{9} = -3$

5.2.2 Finding the Square root

Now we shall find the square root of a number by the division method and by the factorization method when it is

(i) Natural number

(ii) Fraction

(iii) Decimal which are perfect squares

(a) Division Method

(i) Finding the square root of a natural number which is a perfect square

To find the square root of a number, the following are the rules

- Make pairs of two digits of the number starting from right to left
- Find the number whose square is equal to or less than the number in the first pair or digit.

Guided Practice

Find each square root, if possible. $\sqrt{49}$ $\sqrt{64}$

Example 3 Find the square root of 784

Solution

$$\begin{array}{r|l} & 28 \\ \hline 2 & 784 \\ & -4\downarrow\downarrow \\ \hline 48 & 384 \\ & -384 \\ \hline & 0 \end{array}$$

$\sqrt{784} = 28$

(i) Pair the digits from right to the left
(ii). $2 \times 2 = 4 < 7$
(iii). Subtract it from 7.
(iv) Twice the quotient
i.e. 2 (2) = 4 and bring down the next pair i.e. 84.
(v). Find a number 8
such that $48 \times 8 = 384$
(vi). The remainder is zero

Hence 28 is the square root of 784.

Example 4 Find the square root of 6889.

Solution

$$\begin{array}{r|l} & 83 \\ \hline 8 & 6889 \\ & -64 \\ \hline 163 & 489 \\ & -489 \\ \hline & 0 \end{array}$$

$\sqrt{6889} = 83$

(i). Pair the digits as and 89.
(ii). $8 \times 8 = 64 < 68$
(iii). Find reminder which is 4
(iv). Double the quotient
i.e. 2 (8) = 16
(v). Find a number 3
such that $163 \times 3 = 489$
(vi) The remainder is zero (489 - 489)

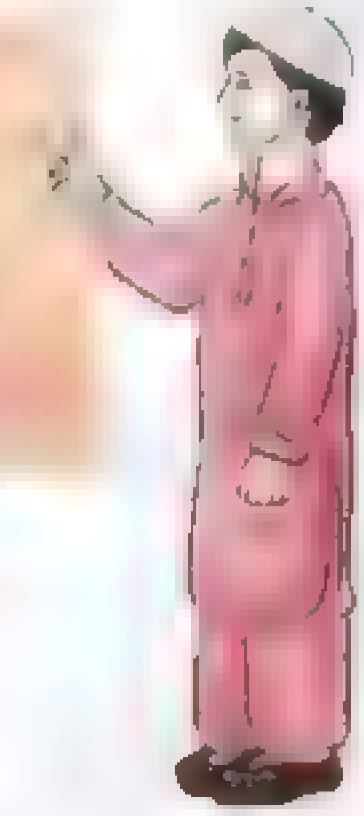
Hence 83 is the square root of 6889

Guided Practice

Find the square root of . 21025 363609

(ii) Finding the square root of a fraction which is a perfect square.

The square root of a fraction is equal to the square root of its numerator divided by the square root of its denominator. The procedure is explained through the following examples:



Example 5 Find the square root of $\frac{784}{841}$

Solution

$$\sqrt{\frac{784}{841}} = \frac{\sqrt{784}}{\sqrt{841}}$$

Square root of the denominator

$$\begin{array}{r|l} & \;\;28 \\ \hline 2 & \overline{7}\,\overline{84} \\ & \pm 4\downarrow\downarrow \\ \hline 48 & \;\;384 \\ & \pm 384 \\ \hline & \;\;\;\;0 \end{array}$$

Square root of the numerator

$$\begin{array}{r|l} & \;\;29 \\ \hline 2 & \overline{8}\,\overline{41} \\ & \pm 4 \\ \hline 49 & \;\;441 \\ & \pm 441 \\ \hline & \;\;\;\;0 \end{array}$$

Hence $\sqrt{\frac{784}{841}} = \frac{\sqrt{784}}{\sqrt{841}} = \frac{28}{29}$ or $\sqrt{\frac{784}{841}} = \frac{28}{29}$

Hence $\frac{28}{29}$ is the square root of $\frac{784}{841}$.

6 Find the square root of $145\frac{144}{169}$

Solution

$$\sqrt{145\frac{144}{169}} = \sqrt{\frac{24649}{169}}$$

Denominator

$$\begin{array}{r|l} & 157 \\ \hline 2 & 2\bar{4}\bar{6}\bar{4}\bar{9} \\ & -1 \\ \hline 25 & 146 \\ & -125 \\ \hline 307 & 2149 \\ & -2149 \\ \hline & 0 \end{array} \qquad \text{and} \qquad \begin{array}{r|l} & 13 \\ \hline 1 & 1\bar{6}\bar{9} \\ & -1 \\ \hline 23 & 69 \\ & -69 \\ \hline & 0 \end{array}$$

Therefore,

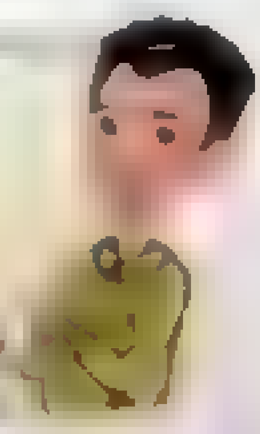
$$\sqrt{145\frac{144}{169}} = \sqrt{\frac{24649}{169}}$$

$$= \frac{\sqrt{24649}}{\sqrt{169}}$$

$$= \frac{157}{13}$$

$$= 12\frac{1}{13}$$

Did you know?



Guided Practice

Find the square roots of the following: $\frac{81}{225}$, ii. $9\frac{49}{64}$

(iii) Finding the square root of a decimal, which is a perfect square.

To find the square root of a decimal the following rules must be followed

- Make pairs of the integral part of the digits from right to left.
- Make pairs of the digits of decimal part from left to right
- If the last digit of the decimal part is only one digit, place zero to its right to complete the pairs.
- Place the decimal point in the quotient after dealing with the integral part of the number.
- Place '0' in the quotient while taking down two pairs at a time

These rules are explained through the examples given.-

Example 7 (i) Find the square root of 30.3601

(ii) Find the square root of 0.00868624

Solution

```
             5.51
        ___________
        | 30.3601
  5     | +25
        |_________
        |   536
  25    | ± 525
        |_________
  1101  |   1101
        |  +1101
        |_________
        |     0
```

Hence, $\sqrt{30.3601} = 5.51$

Solution

```
              0.0932
          _____________
   9     | 0.00868624
         |  +81
         |__________
   183   |   586
         |  ±549
         |__________
   1862  |    3724
         |   +3724
         |__________
         |       0
```

Hence, $\sqrt{0.00868624} = 0.0932$

Guided Practice

Find the square root of i. 0.12321 ii. 84.8241

Exercise 5.2

Find the square root of the following by division method.

(i) 3481 (ii) 2116 (iii) 15129 (iv) $\frac{17161}{169}$

(v) $\frac{31329}{841}$ (vi) $410\frac{1}{16}$ (vii) 50.253921 (viii) 0.0676

(ix) 152.7696 (x) 1.2769

(b) Factorization Method

(i) Finding the square root of a natural number which is a perfect square

To find the square root of a natural number:

- Factorize the number.
- Write the factors in square from.

Example 8 Find the square root of 81.

Solution Factorization of $81 = 3 \times 3 \times 3 \times 3$

$= 3^2 \times 3^2$

3	81
3	27
3	9
	3

Therefore,

$$\sqrt{81} = \sqrt{3^2 \times 3^2}$$
$$= 3 \times 3$$
$$= 9$$

Example 9 Find the square root of 225.

Solution

$$225 = 3 \times 3 \times 5 \times 5$$
$$\sqrt{225} = \sqrt{3^2 \times 5^2}$$
$$= 3 \times 5$$
$$= 15$$

3	225
3	75
5	25
	5

Example 10 Find the square root of 3969

Solution

$$3969 = 3 \times 3 \times 3 \times 3 \times 7 \times 7$$

$$\begin{array}{r|l} 3 & 3969 \\ \hline 3 & 1323 \\ \hline 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

$$\sqrt{3969} = \sqrt{3^2 \times 3^2 \times 7^2}$$

$$= 3 \times 3 \times 7$$

$$= 63$$

(iii) To find the square root of a fraction which is a perfect square.

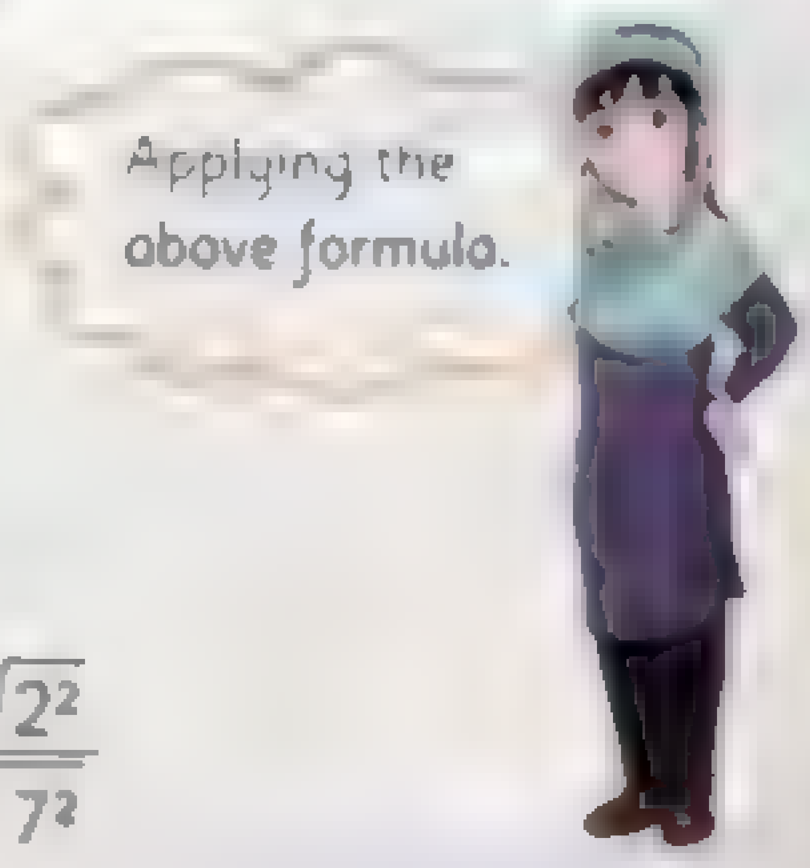
Square root of a fraction can be found out by using the following formula

$$\text{Square root of fraction} = \frac{\text{Square root of numerator}}{\text{Square root of denomenator}}$$

Example 11 Find the square root of $\frac{4}{49}$

Solution

$$\begin{array}{r|l} 2 & 4 \\ \hline & 2 \end{array} \quad \text{and} \quad \begin{array}{r|l} 7 & 49 \\ \hline & 7 \end{array}$$



Therefore,

$$\sqrt{\frac{4}{49}} = \frac{\sqrt{4}}{\sqrt{49}}$$

$$= \frac{\sqrt{2 \times 2}}{\sqrt{7 \times 7}} = \frac{\sqrt{2^2}}{\sqrt{7^2}}$$

$$= \frac{2}{7}$$

Guided Practice

Find the square root of . 2 04 . 1089

Example 12 Find the square root of $\frac{81}{25}$

Solution

$$\begin{array}{r|l} 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array} \quad \text{and} \quad \begin{array}{r|l} 5 & 25 \\ \hline & 5 \end{array}$$

Therefore,

$$\sqrt{\frac{81}{25}} = \frac{\sqrt{81}}{\sqrt{25}}$$

$$= \frac{\sqrt{3 \times 3 \times 3 \times 3}}{\sqrt{5 \times 5}}$$

$$= \frac{\sqrt{3^2 \times 3^2}}{\sqrt{5^2}}$$

$$= \frac{3 \times 3}{5}$$

$$= \frac{9}{5}$$

$$= 1\frac{4}{5}$$

To change the improper fraction $\frac{9}{5}$ to mixed ratio.

$$\begin{array}{r} 1 \\ 5\overline{)9} \\ \underline{5} \\ 4 \end{array}$$

$\frac{9}{5}$ is an improper fraction and

$1\frac{4}{5}$ is its mixed form

Example 13 Find the square root of $6\frac{57}{64}$

Solution $6\frac{57}{64} = \frac{441}{64}$

3	441
3	147
7	49
	7

and

2	64
2	32
2	16
2	8
2	4
	2

Therefore,

$$\sqrt{6\frac{7}{64}} = \sqrt{\frac{441}{64}}$$

$$= \frac{\sqrt{441}}{\sqrt{64}}$$

$$= \frac{\sqrt{3 \times 3 \times 7 \times 7}}{\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2}}$$

$$= \frac{\sqrt{3^2 \times 7^2}}{\sqrt{2^2 \times 2^2 \times 2^2}}$$

$$= \frac{3 \times 7}{2 \times 2 \times 2}$$

$$= \frac{21}{8}$$

$$= 2\frac{5}{8}$$

Guided Practice

Find the square root of i $\frac{625}{64}$ ii $1\frac{87}{169}$

To find the square root of a decimal which is a perfect square First convert it into a common fraction, then solve it

Example 14 Find the square root of 30.25

Solution $30.25 = \dfrac{3025}{100}$

Therefore, $$\sqrt{30.25} = \sqrt{\frac{3025}{100}}$$
$$= \frac{\sqrt{3025}}{\sqrt{100}}$$
$$= \frac{\sqrt{5 \times 5 \times 11 \times 11}}{\sqrt{2 \times 2 \times 5 \times 5}}$$
$$= \frac{\sqrt{5^2 \times 11^2}}{\sqrt{2^2 \times 5^2}}$$
$$= \frac{\cancel{5} \times 11}{2 \times \cancel{5}}$$
$$= \frac{11}{2} = 5.5$$

$$\begin{array}{r|l} 5 & 3025 \\ \hline 5 & 605 \\ \hline 11 & 121 \\ \hline & 11 \end{array} \qquad \begin{array}{r|l} 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$$\begin{array}{r} 5.5 \\ 2\overline{)\,11.0} \\ 10 \\ \hline 10 \\ 10 \\ \hline \times \end{array}$$

Hence, $\sqrt{30.25} = 5.5$

Guided Practice

Find the square root of i) 16.81 ii) 16.4025

Exercise 5.3

Find the square root of the following by factorization method

(i) 169 (ii) 1764 (iii) 1024 (iv) $\frac{36}{25}$ (v) $10\frac{9}{16}$

(vi) $\frac{22500}{324}$ (vii) 1.44 (viii) 19.36 (ix) 10.24 (x) 1030.4

5.2.3 Solving the real life problems involving square

Example 15

What is the length of the side of a squared garden shape whose area is 196 m^2.

Solution

Area of garden $= 196\ m^2$

Length of a side of garden $= ?$

We know that

Area of square $=$ (length of side)^2

or (length of a side)^2 $=$ Area of square.

(length of a side)^2 $= 196m^2$

length of a side $= \sqrt{196 \times m^2}$

$\Rightarrow$ length of a side $= \sqrt{2 \times 2 \times 7 \times 7 m \times m} = 2 \times 7m$

$= 14m$

Hence, the length of a side of the garden is 14m

Guided Practice

What is the length of the side of a square whose area is $279m^2$

Example 16

The area of a squared shape room is $144m^2$. Find the perimeter of the room, also find the cost of chips flooring of the room at the rate of Rs 25 per m^2.

Solution

Area of the room = $144m^2$

Perimeter of the room = ?

Cost of chips flooring of the room = ?

As we know that

Area of the square = $(\text{length of side})^2$

or length of side = $\sqrt{\text{Area of the square}}$

$= \sqrt{12 \times 12 \times m \times m} = 12m$

$= \sqrt{144 \times m^2}$

$= 12m$

Therefore, length of side of room is 12m.

Perimeter of a square = 4 x side

or perimeter of the room = $4 \times 12 = 48$ m

Also

Cost of chips flooring for $1m^2$ = Rs 25

Cost of chips flooring for $144m^2$ = 25×144

= Rs.3600

Guided Practice

The area of squared shape room is $144m^2$. Find the perimeter of the room, also find the cost of chips flooring of the room at the rate of Rs 15 per m^2

Exercise 5.4

1. The area of a squared classroom is $31.36m^2$. Find the length of its side
2. The area of a squared garden is 4624 square kilometers. Find the length of its side.
3. In a garden, 676 trees are planted in rows in such a way that the number of rows equal to the number of trees in a row. How many trees are there in each row?
4. The area of a square shaped farm is $6400m^2$. Find the perimeter of the farm.
5. The area of a squared garden is $121yd^2$. What is the length of its sides?

REVIEW EXERCISE 5

1. Fill in the blanks.

   (i) The square of an even number is ________ number.

   (ii) The square of a proper fraction is ________ than itself.

   (iii) The square of an odd number is ________ number.

   (iv) The square root of 121 is ________

   (v) 625 is the perfect square of ________.

2. Choose the correct answer.

   (i) 169 is the perfect square of

   a 9 b 13 c 19 d 23

   (ii) 28 is the square root of

   a 144 b 742 c 784 d 169

   (iii) The square of any even number is

   a even b odd c primo d negative

(iv) The symbol '$\sqrt{\quad}$' is called

a index b radical c radicant d square root

(v) The area of a square whose length of one side is 8m is

a $16m^2$ b $36m^2$ c $32m^2$ d $64m^2$

(vi) Which of the following is a rational number

a $\sqrt{361}$ b $\sqrt{125}$ c $\sqrt{200}$ d $\sqrt{325}$

(vii) Which of the following is not a perfect square

a $18ft^2$ b $36ft^2$ c $9ft^2$ d $6ft^2$

3. Find the square of

(i) 30 (ii) 65

4. Find the square root of the following by division method.

(i) 7921 (ii) $\frac{3136}{4225}$ (iii) 5.5225

5. Find square root of the following by factorization method

(i) 1764 (ii) $\frac{4624}{1444}$ (iii) 77.44

6. Area of a square shaped garden is $30.25m^2$. find its perimeter

7. Arrange 64 students of 10th class in rows in such a way that the number of rows and number of students are equal. Find the number of students in each row.

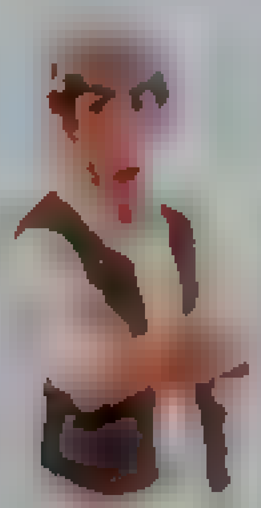
8. Area of a square field is $1600m^2$. How much long wire is required for its boundary?

Glossary

- The perfect square is the product [illegible]
- [illegible] of a natural number [illegible]

[illegible] the error

[illegible] converting $\sqrt{7x}$ to fractional exponents [illegible]



$\sqrt{7x} = (7x)^2$

$\sqrt{7x} = 7x^{\frac{1}{2}}$

Who is correct?

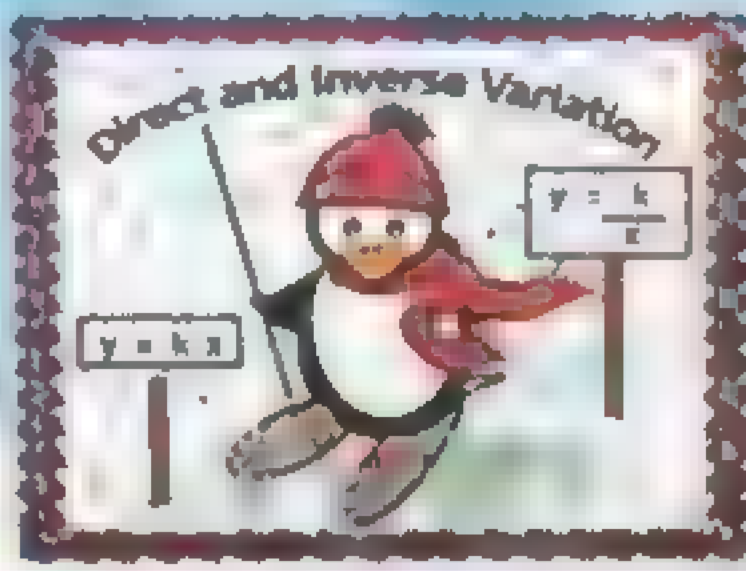
What's your MATH Goal?

- What is your Level now?
- What is your Goal?
- How will you reach that Goal?
- Who can help you reach that Goal?
- What materials do you have to help you?
- How do you know you have reached your Goal?

Unit 6 Direct and Inverse Variation

You'll Learn

- Continued ratio and recall direct and inverse proportion
- Solve the real life problems (involving direct and inverse proportion) using unitary method and proportion method
- Solve the real life problems related to time and work using proportion
- Find a relation between time and distance
- Convert the units of speed (kilometer per hour into meter per second and vice versa).
- Solve variation related problems involving time and distance



Why It's important

The concept of proportionality is the foundation of many branches of mathematics, including geometry, statistics and business math Proportions can be used to solve real-life problems dealing with scale drawings, indirect measurement, predictions and money

6.1 Ratio

A ratio is a comparison of like quantities measured in the same units. For example Aqeel earned Rs. 10,000 and Shakeel earned Rs. 5000, then the ratio between the earning of Aqeel and Shakeel is 10,000 : 5000 or 2 : 1. A ratio a : b of two quantities a and b can also be represented as $\frac{a}{b}$.

6.1.1 Continued Ratio

The comparison of ratios of three or more quantities is called continued ratio

The advantage of finding continued ratio is that we can easily tell about the ratio between the first and the third quantity

Example 1

The ratio between the ages of Amir and Imran are 7 to 5 and the ratios between Imran and Ali are 3 to 2. Find the continued ratio among the ages of Amir, Imran and Ali.

Solution

(i) Ratio between Amir and Imran = 7 : 5

(ii) Ratio between Imran and Ali = 3 : 2

Amir		Imran		Ali
7	:	5		
		3	:	2

We write the two ratios as shown above and multiply the quantities as indicated by arrows

Amir		Imran		Ali
7 × 3	:	5 × 3	:	5 × 2
21	:	15	:	10

Example 2

A construction company is working in an area where sand is available free of cost. To make concrete the ratio of cement to gravel is 2 : 3 and the ratio of gravel to sand is 2 : 5. Find the quantity of gravel and cement required if 1000 m^3 concrete is needed.

Solution

Volume of concrete = 1000 m^3

Ratio among cement gravel and sand.

Cement		Gravel		Sand
2	:	3		
		2	:	5

So their continued ratio is

Cement		Gravel		Sand
4	:	6	:	15

To find out the required quantities we add up the ratios

Sum of the ratios	$= 4+6+15 = 25$
Quantity of the cement	$= 1000 \times \frac{4}{25} = 160 m^3$
Quantity of the gravel	$= 1000 \times \frac{15}{25} = 600 m^3$

6.1.2 Direct proportion

If increase or decrease in one quantity results increase or decrease in the second quantity, then such a proportion is called direct proportion. For example if the cost of 1 note book is Rs. 30 then the cost of 3 note books will be Rs. 90. We see that if we increase one quantity the other also increases. For example

- Proportion between speed and distance covered by a car
- Proportion between temperature and pressure in a tyre
- Proportion between demand and price of tomatoes

Example 3

The relation between time and water pulled by a water pump is direct. If a pump raises 600 litres water in 45 minutes, how much water it will raise in 1 hour.

Solution As the unit of times are different, so first we make them the same.

1 hour = 60 minutes

Time	Volume of water
45	600
60	x

As the proportion is direct, so

$$\frac{45}{60} = \frac{600}{x}$$

$$x = \frac{600 \times 60}{45}$$

$$x = \frac{2400}{3}$$

$$x = 800 \text{ litres}$$

6.1.3 Inverse porportion

If increase in one quantity results in the decrease of the second quantity or vice versa then such proportion is called inverse proportion. For example,

- Proportion between supply of tomatoes and their price.
- Proportion between speed and time required to cover a distance.
- Pressure and volume of gas in a balloon

Can you think of more such examples of direct and inverse proportion?

Example 4

5 masons can build a house in 120 days. How many masons will be required to build the same house in 75 days?

Solution

The relation between the number of masons required and the time spent is inverse

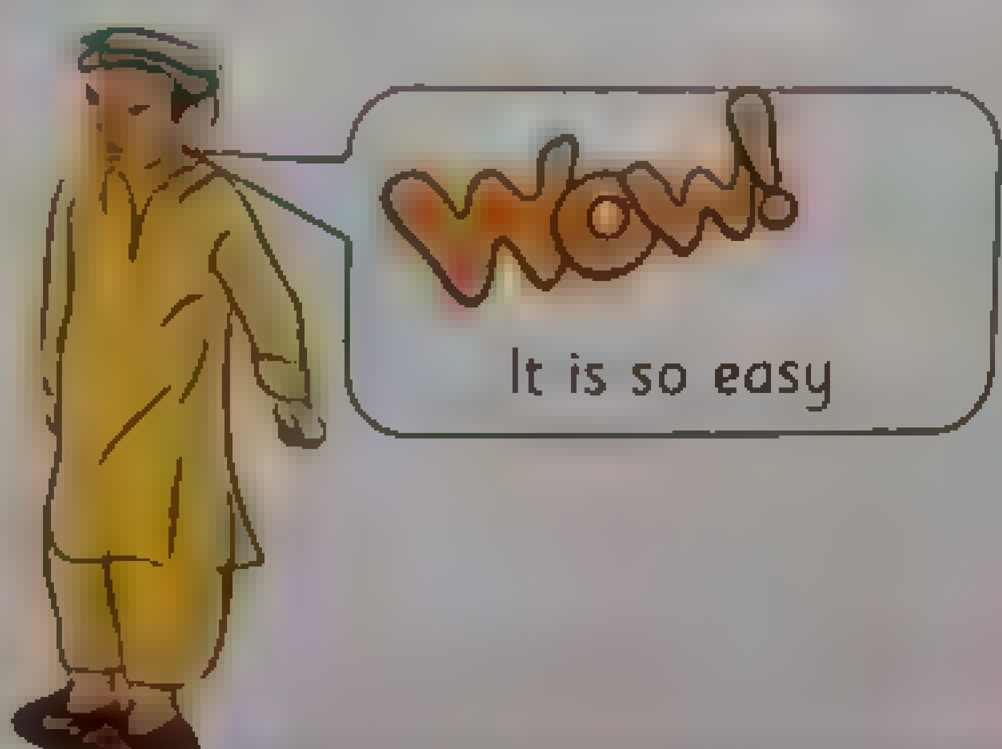
Masons	days
5	120
x	75

As the relation is inverse,

$$\therefore \quad \frac{x}{5} = \frac{120}{75}$$

$$\text{or} \quad x = \frac{120 \times 5}{75}$$

$$x = 8 \text{ masons}$$



6.1.4 Unitary method

The word unitary is derived from unit which means one. In this method first we find out the value of one (unit) item and then multiply it with the number of items.

Eggs, cups, saucers, pencils and hundreds of things are sold in dozens. This method is extremely useful in buying or selling of such things.

Example 5

A street hawker is selling bananas at Rs. 60 per dozen. A man wants to buy 20 bananas. How much he will have to pay? (There are 12 items in 1 dozen)

Solution Price of 1 dozen of Bananas = 60

Price of 1 Banana $= \frac{60}{12}$ = Rs. 5

Price of 20 bananas = 5 × 20 = Rs. 100

He will have to pay Rs 100

Example 6

A man earns Rs 18 000 per month for working 6 hours a day. The office is open for 24 days. He gets an offer from other company to join at the salary of Rs 100 per hour. If the working hours are the same in both offices should he join the new office?

Solution The salary for 24 days = 18000

Salary for 1 day $= \frac{18000}{24}$ = Rs 750

He works for 6 hour, so

The salary for 1 hour $= \frac{750}{6}$ = Rs 125

The person is getting higher salary in his present office (Rs 125 h) than the new offer so he should not accept the new offer.

Exercise 6.1

1. Find 'x' in the following proportion.

   (i) $5 : x = 15 : 60$ (ii) $7 : 14 = 15 : x$

   (iii) $12 : x = 8 : 14$ (iv) $x : 3 = 2.5 : 1.5$

2. Check whether 4,16 and 64 are in proportion

3. Find x if 8, 16 and x one in continued proportion.

4. A survey showed that the colour of cars chosen by people silver, white and black were in the ratio of 7 : 4 : 2 respectively. If a dealer has sold 1300 cars in a year. How many cars of each colour did he sell?

5. Jamil earns Rs. 18000 per month and spends Rs. 16000. Find the ratio in rupees of

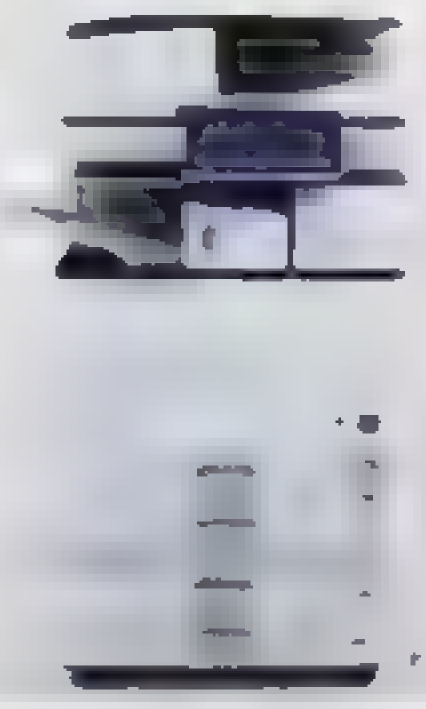
   (i) his income to expenditure

   (ii) his savings to his earnings

6. In an examination hall the ratio of invigilators to the students is 1 : 30. How many invigilators will be required for 210 students?

7. The ratio between the measure of three angles in a triangle is 1 : 2 : 3. Find the measure of each angle.

8. A Printer can print 450 pages in 30 minutes whereas a photocopier can print 30 pages per minute which one is speedy?



6.2 Rate

Why It's important

In daily life and in scientific research we come across different units. For example we take the speed of a bus as 110 km/h or an aircraft as 700 km/h. In science the units used are meter and seconds instead of km and hour for distance and time respectively. For example km/h, m/s, cubic ft/sec.

Note: (cubic ft liquid per second is also called cusec)

Example 7

A painter can paint 250 m^2 wall in 8 hours. How much time will be required to paint 3000m^2

Solution

Area of the wall $= 250\ m^2$

Time taken $= 8$ hours

Time required for painting 1 m^2 $= \frac{8}{250} = 0.032$ hour

Time required for painting 3000 m^2 of the wall $= 0.032 \times 3000 = 96$ hours. 96 hours will be required to paint 3000 m^2 of the wall.

Example 8

The capacity of a car tank is 90 litres and in one full tank it can cover 1260 km. How much this car can travel if there are 5 litres in reserve fuel?

Solution

Distance covered $= 1260$ km

Fuel used $= 90$ litres

Distance covered in 1 litre of fuel $= \frac{1260}{90} = 14$ km / litre

Distance covered in 5 litres of fuel $= 14 \times 5 = 70$.

Guided Practice

Awais goes on a 30 mile bike ride every Saturday. He rides the distance in 4 hours. At this rate, how far can he ride in 6 hours?

How are Time and Distance related?

The time and distance are related with each other as speed. If a car covers 90 km in 1 hour, we say that its speed is 90 km per hour or 90 km/h. The formula is

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Example 9

A missile hit a 3000 km distance target in 45 minutes. Find the speed of the missile in km/h.

Solution

Distance = 3000 km

Time = 45 minutes

As we need speed in km/h so first we convert minutes into hours.

$$\text{Time in hours} = \frac{45}{60}$$

$$\text{Time} = 0.75 \text{ hours.}$$

$$\text{Speed} = ?$$

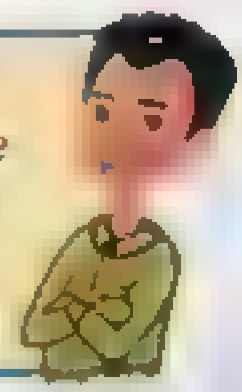
As $$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

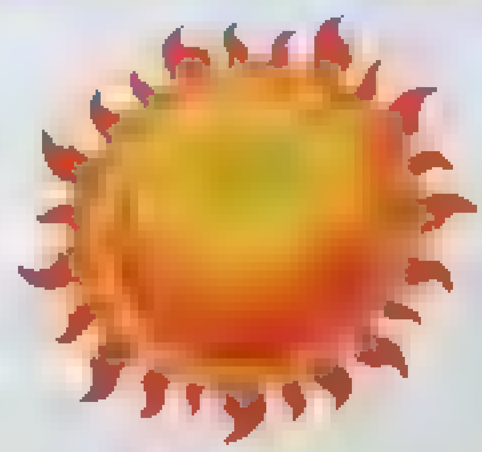
$$\therefore \quad \text{Speed} = \frac{3000}{0.75}$$

$$= 4000 \text{ km/h}$$

The speed of the missile is 4000 km/h

Do you know?
If you drive to the Sun at 55mph, it would take you about 193 years





6.3 Conversion of units of speed

How to convert Units of speed (kilometer per hour into meter per second and vice versa)

To convert m/s into km/h we multiply the speed by $\frac{3600}{1000}$

To convert km/h into m/s we multiply the speed by $\frac{1000}{3600}$

Example 10

A car is moving with a speed of 90 km/hour. What will be its speed in metre per second (m/s).

Solution Speed of the in km/h = 90 km/h

Speed in m/s =?

speed of the car in meter per second (m/s)

so

$$= \frac{90 \times 1000}{3600} = 25/ms$$

Example 11

The speed of sound in air is about 340 m/s. Find the speed in km/h.

Solution Speed of the sound in m/s = 340 m/s,

Speed of the sound on km/h= ?

So, speed of the sound in km/h $= \frac{340 \times 3600}{1000}$

$=1224/kmh$

Exercise 6.2

1. A machine can fill 300 bottles in 4 hours. How much time will be required for six such machines to fill 9000 bottles?
2. A tube well can suck 100 litres per minute. How much water can it draw per hour?
3. A jet fighter is flying at 594 m/s. Show its speed in km/h.
4. The speed of sound at 25 C° is about 340 m/s. Convert this speed in km/h.
5. A bullet train travels at a speed of 450 km/h. Convert this speed into m/s.
6. When a paratrooper jumps from the aircraft before opening the parachute, its speed becomes 50 m/s in 5 seconds. What will be this speed in km/h.
7. The fastest bowling speed of Shoaib Akhtar is 160 km/h. show this speed in m/s.
8. A cheetah runs at 90 km/h for 50 seconds. How much distance will it cover?



Tidbit

Unit Rate is a comparison of a number to one in different units. It is written as fraction. You **divide so simplify** and **always include units** in your answer.

- 120 students in 4 classrooms

$$\frac{120 \text{ students}}{4 \text{ classrooms}} \quad \frac{\div 4}{\div 4} \quad \frac{30 \text{ students}}{1 \text{ classroom}}$$

- 29 grams per cubic centimeter

$$\frac{29 \text{ grams}}{1 cm^3}$$

Unit rate is rate that is reduced to **1unit**

REVIEW EXERCISE 6

1. Choose the correct answer

(i) If $a : b = 3 : 6$ and $b : c = 9 : 12$ then $a : b : c$ will be

a) $3 : 6 : 12$ b) $3 : 6 : 8$

c) $54 : 27 : 72$ d) $27 : 54 : 72$

(ii) $27 : 54$ can also be written as

a) $1 : 2$ b) $2 : 1$ c) 54×27 d) none of these

(iii) The ratio of an hour to a minute is

a) $1 : 60$ b) $60 : 1$ c) $1 : 3600$ d) none of these

(iv) On a line two supplementary angles are in the ratio of $5 : 1$. The two angles will be

a) 20 & 70 b) 150° & 30° c) 1200 & 60° d) non of these

(v) If one quantity increases and other decreases then the two quantities are in

a) direct proportion b) inverse proportion

c) continued proportion d) no proportion

2. Express the following ratios as continued ratios

(i) $a : b = 7 : 9$ and $b : c = 6 : 13$

(ii) $x : y = 2.7 : 5.4$ and $y : z = 6.3 : 9.45$

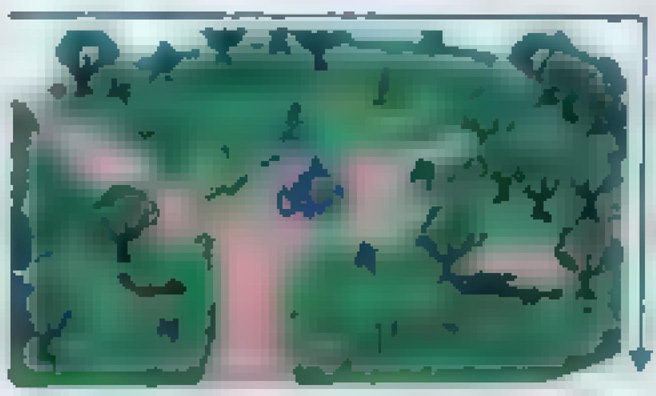
3. A typist can type 60 words per minute. How many typists will be required to type a book of 43,200 words in 6 hours?

4. It is estimated that a building is constructed by 10 masons in 9 months. If 4 new masons join them, how long the same building will take to complete.

5. A food store in a fort is sufficient for 60 days for 300 soldiers. If 200 soldiers are sent on a mission, for how many days the same food will be sufficient.

6. A photograph measuring 5.5 cm by 9 cm is enlarged in the ratio of 7 : 5. Find the new length and width of the picture.

7. Rehan rides bike along the edge of the park, looking the route shown in the diagram. It takes him 4 hours. What is his average speed?



Glossary

- **Ratio.** A comparison of two quantities having the same unit of measurement.
- **Continued Ratio** The comparison of more than two quantities is called continued ratio.
- **Direct proportion** If increase or decrease in one quantity results in the increase or decrease of other quantity they are said to be in the direct proportion e.g. demand and supply of goods, temperature and pressure of air, voltage and current etc.
- **Inverse Proportion** When increase in one quantity results in the decrease of the second quantity and vice versa they are said to be in the inverse proportion e.g. Supply and price of goods, pressure and volume of gas, Resistance and electric current etc.
- **Rate** Rate is the ratio between two different quantities for example kilometer and hour, meter and second etc.
- **Speed** The distance covered in m/s is known as speed.

$$\text{Speed} = \frac{\text{Distance covered}}{\text{time}}$$

Unit 7 Financial Arithmetic

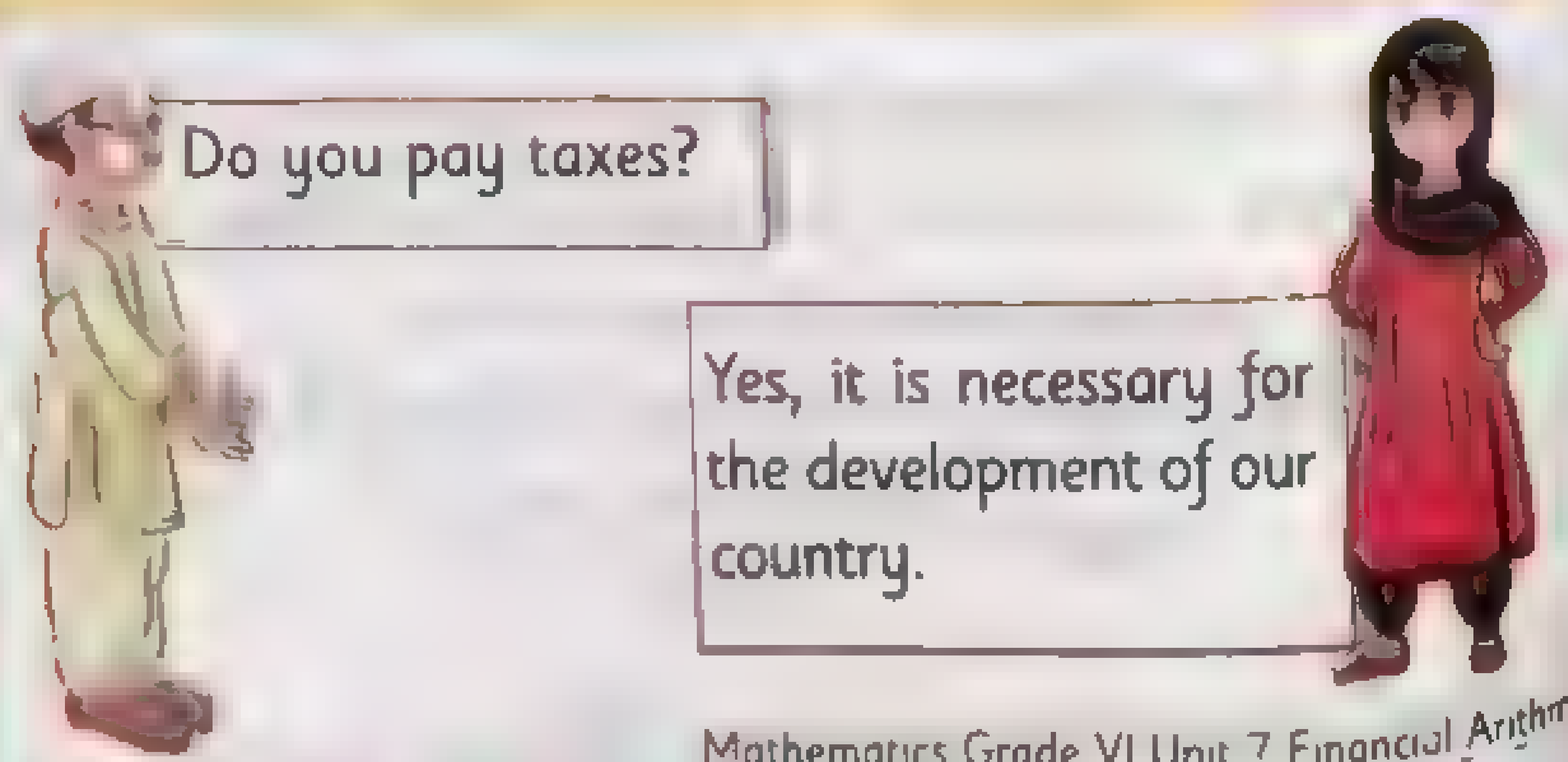
You'll Learn

- Explain property tax and general sales tax
- Solve tax-related problems.
- Explain profit and markup.
- Find the rate of profit per annum.
- Solve the real life problems involving profit/markup
- Define zakat and ushr.
- Solve the problems related to zakat and ushr.

It's Important

The money we pay in taxes goes to many places. In addition to paying the salaries of government workers, it help to support common resources, such as police and firefighters. Taxes fund public libraries and parks.

Simplify with the mony of zakat and ushr needy people of the society are supported.



7.1 Tax

Definition of Tax: The money collected from the citizens to run the country, by the government is called tax.

When human beings started living in a community they felt the need of a governing body or government. The job of a government is to work for the welfare of its citizens. Construction of building schools and hospitals establishing the forces for inner and outer security of the country are some of its functions. For all this the government requires money which is collected in the form of tax. Every citizen should pay the tax to make the country strong. There are a number of taxes, some of which are,

(i). Property tax

(ii). General sales tax (GST)

7.1.1 Property tax

A tax imposed on the real estate, property or building in an urban area is called property tax.

7.1.2 Taxable Area and Tax Rates

Immovable (land) property situated in an urban area measuring at least 1 Kanal or 500 square yards is called a taxable property. The rate of tax per annum (year) is,

(i). Where the value of immovable property is recorded, it is 6% of the total value.

(ii) Where the value of immovable property is not recorded it is Rs. 50 per square yard.

(iii). Residential flats are also charged @ 8% or Rs. 60 per square yard for both recorded or unrecorded value respectively, if the minimum rea is 1800 sq feet.

Example 1

A house of 2 kanal is situated in the cantonment. The worth of land is 2 million rupees per kanal. Calculate the property tax.

Solution Total land = 2 Kanal

Worth of land per kanal = 2 million

Worth of 2 kanal = 2×2 million = 4 million (4000000).

Rate of property tax = 6% or $\frac{6}{100}$

Tax for 2 kanal = $4,000,000 \times \frac{6}{100} = 240,000$/-

Rs. 240, 000/- per annum tax will be charged on this property.

Example 2

Ruaf constructed 4 flats each of 1600 sq feet in an urban area. The worth of land is not recorded. Calculate the property tax due for 3 years.

Solution

Area of one flat	= 1600 Sq feet
Rate of tax	= Rs. 50 per Sq feet
Property tax of 1 flat for one year	= $1600 \times 50 = 80,000$/-
Property tax for 3 years	= 3×80000/-
	= 240, 000/-
Property tax for the 4 flats for 3 years	= $4 \times 240,000$/-
	= 960, 000/-

Rs. 960,000 will be paid as property tax of four flats for three years.

7.2 General Sales Tax (G.S.T)

General Sales Tax is collected when a product is sold to its final consumer. The purpose of GST is to bring a large number of people in the tax network. Its rate is 16.5%.

Note

The rate of Tax is subjected to the government policies

Example 3

A ghee mill manufactures Ghee and oil. The cost of production per kg of ghee and per litre of oil is Rs.115 and Rs. 125 respectively. What will be the selling price after including G.S.T.

Solution

The cost of production of ghee per kg $= \text{Rs. } 115$

Rate of G.S.T. $= 16.5\%$ or $\frac{16.5}{100}$

G.S.T on 1 kg ghee $= 115 \times \frac{165}{1000}$

$= \text{Rs. } 18.97$

The price of 1 kg ghee after including GST $= 115 + 18.97$

$= \text{Rs. } 133.97$

The cost of production of oil per litre $= \text{Rs. } 125$

Rate of GST $= 16.5\%$ or $\frac{16.5}{100}$

GST on 1 litre oil $= 125 \times \frac{165}{1000} = \text{Rs. } 20.62$

The price of 1 litre oil after including GST $= 125 + 20.62 = \text{Rs. } 145.62$.

The selling price of ghee per kg and oil per litre will be Rs. 133.97 and Rs. 145.62 respectively.

Example 4

The retail price of a doll is Rs. 116.5 if 16.5% GST is included in this price. What will be its price without GST: (cost price)

Solution

Price of doll including GST = Rs. 116.5

Rate of GST = 16.5% or $\frac{16.5}{100}$

Cost price (without GST) = ?

The sales price (price including GST) is determined as

Cost price + 16.5% cost price = Sales price

$$CP + \frac{16.5}{100} CP = SP$$

$$CP + 0.165\ CP = 116.5$$

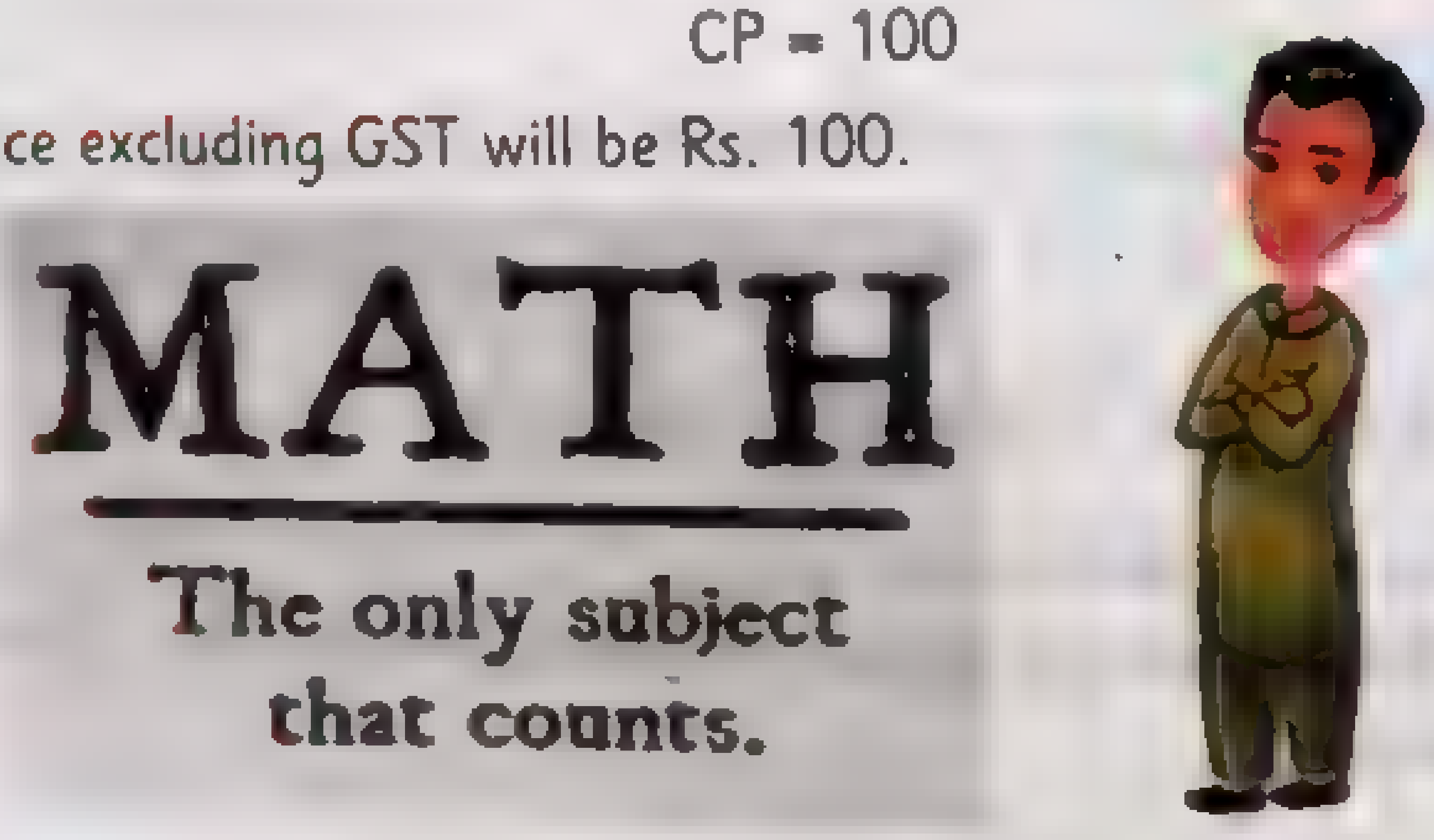
$$CP\ (1 + 0.165) = 116.5 \text{ \{Taking CP as common)}$$

$$CP\ (1.165) = 116.5$$

$$CP = \frac{116.5}{1.165} = \frac{1165 \times 1000}{1165 \times 10}$$

$$CP = 100$$

Cost price excluding GST will be Rs. 100.



Exercise 7.1

1. Naeem has a 2-kanal house in Hayatabad. How much property tax he will have to pay per year at the rate of 6%. If the value per kanal is 3 million rupees.

2. Ajab Khan is living in a flat of 7 marlas in Army flats. The property tax not paid for five years. How much tax is due at the rate of Rs. 50 per square yard (one marla = 272 sq feet)

3. A sugar mill is making sugar at the cost of Rs. 50 per kg. What will be its sales price after including GST at 16.5%?

4. The airfare from Peshawar to Karachi of an airline is Rs. 5700. What will be the selling price of this ticket if Rs. 1500 airport tax and GST at the rate of 16.5% are included?

5. Sales price of a sewing machine is Rs. 4248 (including 16.5% GST). What will be its price without GST?

6. The sale price of a computer is 30,000/- If the government waives of 16.5% GST on this item, what will be the new sale price?

7.3 Profits and markup

(a) Profit

The difference between selling price and cost price is called profit, if it is a positive value. Mathematically it can be written as

Profit = Sales Price (SP) – cost price (CP)

Profit = SP – CP

It is the simplest type of business.

However sometimes we need percent profit

$$\%\text{profit} = \frac{\text{profit}}{\text{cost price}} \times 100$$

Example 5

A shopkeeper purchased five television sets for Rs. 150,000. He sold each set for Rs. 35, 000. How much profit or loss he made in this deal?

Solution

Cost of 5 television set = Rs. 150, 000

Cost of 1 television set = Rs. $\frac{150000}{5}$

= 30, 000

Sale price of 1 television = 35000

Profit = sale price – cost price

= 35000 – 30000

= Rs. 5000

In the sale of the 5 sets he got = 5 × 5000

= Rs. 25000 profit.

(b) Markup

To meet the expenses and earn a profit, a business must sell a product at a higher price. A markup is an amount added to a cost price to calculate the selling price.

cost + markup = selling price

$$M = \frac{RPT}{100}$$

Example 6

Salman bought a bike for Rs. 15,000 on installments at the markup rate of 12% per annum. Find the selling price of the television if time period is 3 years

Solution

Cost price (P) = Rs. 15,000

Markup rate = 12% per annum

Time period (T) = 3 years

Price of the bike = ?

Using the formula,

$$\text{Markup} = \frac{RPT}{100} = \frac{12 \times 15{,}000 \times 3}{100} = \text{Rs. } 5{,}400$$

Price of the bike = cost price + markup

= Rs. 15,000 + Rs. 5,400 = Rs. 20,400

Guided Practice

Fariha bought an oven for Rs. 12000 on installments at the markup rate of 15% per annum find the selling price of the oven if time period is 4 years

Exercise 7.2

1. Daud purchased a house for 1 million and sold for 1.1 million. How much profit did he make?
2. Azam bought 30 dozen eggs of which 30 eggs were rotten. If he bought it for Rs. 1800, what should be the selling price of each egg to get a profit of 1 rupee per egg?
3. Alamgir purchased a car for Rs. 370,000. He spent Rs. 20,000 on its repair and decoration. He sold the car for Rs. 385,000 How much profit or loss did he make?
4. A book seller purchased 1000 books for Rs. 75000. Due to dampness and termites 84 books got destroyed. What should be the selling price of each book to earn a profit of Rs. 25 per book?
5. The cost of a burger is Rs. 90 and it is sold for Rs. 110. What is the percentage of the profit?
6. Find the markup on a bike whose price is Rs. 45,000 for 73 days at the rate of 10% per annum.
7. The markup on a principal amount is Rs. 820 for 6 months at the rate of 12.5% per annum. Calculate the principal amount.

"Our prices are drastically marked down from their drastic markup!"

7.4 Zakat

Zakat is an amount which becomes due at the rate of 2.5% of the savings for a Muslim who has at least specific amount of gold or silver for one complete year.

This specific amount is called Nisab. According to Islamic teaching Nisab is equal 7.5 tola of gold or 52 tola of silver

Example 7

Najma has 20 tola gold and 120 tola silver. How much Zakat will she has to pay after one year. The price per tola of gold and silver is Rs. 60,000 and Rs 900 respectively.

Solution

Quantity of gold = 20 tola

Value of gold = $20 \times 60,000$

= Rs. 1,200,000

Quantity of silver = 120 tola

Value of silver = 120×900

= Rs. 108, 000

Total Value = 1,200,000 + 108,000

= Rs. 1,308, 000

Rate of Zakat = 2.5 % or $\frac{25}{100}$

Payable amount of zakat = $\frac{25}{100} \times 1,308,000$

= Rs. 327, 000

Example 8

Neelam paid Rs. 11,000 as Zakat on gold. How much gold she has?
(Price of gold Rs. 60,000 per tola)

Solution

Let the value of gold $= x$ rupees

Rate of Zakat $= 2.5\%$ or $\frac{25}{1000}$

Zakat paid $=$ Rs. 11,000

$$2.5\% \text{ of } x = 11,000$$

$$\frac{25}{1000} \times x = 11,000$$

$$x = 11000 \times \frac{1000}{25}$$

$$x = \frac{11,000,000}{25}$$

$$x = 440,000$$

The value of gold $=$ Rs. 440,000

Quantity of gold $= \frac{\text{total value}}{\text{price of 1 tola gold}}$

$$= \frac{440,000}{60,000}$$

$$= 7.33 \text{tola}$$

Neelam has 7.33 tola gold

Remember Zakat is one of the five pillars of Islam.

7.5 Ushr

Ushr is Zakat on agricultural products e.g crops, fruit, vegetable etc. Ushr is paid after every harvest. Its rate is 5% of yield where labour is employed for digging wells, canals and bringing water from a distance and 10% where no manual labour is needed for irrigation. Ushr can be paid both in cash or in kind.

Example 9

Tahir obtained a yield of 50,000 kg of strawberry from his tube well irrigated land. How much Ushr will he pay if it was sold for Rs. 25 per kg?

Solution

Total yield of strawberry	= 50,000 kg
Sale price/kg	= Rs. 25
Total sale price	$= 50000 \times 25$
	= Rs. 1, 250,000
Rate of Ushr	= 5% (as land is tube well irrigated)
Amount of Ushr payable	$= 1,250,000 \times \frac{5}{100}$
	= Rs. 62500

Tahir will pay Rs. 62500 as Ushr.

Exercise 7.3

1. Shehla had 15 tola gold and 140 tola silver for more than one year. How much Zakat will she have to pay if the market value of gold is Rs 60,000 and silver is Rs. 1200 per tola.
2. Usman had some jewellery and cash. He paid Rs. 25000 as Zakat. How much was his savings.
3. Irum has gold worth Rs. 280,000 and silver jewellery worth Rs. 62,400 How much zakat will she have to pay?
4. Ali paid Rs. 49500 as Zakat. If he has 40 tola gold, how much cash he has? (Gold is Rs. 40,000 per tola).
5. Yunas has 30 jarib barani land. He obtained 1000 kg wheat per jarib. How much wheat as Ushr will he have to pay.
6. Hayat Khan cultivated sugarcane on 10 jarib canal irrigated land. He sold the sugarcane for Rs. 120,500. How much Ushr will he have to pay

REVIEW EXERCISE 7

1. Colour the correct answer.

i. Ushr is payable after

a. One year   b. Six months   c. Every harvest   d. All of these

ii. The rate of property tax for recorded value is

a. 6%   b. 20%   c. 18%   d. 0 2%

iii. Property tax is levied on

a. Urban property   b. Rural property
c. None of these   d. Both urban and rural properties

iv. The rate of Ushr for barani land is

a. 5%   b. 10%
c. 20%   d. 2%

v. GST is levied on

- a. Factory owner
- b. Whole saler
- c. Final consumer
- d. All of these

vi. The rate of GST is

- a. 20 %
- b. 2%
- c. 108 %
- d. 16.5%

vii. The rate of Ushr for canal irrigated land is

- a. 5%
- b. 10%
- c. 20%
- d. 2%

viii. If the cost price is more than the sale price then the seller has made

- a. Profit
- b. Loss
- c. Neither profit nor loss
- d. All of these

ix. Profit can be calculated in

- a. Match factory
- b. Restaurant
- c. Banks
- d. All of these

x. 20% of 500 is

- a. 1000
- b. 100
- c. 10
- d. None of these

xi. The payment of Zakat becomes compulsory on a person if 7.5 tola gold is in his or her possession for

- a. One month
- b. One week
- c. One year
- d. 355 days

Ali owns a house worth 7.2 million rupees. How much property tax will he have to pay after one year if the rate of property tax is 2%.

3. Anees is living in army flats. He paid Rs. 90,000 as property tax. What is the are of the flat if it is charged at Rs. 50 per square yard

4. The cost price of a tube well is Rs. 50,000. What will be its selling price after adding 16.5% GST?

5. A shopkeeper purchased 200 tube lights for Rs. 7600. He sold each one for Rs 45 How much profit he obtained on the sale of tube lights. Also calculate percentage profit of per tube light.

Glossary

- [illegible] The money collected from the citizens to run the country, by the government is called tax.
- [illegible] The tax levied upon buildings and residential areas is called property tax.
- [illegible] The tax on items purchased is called general sales tax.
- [illegible] An amount payable by every adult muslim if he or she has at least an amount of wealth called Nisab.
- [illegible] 7.5 tola gold (87 gram) or 52 tola (603 gram) silver is the nisab of gold and silver. No Zakat is due on wealth until one year passes.
- [illegible] A type of Zakat payable on agricultural products after each harvest
- [illegible] The Muslim having the required quantity of Nisab. for such a muslim Zakat becomes obligatory.

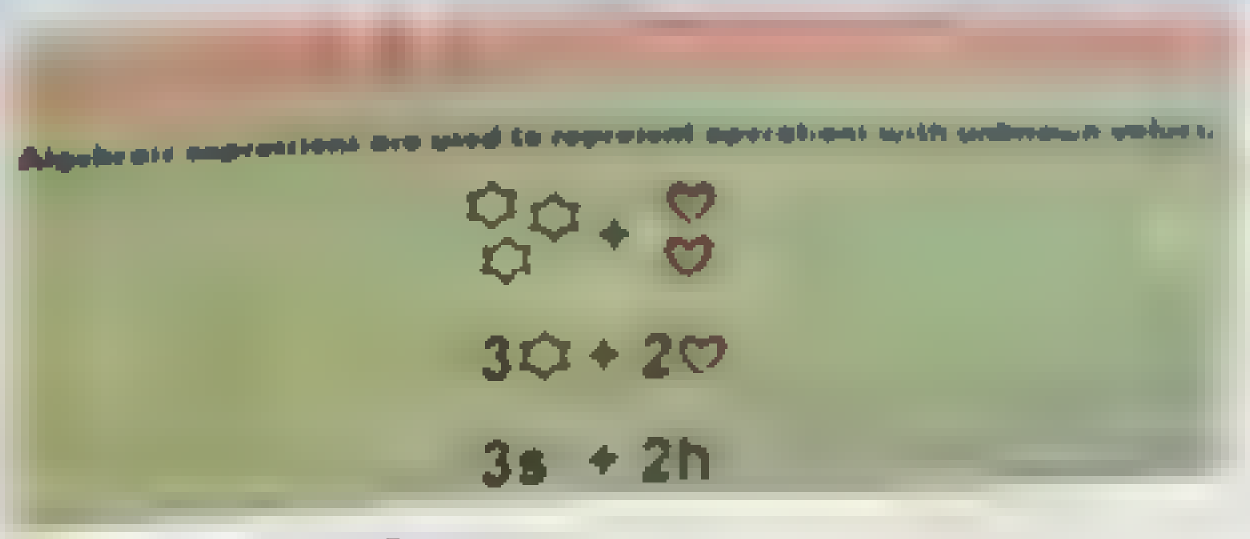
ACTIVITY

Calculate the GST for 10 commodities used in your home.
Make a list of it.

Unit 8 Algebraic Expressions

What You'll Learn

- Define a constant as a symbol having a fixed numerical value
- Recall variable as a quantity which can take various numerical values
- Recall literal as an unknown number represented by an alphabet
- Recall algebraic expression as a combination of constants and variables connected by the signs of fundamental operations
- Define polynomial as an algebraic expression in which the powers of variables are all whole numbers
- Identify a monomial, a binomial and a trinomial as a polynomial having one term, two terms or a three terms respectively
- Add two or more polynomials.
- Subtract a polynomial from another polynomial
- Find the product of
  - monomial with monomial,
  - monomial with binomial/trinomial,
  - binomials with binomial/trinomial
- Simplify algebraic expressions involving addition, subtraction [illegible]
- Recognize and verify the algebraic identities
  - $(x + a)(x + b) = x^2 + (a + b)x + ab$,
  - $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$,
  - $(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$,
  - $a^2 - b^2 = (a - b)(a + b)$
- Factorize an algebraic expression (using algebraic identities)
- Factorize an algebraic expression (making groups)



8.1 Algebraic Expressions

Recal that,

8.1.1 Constant

8.1.2 Variable

8.1.3 Literals

8.1.4 Algebraic expression

8.1.5 Polynomials

A polynomial of degree n in one [illegible] has terms of the form ax [illegible] a is constant and n is a whole num[illegible]

The parts of a polynomial separated [illegible] terms

For example [illegible]+5, 3[illegible]+2[illegible]+5 [illegible] 2[illegible]+4 [illegible]

[illegible] as $\frac{1}{x}$, $2x-\frac{1}{x^2}$ and $\sqrt{x}+2$ are not polynomials as the [illegible] of every variable is not a positive integer.

The degree of polynomial is the highest degree of its te[illegible]

A polynomial may also be in two or three [illegible] $x^2+3x^2y+3xy^2+y^3$ are polynomials in two [illegible] is a polynomial in three variables [illegible]

[illegible] 1

Find the degree of each polynomial

Polynomial	[illegible]	Degree of Each Term	Degree of Polynomial
[illegible]	[illegible]	[illegible]	3
[illegible]	[illegible]	4, 2, 0	4
$3a+7ab-2a^2b+16$	3a, 7ab, [illegible] 16	1, 2, 3, [illegible]	3

Remember

Polynomial means an [illegible] terms "[illegible]" means "many" and "nomial" means "terms"

8.1.6 Kinds of polynomial according to number of terms

- **Monomial:** A polynomial having only one term is called a monomial. For example, $P(x) = 3$, $P(x) = 3x$ are monomials.
- **Binomials:** A polynomial having two terms is called binomial. For example, $P(x) = x+2$, $P(x) = 3y-7$ are binomials.
- **Trinomial:** A polynomial having three terms is called trinomial. For example x^2+2x+1, y^2-2y+4 are trinomials.

Monomial	Binomial	Trinomial
7	$3+4y$	$x+y+z$
$13n$	$2a+3c$	p^2+5p+4
$5z^3$	$6x^2+3xy$	$a^2-2ab-b^2$
$4ab^3c^4$	$7pqr+pq^2$	$3v^2-2w+ab^3$

Example 2

State whether each expression is a polynomials. If it is a polynomial, identify it as a monomial, binomial, or trinomial.

Expression	Polynomial	Monomial, Binomial or Trinomial
$2x-3yz$	Yes	binomial
$8n^3+5n^{-2}$	No	
-8	Yes	monomial
$4a^2+5a+a+9$	Yes The Expression simplifies to $4a^2+6a+9$	trinomial

Exercise 8.1

1. Write the constants and variables of the following algebraic expressions

$x-3$ $\qquad$ $y+5$ $\qquad$ $x^2+2xy+4$

$2x-25+y^2$ $\qquad$ z^2-5z+8 $\qquad$ $2t^2+3t+7$

2. Find which of the following expressions are polynomial

(i) $2x+1$ $\qquad$ (ii) $y-\frac{1}{y}$ $\qquad$ (iii) 3

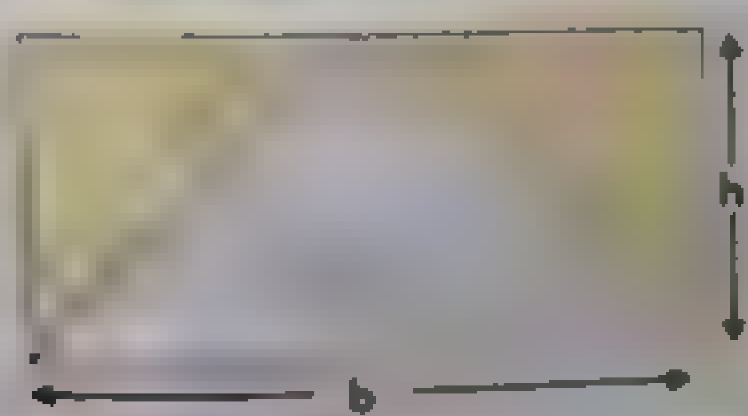
(iv) $3x^2-2x+5$ $\qquad$ (v) $\frac{z^2+5}{z}$ $\qquad$ (vi) $\frac{1}{x}-\frac{2}{x+1}$

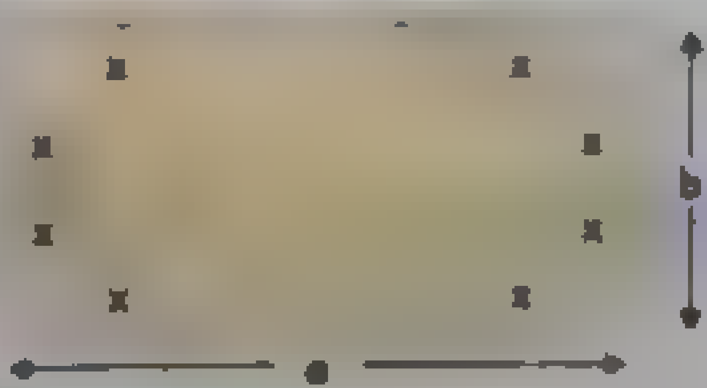
3. Write down the names of the following polynomials

$2x$ $\qquad$ $8x+4$ $\qquad$ 4 $\qquad$ x^2-y^2 $\qquad$ $x-y+z$

x^3-1 $\qquad$ x^2+5x+6 $\qquad$ $-7x$ $\qquad$ $2xy+3yz$ $\qquad$ $\frac{1}{2}x+\frac{1}{5}y+3$

4. Write a polynomial to represent the area of each shaded region





8.2 Operations on Polynomials

8.2.1 Addition of two or more polynomials

Addition of polynomials can be done by horizontal and vertical methods

Method 1 Vertical Method

- Arrange the polynomials in descending order
- Write the like terms in the same column
- Add the co-efficients of like terms

Method 2 Horizontal Method

- Use brackets and group the like terms
- Add the co-efficents of the like terms
- These two methods are explained with the help of the following examples

Example 3 Add $2x^2+3x+5$ and $5x^2-4x-2$

Solution

Method 1 Vertical Method

$$\begin{array}{r} 2x^2+3x+5 \\ 5x^2-4x-2 \\ \hline 7x^2-x+3 \end{array} \qquad \begin{array}{l} 2+5=7 \\ 3-4=-1 \\ 5-2=3 \end{array}$$

Method 2 Horizontal Method

$(2x^2+3x+5)+(5x^2-4x-2)$

$=2x^2+5x^2+3x-4x+5-2$

$=7x^2-x+3$

Example 4 Add $3x^3+x-2$, $-5x^3+7$ and $4x^2-6x+5$

Solution

Method 1 Vertical Method

$$\begin{array}{r} 3x^3+0x^2+x-2 \\ -5x^3+0x^2+0x+7 \\ 0x^3+4x^2-6x+5 \\ \hline -2x^3+4x^2-5x+10 \end{array}$$

Method 2 Horizontal Method

$(3x^3+x-2)+(-5x^3+7)+(4x^2-6x+5)$

$=3x^3-5x^3+4x^2+x-6x-2+7+5$

$=-2x^3+4x^2-5x+10$

8.2.2 Subtraction of a Polynomial from another polynomial

Subtraction of a Polynomials can also be done by vertical and horizontal methods.

Method 1 Vertical Method

- Arrange the polynomials in descending order
- Write the like terms in the same column
- Change the sign of each term in the second polynomial
- Add the co-efficient of two polynomials

Method 2 Horizontal Method

- Use brackets then change the sign of each term to be subtracted from the given polynomial
- Group the like terms
- Add the co-efficient of the like term
- Both the methods are explained with the help of the following examples

Guided Practice

i. $(5x^2-2)+(x^2-x+11)+(2x^2-5x+7)$ ii. $(2x^3-3x+13)-(6x^2-5x)+(2x^3-x^2-8x+4)$

Example 5 Subtract $2x^2-3x+4$ from $4x^2+5x-6$

Solution

Vertical Method		Horizontal Method
$4x^2+5x-6$	$4-2=2$	$4x^2+5x-6-(2x^2-3x+4)$
$\pm 2x^2 \mp 3x \pm 4$	$5+3=8$	$=4x^2+5x-6-2x^2+3x-4$
$2x^2+8x-10$	$-6-4=-10$	$=(4x^2-2x^2)+(5x+3x)+(-6-4)$
		$=2x^2+8x-10$

Guided Practice

1. Subtract $3x-2x^2-5$ from $5x^2+2x-9$
2. $(x^3-7x+4x^2-2)-(2x^2-9x+4)$

Exercise 8.2

Tidbit: We can only add like terms

1. Add the following polynomials.

   (i) x^2+3x+4 , $3x^2-x+2$

   (ii) $3x^3-2x^2+4x+5$, x^3+x^2-2x-9

   (iii) $-x+5x^2+4$, $10-x+2x^2$, $4x^2-3x+5$

   (iv) $4y+2-3y^2+2y^3$, $4y^2-5y+7$, $2y^2-5$

   (v) $p+2q-3r$, $4p-3q+4r$

2. Subtract the second polynomial from the first polynomial

   (i) x^2+2x+4 , $4x^2+6x-5$

   (ii) x^3-x^2+x-5 , $4x^2+6x+8$

   (iii) $a+3b-c$, $3a-4b+2c$

3. Add the following

   (i)
   $$\begin{array}{r} 5y^3-4y^2-3y+4 \\ -4y^3-2y^2+6y+15 \\ \hline \end{array}$$

   (ii)
   $$\begin{array}{r} 3x^4-6x^3y+7x^2+10 \\ -10x^4+0x^3y+2x^2+5 \\ \hline \end{array}$$

8.2.3 Product of Polynomials

(a) Product of monomials with monomials

Example 6 Find the product of $(-3x^2y)(2xy^2)(-5x^3y^2)$.

Solution

$$(-3x^2y)(2xy^2)(-5x^3y^2)$$
$$=(-3)(2)(-5)(x^2y)(xy^2)(x^3y^2)$$
$$=30x^{2+1+3}y^{1+2+2}$$
$$=30x^6y^5$$

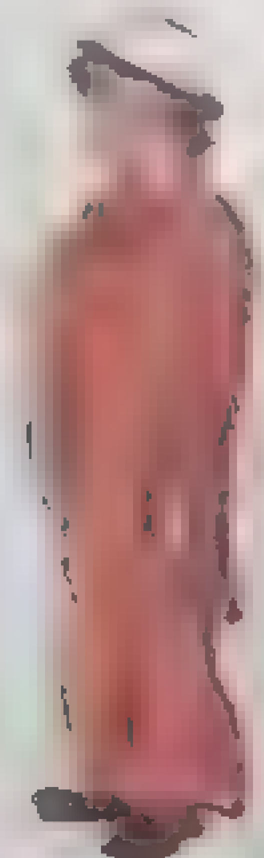
(b) Product of a monomial with a binomial/trinomial

Here we shall use distributive property of multiplication.,

Example 7 Find the product of $2x^2(3x+4y)$.

Solution

$$2x^2(3x+4y)$$
$$=(2x^2)(3x)+(2x^2)(4y)$$
$$=6x^{2+1}+8x^2y=6x^3+8x^2y$$



Find $-2x^2(3x^2-7x+10)$

➡ Method 1 Vertical Method

$$\begin{array}{r} 3x^2-7x+10 \\ \times \qquad -2x^2 \\ \hline -6x^4+14x^3-20x^2 \end{array}$$

Distributive Property

Multiply

➡ Method 2 Horinzontal Method

$2x^2(3x^2-7x+10)$	
$=-2x^2(3x^2)-(-2x^2)(7x)+(-2x^2)(10)$	Distributive Property
$=-6x^4-(-14x^3)+(-20x^2)$	Multiply
$=6x^4+14x^3-20x^2)$	Simplify

(c) Product of a binomial with a binomial/trinomial

In order to find the product of a binomial with another binomial/trinomial multiply each term of the first polynomial with all the terms of the 2nd polynomial then add the co-efficents of the like terms. This method is explained with the help of the following examples.

Example 8 Multiply $x+1$ by $2x+3$

Solution

$$\begin{array}{r} 2x+3 \\ x+1 \\ \hline 2x^2+3x \\ +2x+3 \\ \hline 2x^2+5x+3 \end{array}$$

Example 9 Simplify $(x+2)(x^3-3x+4)$

Solution Horizontal Method

$(x+2)(x^3-3x+4)$

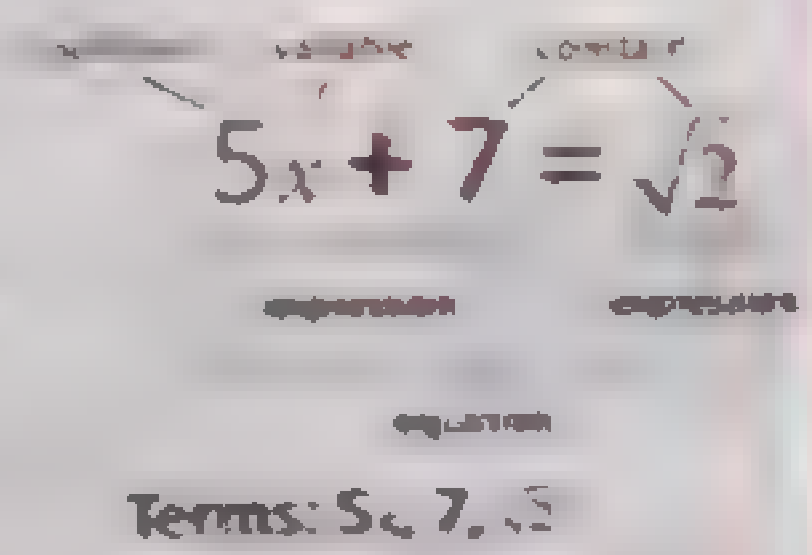
$=x(x^3-3x+4)+2(x^3-3x+4)$

$=x^4-3x^2+4x+2x^3-6x+8$

$=x^4+2x^3-3x^2+(4x-6x)+8$

$=x^4+2x^3-3x^2+(4-6)x+8$

$=x^4+2x^3-3x^2-2x+8$



Guided Practice

Find each product

i. $-3y(5y+2)$ ii. $2x(4a^4-3ax+6x^2)$

Exercise 8.3

Simplify.

[illegible]

8.2.4 Simplification of algebraic expressions involving addition, subtraction and multiplication of Polynomials.

Example 10 Find $(x+3)(x+2)$

- Method-1: Vertical Method

Step I: Multiply by 2

$$\begin{array}{r} x+3 \\ \times\ x+2 \\ \hline 2x+6 \end{array}$$

Step II: Multiply by x

$$\begin{array}{r} x+3 \\ \times\ x+2 \\ \hline 2x+6 \\ x^2+3x \\ \hline \end{array}$$

Step III: Add like terms

$$\begin{array}{r} x+3 \\ \times\ x+2 \\ \hline 2x+6 \\ x^2+3x \\ \hline x^2+5x+6 \end{array}$$

- Method-2: Horizontal Method

$$\begin{aligned}(x+3)(x+2) &= x(x+2)+3(x+2)\\ &= x(x)+x(2)+3(x)+3(2)\\ &= x^2+2x+3x+6\\ &= x^2+5x+6\end{aligned}$$

Example 11 Simplify $4(3d^2+5d)-d(d^2-7d+12)$

$$\begin{aligned}&4(3d^2+5d)-d(d^2-7d+12)\\ &= 4(3d^2)+4(5d)-d(d^2)-(-d)(7d)+(-d)(12)\\ &= 12d^2+20d+(-d^3)-(-7d^2)+(-12d)\\ &= 12d^2+20d-d^2+7d^2-12d\\ &= -d^3+(12d^2+7d^2)+(20d-12d)\\ &= -d^3+19d^2+8d\end{aligned}$$

Example 12

Simplify [illegible]

Solution

$$[illegible]$$
$$=x^2-xy-2yx-2y^2+2xy$$
$$=x^2-2y^2+(-xy-2yx+2xy)$$
$$=x^2-2y^2-xy$$

Example 13

Simplify $x(x^2+y^2-1)+y(y^2-x^2+1)-(x-y)(x^2+y^2)$

Solution

$$x(x^2+y^2-1)+y(y^2-x^2+1)-(x-y)(x^2+y^2)$$
$$= x^3+xy^2-x+y^3-yx^2+y-(x^3+xy^2-yx^2-y^3)$$
$$= x^3+xy^2-x+y^3-yx^2+y-x^3-xy^2+yx^2+y^3$$
$$= y^3+y^3-x+y$$
$$= 2y^3-x+y$$

Exercise 8.4

Simplify the following expressions.

(i) $x(x-2y)+y(3x+2y)-(x^2+y^2)$ (ii) $(x+y)(x-y)-2xy$

(iii) $ab(a+b)-(a^3+b^3+a^2b+ab^2)$ (iv) $(2a+3b)(2a-3b)+(a^2-b^2)$

(v) $2x^4-3x^3+(2x-1)(x^2+5x+11)$ (vi) $3-x^2+x(x+1)$

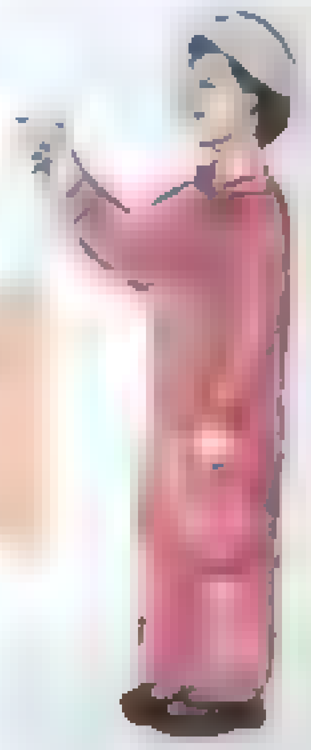
(vii) $(y^2-5)+(9+5y^3-y)(2y^2-7y+4)$

Guided Practice

Simplify.

i) $4y(y^2-8y+6)-3(2y^3-5y^2+2)$ ii) $4(x+2)+3x(5x^2+2x-6)-5(3x^2-4x)$

8.3 Algebraic Identities



Now we shall consider some important algebraic identities which make the process of multiplication easy and short.

Identity No. 1 $(x+a)(x+b)=x^2+(a+b)x+ab$

Proof: L.H.S $= (x+a)(x+b)$

$= x(x+b)+a(x+b)$

$= x^2+xb+ax+ab$

$= x^2+ax+bx+ab$

$= x^2+(a+b)x+ab$

$=$ R.H.S

Thus $\boxed{(x+a)(x+b)=x^2+(a+b)x+ab}$

Example 14 Simplify with the help of algebraic identity.

(i). $(x+5)(x+3)$ (ii). $(x-2)(x+7)$

Solution

(i) $(x+5)(x+3) = x^2+(5+3)x+5x3$

$= x^2+8x+15$

(ii) $(x-2)(x+7) = x^2+(-2+7)x+(-2)(7)$

$= x^2+5x-14$

Guided Practice

Find each product.

i. $(x+5)\,(x+3)$ ii. $(x-2)\,(x+7)$ iii. $(x-4)\,(x+5)$

Identity No. 2 $(a+b)^2 = (a+b)(a+b) = a^2+2ab+b^2$

Proof: $\quad$ L.H.S $= (a+b)^2$

$= (a+b)(a+b)$

$= a(a+b)+b(a+b)$

$= a^2+ab+ba+b^2$

$= a^2+ab+ab+b^2$

$= a^2+2ab+b^2 =$ R.H.S

As $\quad$ L.H.S = R.H.S

Thus $\boxed{(a+b)^2=a^2+2ab+b^2}$ or (sum of two terms)^2

$= (1^{st}\text{term})^2 + 2(1^{st}\text{term} \times 2^{nd}\text{term}) + (2^{nd}\text{term})^2$

Example 15 Simplify with the help of algebraic identity.

(i). $(x+5)^2$ $\quad$ (ii). $(2x+3y)^2$

Solution (i) $(x+5)^2 = (x)^2+2(x)(5)+(5)^2 \quad = x^2+10x+25$

(ii) $(2x+3y)^2 = (2x)^2+2(2x)(3y)+(3y)^2 \quad =4x^2+12xy+9y^2$

Guided Practice

Find each product. i. $(3g+5)^2$ $\quad$ ii. $(2x+3)^2$ $\quad$ iii. $(4x+1)^2$

Identity No. 3 $(a-b)^2 = (a-b)(a-b) = a^2-2ab+b^2$

Proof: $\quad$ L.H.S $= (a-b)^2$

$= (a-b)(a-b)$

$= a(a-b)-b(a-b)$

$= a^2-ab-ba+b^2$

$= a^2-ab-ab+b^2 \quad \because ba = ab$

$= a^2-2ab+b^2 =$ R.H.S

As $\quad$ L.H.S = R.H.S

Thus $\boxed{(a-b)^2 = a^2-2ab+b^2}$ or (difference of two terms)^2

$= (1^{st}\text{term})^2-2(1^{st}\text{term})(2^{nd}\text{term}) + (2^{nd}\text{term})^2$

[illegible] **16** [illegible]

$(x-3)^2$ (ii). $(5x-y)$ [illegible]

Solution [illegible] $= x^2 - 2(x)(3) + (3)^2$ [illegible]

$= x^2-6x+9$

$(5x-4y)^2 = (5x)^2-2(5x)(4y)+(4y)^2$

$= 25x^2-40xy+16y^2$

Guided Practice

[illegible]

i. $(7-4y)^2$ ii. $(4-6h)$ [illegible]

Identity No. 4 [illegible]

Proof

$$\begin{aligned} LHS &= (a+b)(a-b) \\ &= a(a-b)+b(a-b) \\ &= a^2-ab+ba-b^2 \quad \because ba = ab \\ &= a^2-b^2 = RHS \end{aligned}$$

As $LHS = RHS$

Thus $a^2-b^2 = (a+b)(a-b)$

or

[illegible] of the difference of two terms = the product of their sum and their difference

Example (17) [illegible] the help of identity No 4

(i). x^2-25 (ii). $81y^2-64x^2$

Solution (i). $x^2-25=(x)^2-(5)^2$

$=(x+5)(x-5)$

(ii). $81y^2-64x^2=(9y)^2-(8x)^2$

$=(9y+8x)(9y-8x)$

Guided Practice

Simplify; i. $9n^2-4$ ii. $121u^2-64w^4$

[illegible] 8.5

1. [illegible] concerned identity

(i) $(x+2)(x+5)$ (ii) $(x+3)(x-7)$

(iii) $(x+1)(x-3)-(x+4)(x+2)$

2. Expand by using suitable identity

(i) $(2a+3b)^2$ (ii) $\left(\frac{1}{2}x+3y\right)^2$

(iii) $(x-2y)^2$ (iv) $\left(\frac{3}{2}a-\frac{5}{4}b\right)^2$

(v) $(3a+4b)^2-(2a-5b)^2$ (vi) $(2x-5y)^2+(3x+4y)^2$

3. [illegible] suitable identity

(i) [illegible]

(ii) [illegible]

(iii) [illegible]

(iv) [illegible]

8.4 Factorization of Algebraic Expressions

Writing an algebraic expression as the product of two or more algebraic expressions is called factorization e.g. $2x+6 = 2(x+3)$

This shows that $2(x+3)$ is the factorization of expression $2x+6$ whereas 2 and $x+3$ are called factors of the given expression. Now we shall consider some important cases of factorization

8.4.1 Factorization of the type $a^2 \pm 2ab + b^2$ and $a^2 - b^2$

This type of expression can be written in the form of perfect square and the factors become clear

We know that $a^2+2ab+b^2 = (a+b)(a+b) = (a+b)^2$

$$a^2-2ab+b^2 = (a-b)(a-b) = (a-b)^2$$

Example 18 Factorize $x^2+16x+64$

Solution

$x^2+16x+64$

$= (x)^2+(2)(x)(8)+(8)^2$ Using $a^2+2ab+b^2=(a+b)^2$

$= (x+8)^2$

$= (x+8)(x+8)$

Example 19 Factorize $45a^2-60a+20$

Solution

$45a^2-60a+20$

$= 5[9a^2-12a+4]$ Taking 5 as common

$= 5[(3a)^2-(2)(3a)(2)+(2)^2]$ Using $a^2-2ab+b^2=(a-b)^2$

$= 5(3a-2)^2$

$= 5(3a-2)(3a-2)$

Example 20 Factorize $25x^2-36y^2$

Solution

$$25x^2-36y^2 = (5x)^2-(6y)^2$$ [illegible]

$$= (5x+6y)(5x-6y)$$

Example 21 Factorize $5ab^2-125a$

Solution

$$5ab^2-125a = 5a(b^2-25)$$

$$= 5a[(b)^2-(5)^2] \quad \text{(taking 5a as common [illegible])}$$

$$= 5a(b+5)(b-5) \quad \text{(using } a^2+2ab+b^2 = \text{ [illegible])}$$

Example 22 Evaluate $(36)^2-(25)^2$

Solution

$$(36)^2-(25)^2 = (36+25)(36-25) \quad \text{(using } a^2+2ab+b^2 = \text{ [illegible])}$$

$$= (61)(11)$$

$$= 671$$

Exercise 8.6

Factorize the following

(i) x^2+4x+4

(ii) $16-24x+9x^2$

(iii) $9x^2+30x+25$

(iv) $16-x^2$

(v) $4x^2+12xy+9y^2$

(vi) a^2b-ab^2

(vii) y^2-6y+9

(viii) x^2-y^2

(ix) $-2x^2+16xy-32y^2$

(x) $(123)^2-(120)^2$

8.4.2 Factorization of an algebraic expression (making groups)

To factorize the expression x^2+px+q

- Write the expression in descending order.
- Find two numbers whose sum is p and product is q then factorize after grouping different terms.

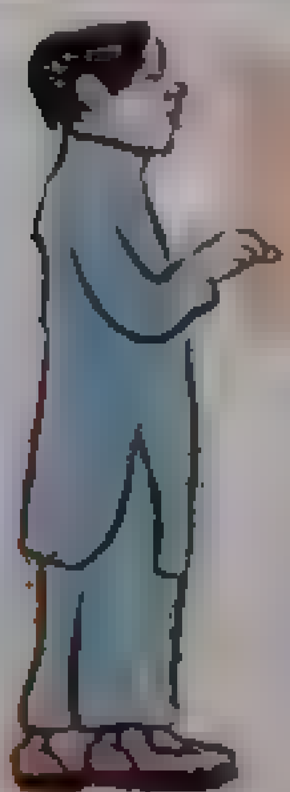
For example:

$$x^2+5x+6$$

Here $p = 5$ and $q = 6$

Possible factors of 6 are as under.

Factorization of 6	sum of factors of 6
$(1)(6)$	$1+6 = 7$
$(2)(3)$	$2+3 = 5$
$(-1)(-6)$	$-1+(-6) = -7$
$(-2)(-3)$	$-2+(-3) = -5$



Thus we find the two numbers 2 and 3 whose sum is 5 and their product is 6.

i.e. $x^2+5x+6 = x^2+2x+3x+6$

$$= (x^2+2x)+(3x+6)$$

$$= x(x+2)+3(x+2)$$

$$= (x+2)(x+3)$$

Example 23 Factorize $x^2-8x+15$.

Solution

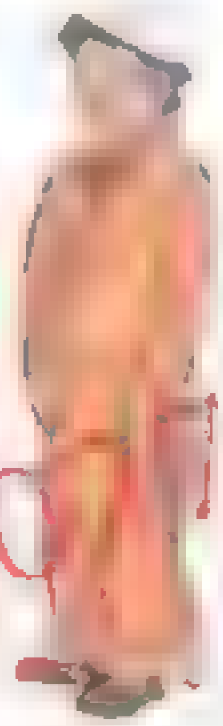
Here -3 and -5 are the two numbers whose sum is -8 and product is $+15$

Therefore, $x^2-8x+15 = x^2-3x-5x+15$

$$= (x^2-3x)-(5-15)$$

$$= x(x-3)-5(x-3)$$

$$= (x-3)(x-5)$$



Example 24 Factorize y^2-y-20

Solution

Here $+4$ and -5 are the two numbers whose sum is -1 and whose product is -20.

Therefore, $y^2-y-20 = y^2+4y-5y-20$

$$= (y^2+4y)-(5y+20)$$

$$= y(y+4)-5(y+4)$$

$$= (y+4)(y-5)$$

Example 25 Factorize $2t^2-2t-40$.

Solution

$$2t^2 + 2t - 40$$

$$= 2(t2 + t - 40)$$

$$= 2\{t^2 + 5t - 4t - 40\}$$

$$= 2\{t(t + 5) - 4(t + 5)\}$$

$$= 2(t + 5)(t - 4)$$

Exercise 8.7

Factorize the following algebraic expressions.

(i) x^2+4x+3 (ii) x^2+6x+8 (iii) [illegible] $y-27xy$

(iv) $x^2+2x-15$ (v) $a^2-2a-15$ (vi) [illegible] $-2a+10a-8a$

(vii) y^2-5y+6 (viii) [illegible] $x-12$ (ix) $x^2+5x-24$

REVIEW EXERCISE 8

1. Fill in the blanks.

(i) A symbol whose value does not remain constant is called __________

(ii) In $x+5$, 5 is called __________

(iii) x^2+2 is __________

(iv) x^2+2x+3 is a __________ in one variable

(v) $(a-b)^2=$ __________

2. Read the following statements carefully and write 'T' in front of true statement and 'F' in front of false statement

(i) $x-\frac{1}{x}$ is a polynomial ☐

(ii) $x^2+x=3$ is binomial ☐

(iii) $(x+a)(x+b)=x^2+(a+b)x+ab$ ☐

(iv) $(x+3)(x-2)=x^2+5x-6$ ☐

(v) $a^2-b^2=(a+b)(a-b)$ ☐

3. Choose the correct answer

(i) $(a+b)^2=$ __________

a) $a^2-2ab+b^2$ b) $a^2+2ab-b^2$

c) $a^2+2ab+b^2$ d) $b^2+2ab-a^2$

(ii) $x^2+5x+6 =$ ________

a $(x-3)(x-2)$ b $(x+3)(x+2)$
c $(x+3)(x-2)$ d $(x-3)(x+2)$

(iii) $(x+4)(x-4) =$ ________

a x^2-8 b x^2+16 c x^2+8 d x^2-16

(iv) $p(x)=x^2+5x+4$ is a

a Monomial b binomial
c trinomial d none of these

(v) $(-2x^3)(4x^2) =$ ________

a $-8x^5$ b $-6x^5$ c $-8x^6$ d $6x^6$

(vi) The product of $2x^3$ and $4x^3$ is

a $8x^{12}$ b $6x^6$ c $6x$ d $8x$

(vii) If $a^2-2ab+b^2 = 36$ and $a^2-3ab+b^2 = 22$, find ab

a 6 b 8 c 12 d 14

(viii) When x^2-x+1 is subtracted from $3x^2-4x+5$, the result will be

a $2x^2-3x+4$ b $2x^2-4x+4$
c $3x^2-6x+6$ d $4x^2-6x+6$

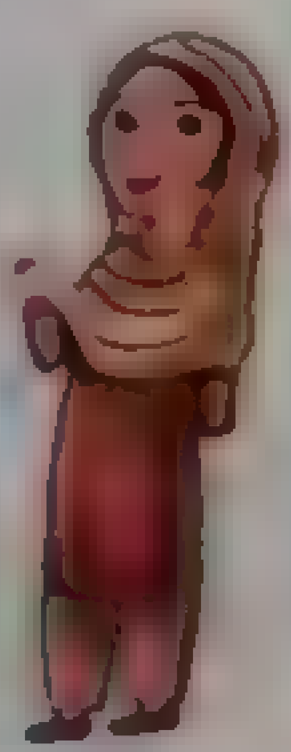
4. Add $2x^2+x-2$ and $-4x^2+5x+6$

5. Subtract x^2y+7x^3y-5 from $-3x^2y+10+2x^3y$

6. Multiply (x^2-x+1) with $x+1$.

7. If $A = x+y+2$, $B = x+1$, $C = y-1$ then find
(i) $A+B-C$ (ii) $A-B+C$ (iii) $A+BC$

8. Factorize
(i) $x^2+16x+64$ (ii) $16x^2-25y^2$ (iii) x^2-x-42



Glossary

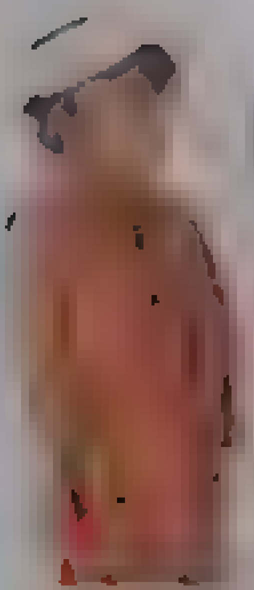
- A symbol or letter having fixed numerical value is called constant
- A symbol or letter whose value does not remain constant is called variable
- The combination of constants and variables a, b, c ... x, y, z. connected by fundamental operations +, –, ×, ÷ is called algebraic expression
- An unknown number represented by an English alphabet is called literal number.
- An algebraic expression in which the powers of the variables are all whole numbers is called polynomial.
- A polynomial having only one term is called monomial
- A polynomial having two terms is called binomial.
- A polynomial having three terms is called trinomial.
- Algebraic identities
  - $(x+a)(x+b) = x^2+(a+b)x+ab$
  - $(a+b)^2 = a^2+2ab+b^2$
  - $(a-b)^2 = a^2-2ab+b^2$
  - $a^2-b^2 = (a+b)(a-b)$

Tidbit

Use of brackets minimizes chances of mistakes.

- Writing an algebraic expression as the product of two or more algebraic expressions is called factorization.

Abdullah and Nadia are subtracting $4x-5$ from x^2+3x-5.



Abdullah	Nadia
$x^2+3x-5-4x-5$	$x^2+3x-5-(3x-5)$
$=x^2-x-10$	$=x^2-3x-5-4x+5$
	$=x^2-x$

Who is correct?

Unit 9 Linear Equations

- Define a linear equation in one variable.
- Demonstrate different techniques to solve linearequations
- Solve linear equations of the type
  - $ax + b = c$
  - $\dfrac{ax + b}{cx + d} = \dfrac{m}{n}$
- Solve the real life problems involving linear equations.

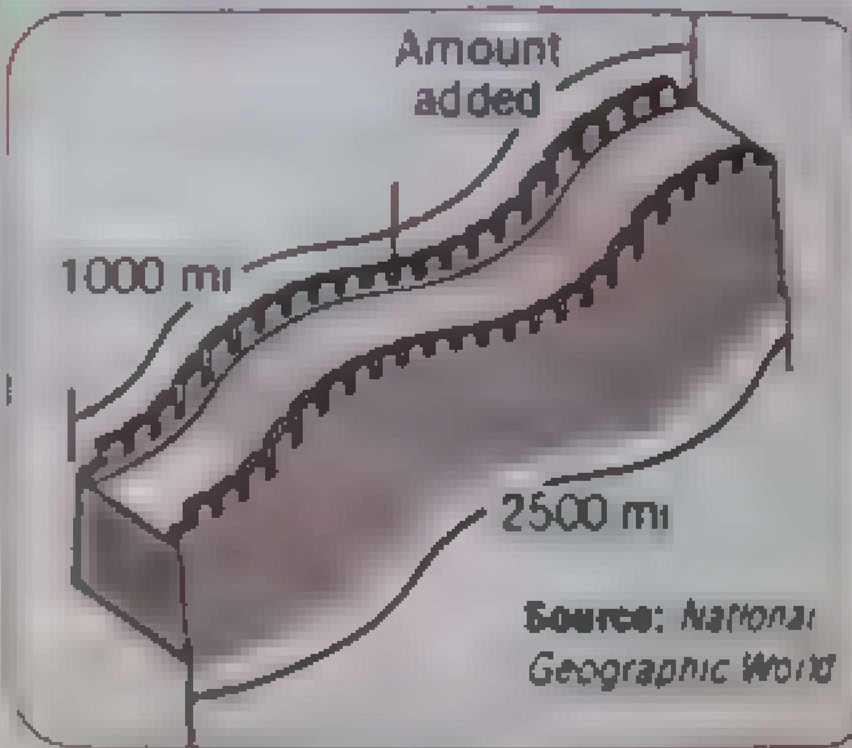
It's Important

Linear equations can be used to solve problems in every walk of life from planning a garden, to investigating data. One of the most frequent uses of linear equations is solving problems involving motion.

When can I use linear equation of motion?

When acceleration is constant, motion is in straight line.

The Great Wall of China



In the fourteenth century, the part of The Great Wall of China that was built during Qui Shi Huangdi's time was repaired, and the wall was extended. It was 1000 miles before the extension. When the wall was completed, it was 2500 miles long. How much of the wall was added?

If we let x = the additional length. Then situation can be shown by theequation

$$1000 + x = 2500$$

Solving for x, we get

$$x = 2500 - 1000 = 1500$$

The Great Wall of China was extended 1500 miles in fourteenth century.

Tidbit

An equation is like a scale to keep the scale balanced, you will have to put same weights on both pans.

9.1 Equations

Definition

An **equation** is a statement that two expressions are equal

9.1.1 Linear Equations

A linear equation in one variable has the form of $ax + b = 0$, where a and b are real numbers $a \neq 0$

Example 1 Write five example of linear equations

Solution

Each of the following is a linear equation

(i). $x - 10 = 0$ (ii). $y + 4 = 0$ (iii). $\frac{x}{3} = 7$

(iv). $2x + 3 = 0$ (v). $5z = 4$

9.2 Solution of Linear Equations

The value of the variable which converts the given equation into a true sentence is called solution or root of the equation.

For example $x=2$ is the solution of the equation $2x+3=7$ because replacing x by 2, reduces the equation to true sentence on each side and the equality holds.

i.e. $4 + 3 = 7$

or $7 = 7$

L.H.S = R.H.S

9.2.1 Different techniques to solve linear equations

Certain techniques are used to solve linear equations in one variable which are illustrated with the help of the following examples.

Example 2 Solve each equation mentally.

(i) $5x = 30$

$5 \times 6 = 30$

$x = 6$ The solut

$\frac{72}{d} = 8$

$d = 9$ The solution is 9

Guided Practice

For what value of x the equation $5x - 2 = 8$ is true?

Solve each equation mentally.

$a + 8 = 13$ $12 - d = 9$ $3x = 18$ $4 = \frac{36}{t}$

Tidbit

When solving an equation, we can

(a) Add the same number to both sides of the equation.

(b). Subtract the same number from sides of the equation;

(c). Multiply both sides of the equation by the same number.

(d). Divide both sides of the equation by the same non-zero number.

u know?

Do you notice anything interesting in the following multiplication?

138 X 42 = 5796

Example 3 Solve. $5x + x = 4x + 2$

Always write solution set in { } i.e. within the brackets.

Solution

$$5x + x - 4x = 2 \quad \text{(Collecting like terms)}$$
$$2x = 2$$
$$\frac{2x}{2} = \frac{2}{2} \quad \text{(Dividing both sides by 2)}$$
$$x = 1 \quad \text{The solution set} = \{1\}$$

Verification:

If we put $x = 1$, in the original equation we get

$$5(1) + 1 - 4(1) = 2$$
$$5 + 1 - 4 = 2$$
$$6 - 4 = 2 \qquad 2 = 2 \qquad \text{Which is true.}$$

Example 4 Solve. $3(9 + 2x) = 5x$ (Using Distributive law)

Solution

$$3(9 + 2x) = 5x$$
$$27 + 6x = 5x \quad \Rightarrow \quad 6x = 5x - 27$$
$$27 + 6x - 27 = 5x - 27$$
$$6x - 5x = (5x - 27) - 5x$$
$$x = 5x - 27 - 5x$$
$$\therefore \quad x = -27 \qquad \text{Hence the solution set} = \{-27\}$$

Verification:

Substituting the value of x in the original equation, we get

$$3[9 + 2(-27)] = 5x(-27)$$
$$3(9 - 54) = -135 \quad \Rightarrow 3(-45) = -135$$
$$-135 = -135, \text{ which is true.}$$

Example 5 Solve $\frac{4}{5}x + \frac{1}{4} = x + \frac{2}{5}$

Solution

$$\frac{4}{5}x + \frac{1}{4} = x + \frac{2}{5}$$

Multiplying all terms by 20 (the LCM of the denominators)

$$20 \times \frac{4}{5}x + 20 \times \frac{1}{4} = 20x + 20 \times \frac{2}{5}$$

$$16x + 5 = 20x + 8$$

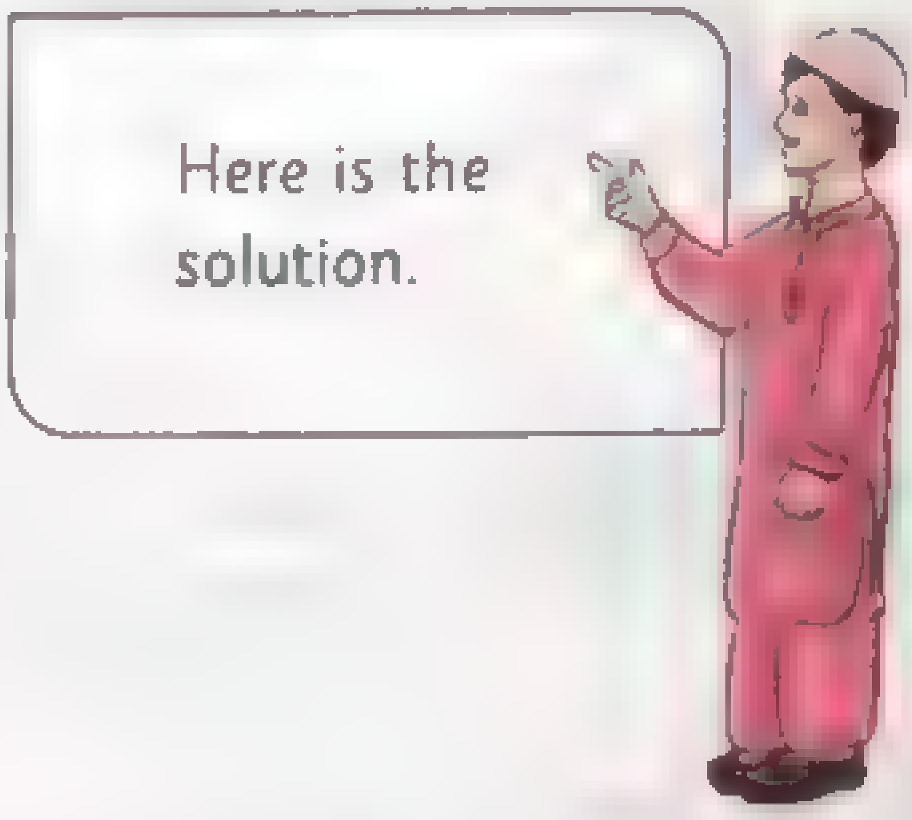
$$16x + 5 - 5 = 20x + 8 - 5$$

$$16x = 20x + 3$$

$$16x - 20x = 20x + 3 - 20x$$

$$-4x = 3$$

$$\frac{-4x}{-4} = \frac{3}{-4}$$



Hence the solution set $= \left\{\frac{-3}{4}\right\}$

Verification

Substituting $x = \frac{-3}{4}$ in the original equation we get,

$$\frac{4}{5}\left(\frac{-3}{4}\right) + \frac{1}{4} = \frac{-3}{4} + \frac{2}{5}$$

$$\frac{-3}{4} + \frac{1}{4} = \frac{-3}{4} + \frac{2}{5}$$

$$\frac{-12+5}{20} = \frac{-15+8}{20}$$

$$\frac{-7}{20} = \frac{-7}{20}$$, which is true.

Example 6 Solve $0.8x + 0.25 = -0.1x + 0.7$

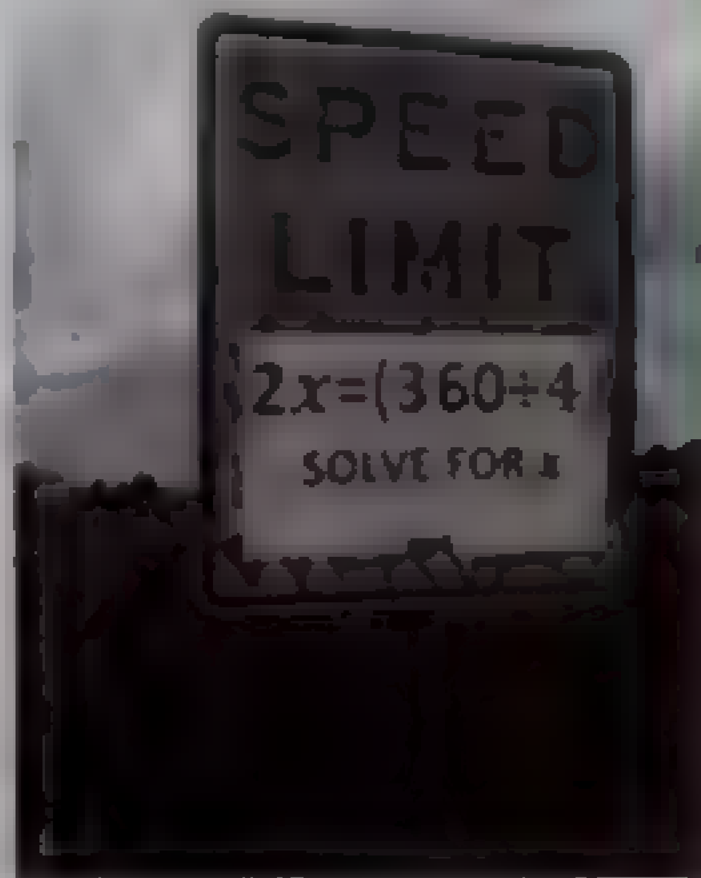
Solution

Converting decimal fractions to common fractions, we get

$$\frac{8x}{10} + \frac{25}{100} = \frac{-x}{10} + \frac{7}{10}$$

Multiplying both sides by 100

$$100 \times \frac{8x}{10} + 100 \times \frac{25}{100} = 100 \times \frac{-x}{10} + 100 \times \frac{7}{10}$$



$$80x + 25 = -10x + 70$$

$$80x + 25 = -10x + 70$$

$$80x + 25 - 25 = -10x + 70 - 25$$

$$80x = -10x + 45$$

$$80x + 10x = -10x + 45 + 10x$$

$$90x = 45$$

$$\frac{90x}{90} = \frac{45}{90}$$

$$x = \frac{1}{2}$$

or $\quad x = 0.5$

Hence solution set = {0.5}

Verification

Substituting $x = 0.5$ in the original equation we get,

$$0.8(0.5) + 0.25 = -(0.1)(0.5) + 0.7$$

$$0.4 + 0.25 = -0.25 + 0.7$$

$$0.65 = 0.65, \quad \text{which is true}$$

Example 7 Solve $250x + 75 = 125 + 175x$

Solution

$$250x + 75 = 125 + 175x$$

HCF of the coefficients is 25, we divide all the terms by 25 and get,

$$10x + 3 = 5 + 7x$$

$$10x + 3 - 3 = 5 + 7x - 3$$

$$10x = 7x + 2$$

$$10x - 7x = 7x + 2 - 7x$$

$$3x = 2$$

$$\frac{3x}{3} = \frac{2}{3}$$

$$x = \frac{2}{3}$$

Tidbit

Good mathematics is not about how many answers you know .... It's how you behave when you don't know.

Hence, the solution set $= \left\{\frac{2}{3}\right\}$

Verification

Substituting $x = \frac{2}{3}$ in the original equation we get,

$$250 \times \frac{2}{3} + 75 = 125 + \frac{2}{3} \times 175$$

$$\frac{500}{3} + 75 = 125 + \frac{350}{3}$$

$$\frac{500 + 225}{3} = \frac{375 + 350}{3}$$

$$\frac{725}{3} = \frac{725}{3}$$, which is true.

9.2.2 Solving linear equations of the type

$ax + b = c$ (ii). $\dfrac{ax + b}{cx + d} = \dfrac{m}{n}$

Example 8 Solve $2x + 4 = 12$.

Solution

$$2x + 4 = 12 \text{ ———— } (1)$$

$$2x + 4 - 4 = 12 - 4 \quad \text{[illegible]}$$

$$2x = 8 \quad \text{(Simplify both sides)}$$

$$\frac{2x}{2} = \frac{8}{2} \quad \text{(Dividing both sides by 2)}$$

$$x = 4 \quad \text{(Simplify both sides)}$$

Hence the solution set = {4}

Example 9 Solve $\dfrac{5x + 1}{3} = 7$.

Solution

$$\frac{5x + 1}{3} = 7 \text{ ————}$$

$$3 \times \frac{5x + 1}{3} = 3 \times 7 \quad \text{(Multiplying both sides by 3)}$$

$$5x + 1 = 21 \quad \text{(Simplify both sides)}$$

$$5x + 1 - 1 = 21 - 1 \quad \text{(Subtracting 1 from both sides)}$$

$$5x = 20 \quad \text{(Simplify both sides)}$$

$$\frac{5x}{5} = \frac{20}{5} \quad \text{(dividing both sides by 5)}$$

$$x = 4. \quad \text{(Simplifying both sides)}$$

Hence the solution set = {4}

Example 10 Solve $\frac{x-1}{3x+2} = \frac{1}{5}$

Solution

$$\frac{x-1}{3x+2} = \frac{1}{5} \quad \text{———— (1)}$$

$5(x - 1) = 1(3x + 2)$ (By cross multiplication)

$5x - 5 = 3x + 2$ (Using distributive property)

$5x - 5 - 3x = 3x + 2 - 3x$ (Subtracting 3x from both sides)

$2x - 5 = 2$ (Simplify both sides)

$2x - 5 + 5 = 2 + 5$ (Add 5 on both sides)

$2x = 7$ (Simplify both sides)

$\frac{2x}{2} = \frac{7}{2}$ (Divide both sides by 2)

$x = \frac{7}{2}$

Hence the solution set = $\left\{\frac{7}{2}\right\}$

Look for error: Sultan and Nosheen are solving $8n = -72$

Sultan
$8n = -72$
$8n\,(8) = -72\,(8)$
$n = -576$

Nosheen
$8n = -72$
$\frac{8n}{8} = \frac{-72}{8}$
$n = -9$

Who is correct?

Exercise 9.1

Solve the following equation.

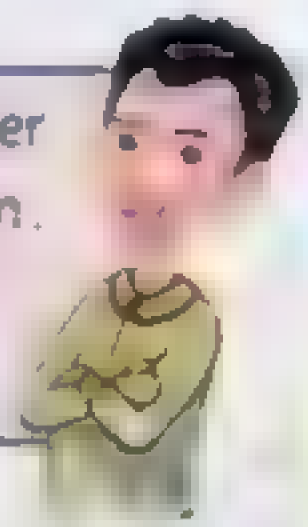
$x + 7 = -5$ (ii) $x - 5 = 2$ $\frac{x}{2} = 5$ $5x$ [illegible]

$8x - 2 = 14$ $\frac{x-1}{5} = \frac{5}{4}$ $7(x - 2) = 21$ $\frac{2y - 6}{y + 1} = \frac{2}{3}$

$\frac{2x - 2}{2x + 1} = \frac{4}{5}$ x) $\frac{3y + 1}{2y + 1} = \frac{3}{4}$ $0.3x + 0.2(10 - x) = 0.15(30)$

9.2.3 Real life problems

The difference of a number and ten is seventeen. Find the number.

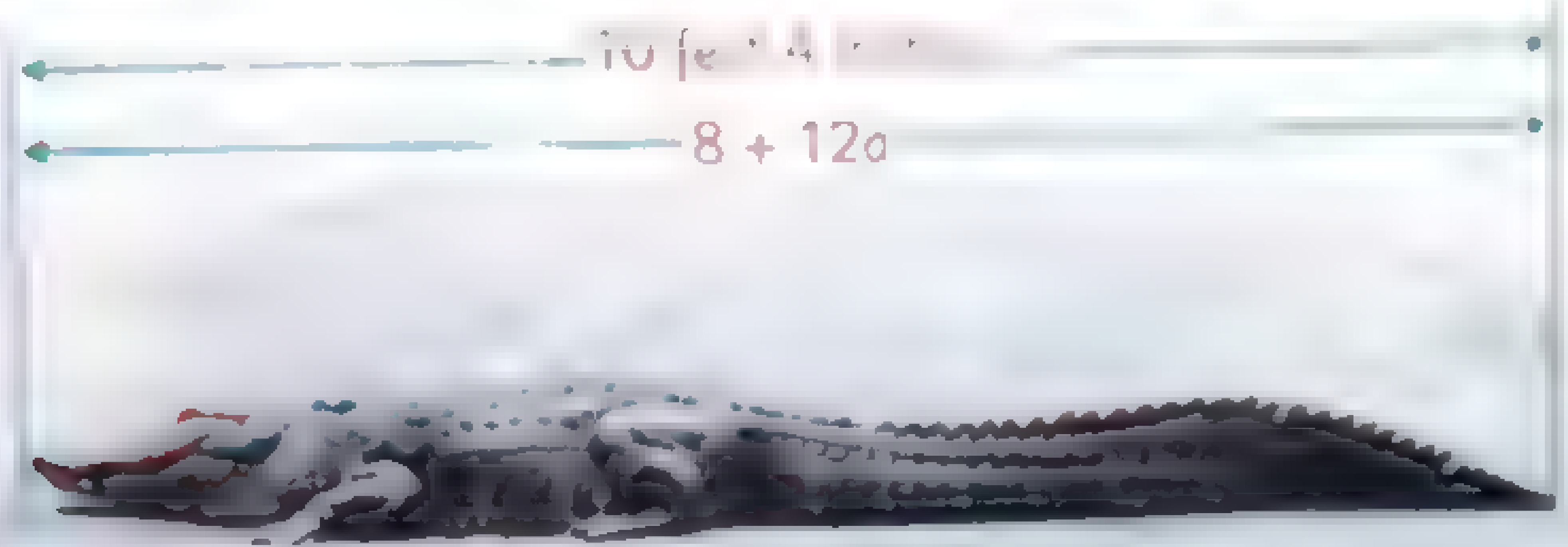


Words	The difference of a number and ten is seventeen		
Variables	Let n = the number Define the variable		
	The difference of a number and ten	is	seventeen
Equation	$n - 10$	$=$	17

$$n - 10 = 17$$
$$n - 10 + 10 = 17 + 10$$
$$n = 27$$

Example 11

An American alligator hatchling is about 8 inches long. These alligators grow about 12 inches per year. Estimate the age of the alligator in the photograph.



Solution

The expression $8 + 12a$ represents the length in inches of an alligator that is a years old.

Since 10 feet 4 inches equals $10(12) + 4$ or 124 inches, here the equation

$$8 + 12a = 124$$

$$8 + 12a - 8 = 124 - 8$$

$$12a = 116$$

$$\frac{12a}{12} = \frac{116}{12} = 9\frac{8}{12}$$

$$\begin{array}{r} 9 \\ 12\overline{)\,116} \\ \underline{108} \\ 8 \end{array}$$

$a = 9\frac{2}{3}$ years or 9 years 8 months

Tidbit

There is no magic formula for becoming a good problem solver. But it does seem that successful problem solvers do a lot of problems; they practice.

12

The length of a rectangular farm is twice of its breadth. Find the length and breadth of the rectangular farm if its perimeter is 30metre

Solution

Let the breadth of rectangular farm = x metre

The length of rectangular farm = $2x$ metre

Perimeter of the rectangular farm = 30metre

x

$2x$

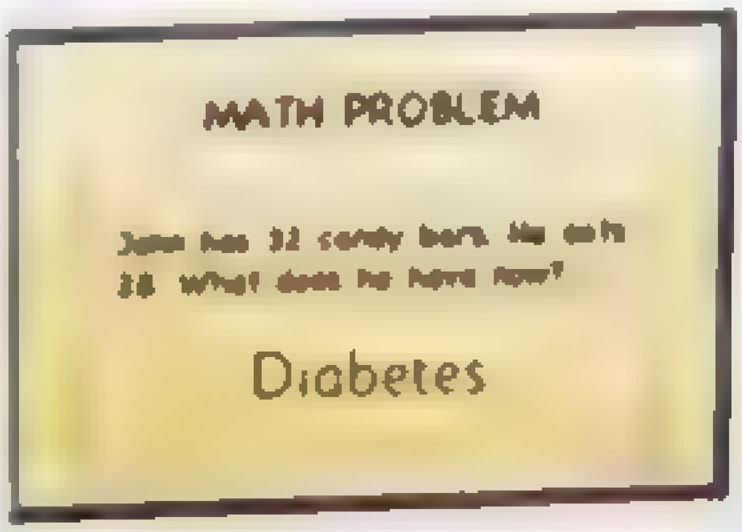
We know that

Perimeter of a rectangle = 2 (length + breadth)

$$30 = 2\,(2x + x)$$

$$30 = 2\,(3x)$$

or $30 = 6x$ or $6x = 30$



Dividing both sides by 6, we get

$$\frac{6x}{6} = \frac{30}{6}$$

$$x = 5$$

Hence breadth of the rectangular farm = 5metre

length of the rectangular farm = 2×5 = 10metre

Guided Practice

Najma's scarf is 15cm longer than its width. If its area is 1350cm, find its dimensions.

Example 13

Age of the brother is twice the age of the sister, three years back, age of the brother was three times the age of the sister. Find their present ages

Solution

Let the present age of the sister = x years

According to condition the present age of the brother = $(2x)$ years

Three years back the age of the sister = $(x - 3)$ years

Three years back the age of brother = $(2x - 3)$ years

According to the second condition

Age of brother = 3 (age of the sister)

or $2x - 3 = 3(x - 3)$

$\Rightarrow \quad 2x - 3 = 3x - 9$

$\Rightarrow \quad 2x - 3 - 2x = 3x - 9 - 2x$ (Subtract $2x$ from both the sides

$-3 = x - 9$

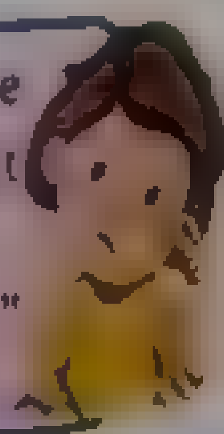
Add 9 on both sides

$-3 + 9 = x - 9 + 9$

$\Rightarrow \quad 6 = x$

or $\quad x = 6$

I am an odd number, Take away one letter and I becomes even What number am I?
Answer Seven, take away the "S" and it becomes "even"



Hence

The present age of the sister = 6 years

$\therefore$ The present age of the brother = $2 \times 6 = 12$ years

Guided Practice

i. A number increased by 8 is 23. Find the number.

ii. Twenty five is 10 less than a number Find the number

Exercise 9.2

1. Ali thinks of a number, adds 5 to it, subtracts 7 from the double of the sum, the result is 9. Find the number Ali thought.
2. In a class of 45 students, the number of girls is $\frac{7}{8}$ of the number of boys. Find the number of girls in the class.
3. A man has Rs. x. From which he spends Rs 6. If twice of the amount left with him is Rs. 86, find x.
4. Afridi and Shehzad gave 69 runs opening start to Pakistan. If Afridi's score is doulbe of Shehzad's score then he needs how many runs to complete his half century?
5. Perimeter of a rectangular play ground is 32 meter and its length is 4 m more than its breadth. Find the length and breadth of the rectangular play ground.
6. Age of a mother is 3 times the age of her daughter, after 4 years the sum of their ages will 60 years. Find their present ages.

REVIEW EXERCISE 9

1. Read the following statements carefully and write 'T' in front of true statement and 'F' in front of false statement.
   (i) $ax + b = 0$ where $a \neq 0$ is a linear equation in one variable. ____
   (ii) The solution set of $x - 10 = 0$ is $\{-10\}$. ____
   (iii) The solution set of $4x = 24$ is $\{6\}$. ____
   (iv) The solution set of $\frac{x}{5} = 4$ is $\{9\}$. ____
   (v) $x^2 + 1$ is a linear equation in one variable. ____

2. Fill in the blanks

(i) A value of variable which makes the equation a true statement is called ________ of the equation

(ii) To solve a linear equation subtract ________ number from both sides of the equation.

(iii) The solution set of $\frac{x}{5} = 2$ is ________.

(iv) $\frac{ax + b}{cx + d} = \frac{m}{n}$ is a ________ equation in ________ variable

(v) The value of x of a linear equation $3x - 10 = 2$ is ________.

3. Choose the correct option in the (a), (b), (c) or (d) form

(i) Which equation is not equivalent to $b - 15 = 32$?

a) $b + 5 = 52$
b) $b - 20 = 27$
c) $b - 13 = 30$
d) $b = 47$

(ii) What is the solution of $x - 167 = -52$?

a) 115
b) 219
c) –115
d) –219

(iii) Solve $8x - 3 = 5(2x + 1)$.

a) 4
b) 2
c) 2
d) –4

(iv) Which of the following equations has the same solution as $8(x + 2) = 12$?

a) $8x + 2 = 12$
b) $x + 2 = 4$
c) $8x = 10$
d) $2x + 4 = 3$

(v) Which equation has a solution of –5?

a) $2a - 6 = 4$
b) $3a + 7 = 8$
c) $\frac{3a - 7}{4} = 2$
d) $\frac{3}{5}a + 19 = 16$

(vi) Solve $2(b-3)+5=3(b-1)$.

a -2 b 2 c -3 d 3

(vii) Solve $75-9t=5(-4+2t)$.

a -5 b -4 c 4 d 5

4. Solve the following linear equations.

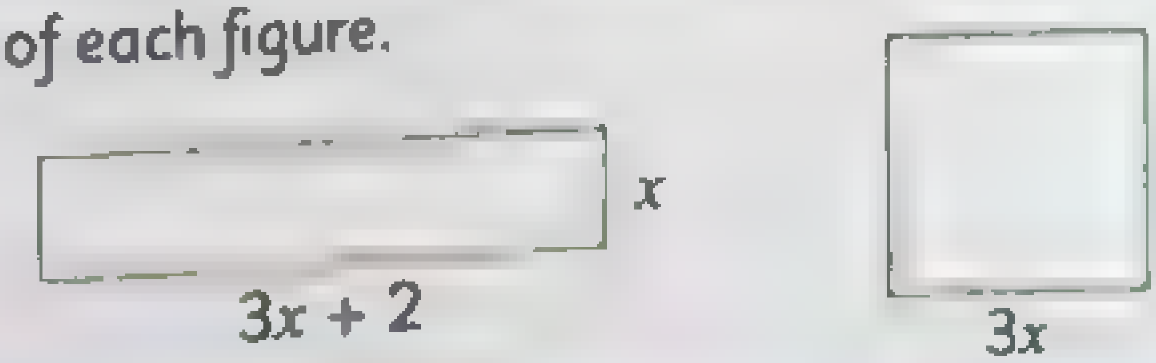
(i) $5(3x-2)-2=-2(1-7x)$ (ii) $\frac{3x+1}{3x+3}=\frac{4}{5}$

5. Perimeter of a squared filed is 20 m. Find the length of each side of the field

6. The price of 2 tables and 3 chairs ins Rs. 340, but a table costs Rs. 20 more than a chairs. Find the price of each.

7. A father is 3 times as old as his son In 10 years time he will be double of his son's age. Find their present ages.

8. The rectangle and square shown below have the same perimeter. Find the dimensions of each figure.



Glossary

- **Linear equation** An equation written in the form of $ax+b=0$, where a and b are real numbers and $a \neq o$ is called a linear equation in one variable.
- **Solution** A value of the variable which makes the equation a true statement is called solution or root of the linear equation in one variable.
- **Techniques to solve linear equation in one variable**

  To solve a linear equation in one variable basic techniques are

  1 Add the same number both sides of the equation.
  2 Subtract the same number from both sides of the equation.
  3 Multiply both sides of the equation by the same number.
  4 Divide both sides of the equation by the same number.
  5 Value of variable, which is single numerical value is obtained.
  6 Verify this value by putting in original equation.

Unit 10 Fundamentals of Geometry

You'll Learn

- Define adjacent angles, complementary and supplementary angles
- Define vertically opposite angles
- Calculate unknown angles involving adjacent angles, complementary, supplementary angles and vertically opposite angles
- Calculate unknown angle of a triangle
- Identify congruent and similar figures
- Recognize the symbol of congruency
- Apply the properties for two figures to be congruent or similar
- Apply the following properties for two figures to be congruent or similar

$$SSS \cong SSS$$
$$SAS \cong SAS$$
$$ASA \cong ASA$$
$$HS \cong HS$$

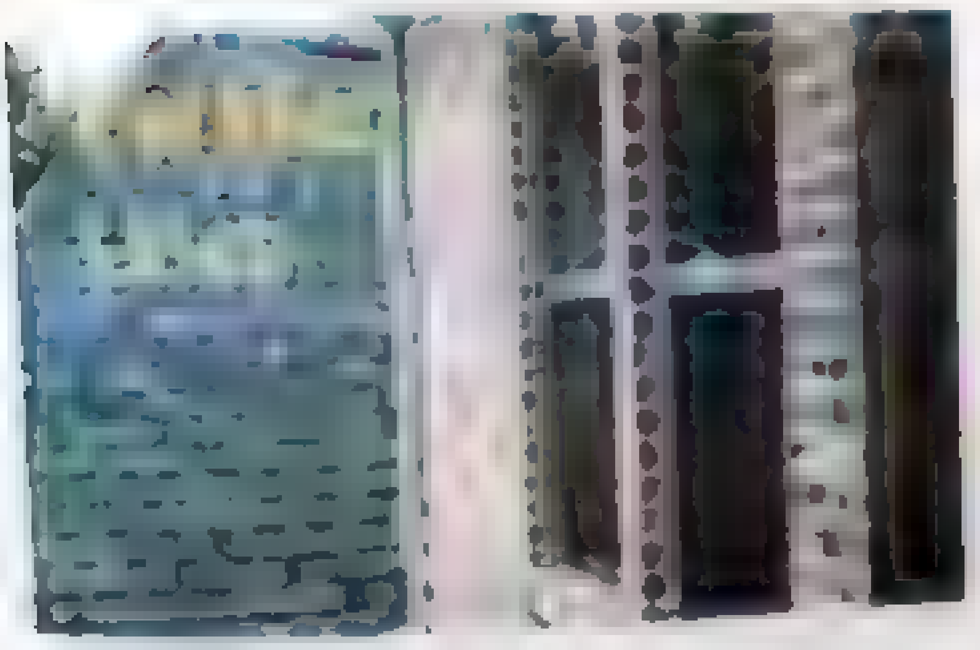
- Draw segment of a circle and demonstrate the property, the angles in the same segment of circle are equal.
- Draw a semicircle and demonstrate the property, the angle in the semicircle is a right angle.
- Describe a circle and its center, radius, diameter, chord, arc, major arc and minor arcs, semicircle and segment of a circle.

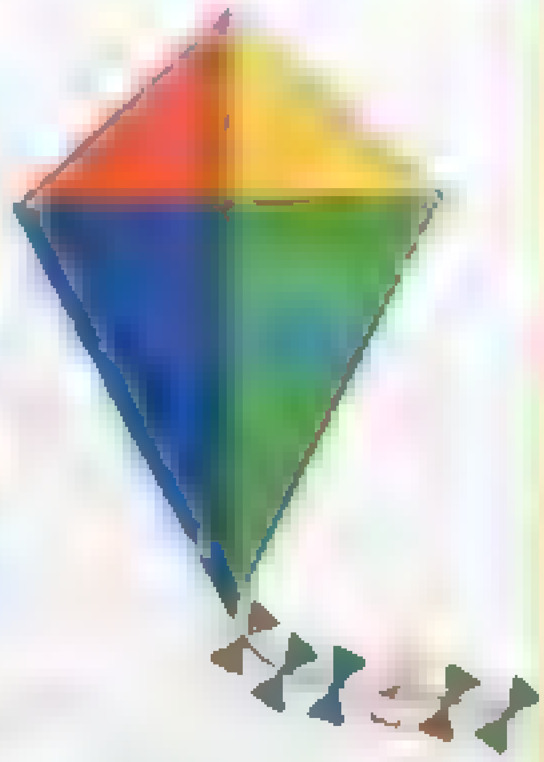
"There is no Royal Road to Geometry"

— Euclid

It's Important

Geometry is everywhere. Angles, shapes, lines, line segments, curves, and other aspects of geometry are every single place you look, even on this page. Letters themselves are constructed of lines, line segments, and curves! Take a minute and look around the room you are in, take note of the curves, angles, lines and other aspects which create your environment.



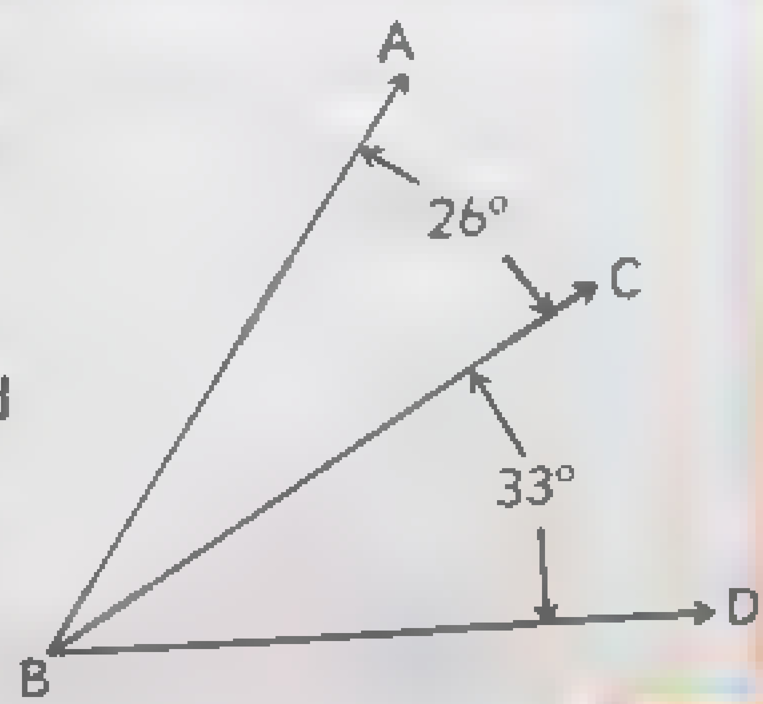


10.1 Adjacent Angles

Two angles are adjacent if they have a common side and a common vertex (corner point) and their intersection is null set

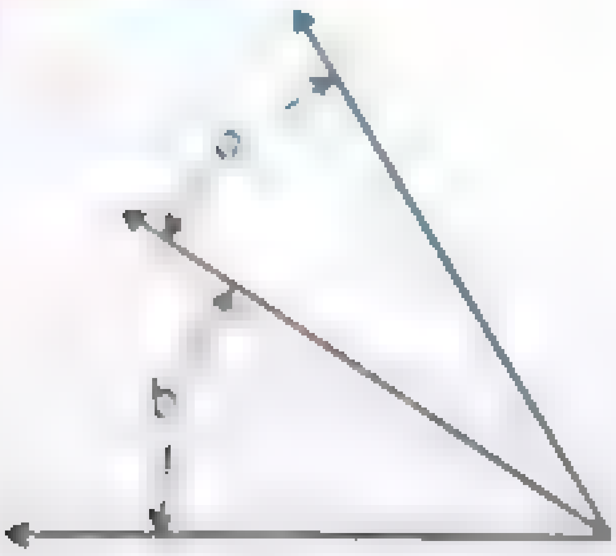
In the figure the angle ABC is adjacent to angle CBD because:

- they have a common side $\overline{BC}$
- they have a common vertex (point B) and
- one angle is not contained in the other i.e. their intersection is a null set.



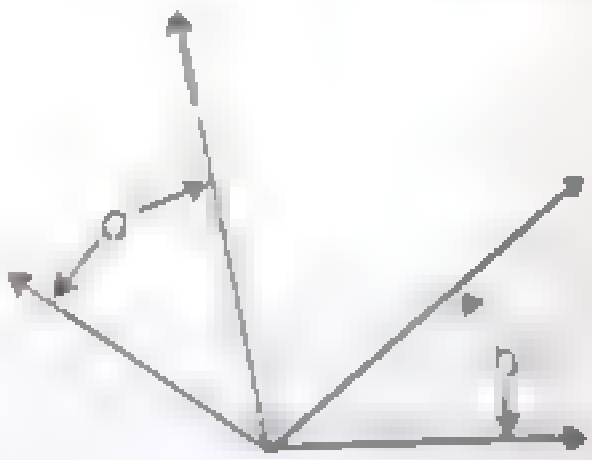
Example 1 Which angles are adjacent and which are not?

(i)



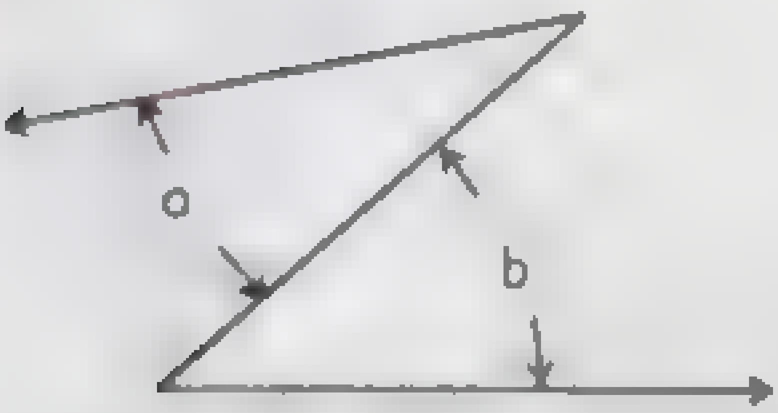
These are adjacent angles

(ii)



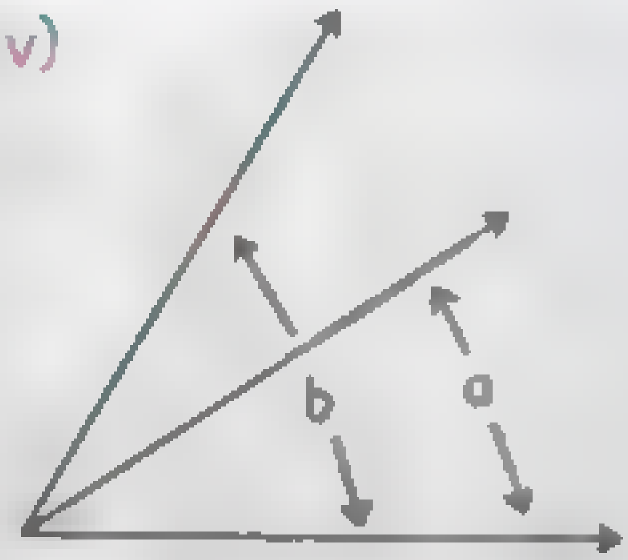
These are not adjacent angles as they do not share a side

(iii)



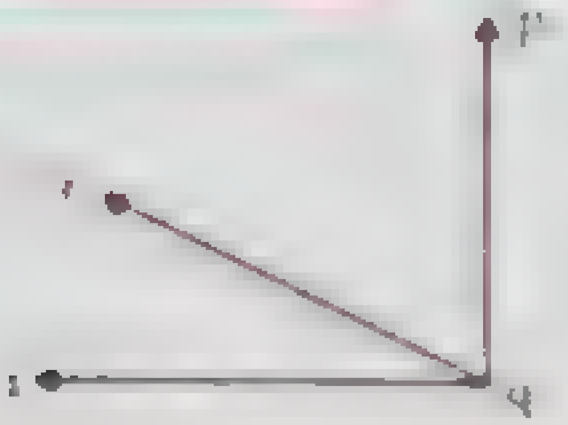
They are not adjacent angles as they do not share a vertex

(iv)



They are not adjacent angles as their intersection is not a null set

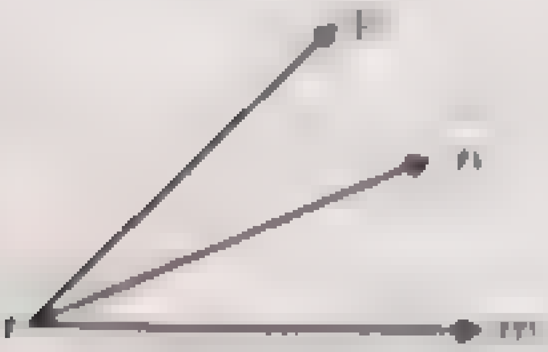
Guided Practice

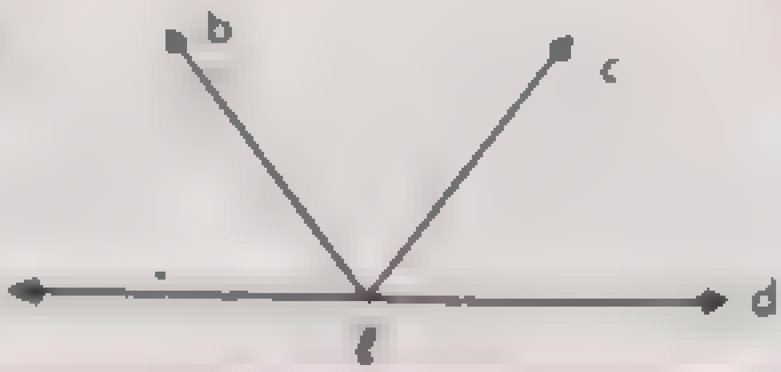


Adjacent angles $\angle pqr$ and $\angle rqs$

Common vertex q

Common arm $\overrightarrow{qr}$





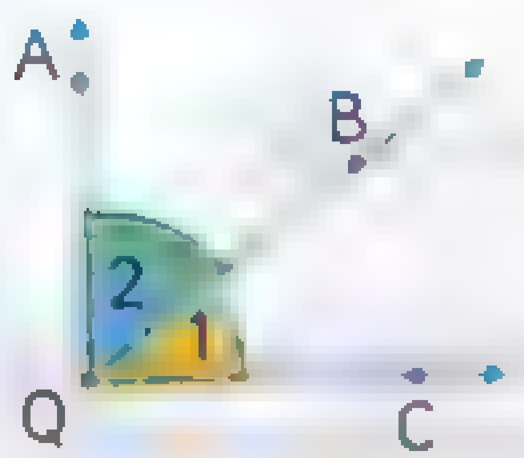
10.2 Complementary Angles

Definition: A pair of angles whose sum is 90°

Example 2 $m\angle 1 = 40^\circ$, $m\angle 2 = 50^\circ$

Remember

If the sum of two angles is 90° we say they are "Complement" to each other

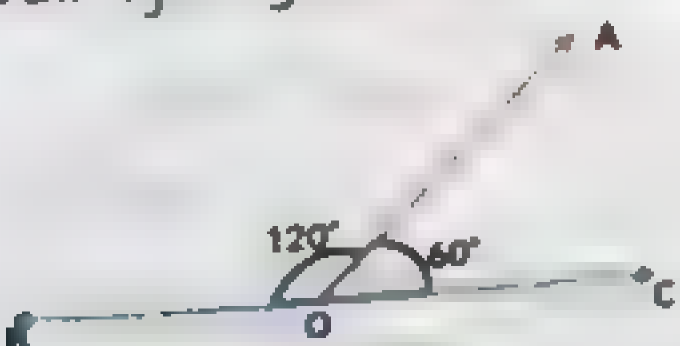


Adjacent Complementary Angles

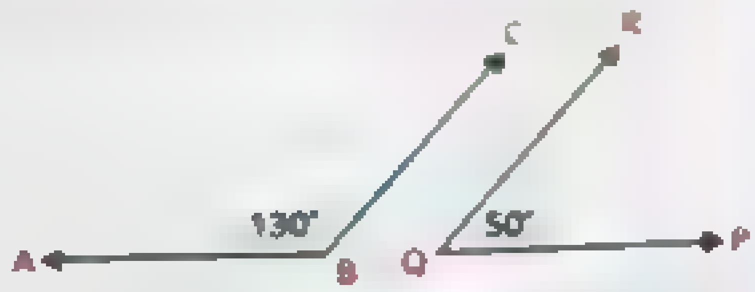
Non-Adjacent Complementary Angles

10.3 Supplementary Angles

A pair of angles whose sign is 180°



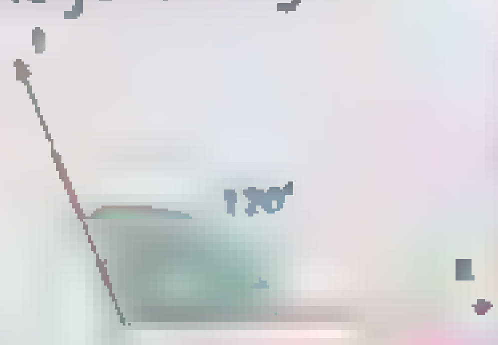
Adjacent Supplementary Angles



Non-adjacent Supplementary Angles

Guided Practice

Find complementary and supplementary angles in the following.



10.4 Vertically Opposite Angles

Vertical opposite Angles are two angles whose sides form two pairs of opposite rays (straight lines).These angles are **not** adjacent. They are always congruent (i.e. of same measure).

$\angle 1$and $\angle 3$ are vertical angles.

$\angle 2$ and $\angle 4$ are vertical angles.

$\angle 1$ and $\angle 2$ are not vertical but are supplementary

10.5 Calculate the measure of unknown angles involving adjacent, complementary, supplementary and vertical angles

Example 3

Two complementary angles measure "z"and 80°. What is the value of z?

Solution

We know that sum of the complementary angles is 90°.

$$\text{Hence, } z+80^\circ = 90^\circ$$
$$z = 90^\circ - 80^\circ$$
$$z = 10^\circ$$

Example 4

Two Supplementary angles measure "$2x$"and 136°. What is the value of x?

Solution

We know that sum of the supplementary angles is 180°

$$\text{Hence, } 2x+136^\circ = 180^\circ$$
$$2x = 180^\circ - 136^\circ$$
$$2x = 44^\circ$$
$$x = 22^\circ$$

Example 5

Two vertical angles measure 80° and $4x$. How many degrees are there in x?

Solution

We know that vertical angles are equal

$$\text{Hence } 4x = 80^\circ$$
$$x = \frac{80^\circ}{4}$$
$$x = 20^\circ$$

Example 6

Two supplementary angles measure $(3x+5)$ and $(9x-5)$. What is the value of x?

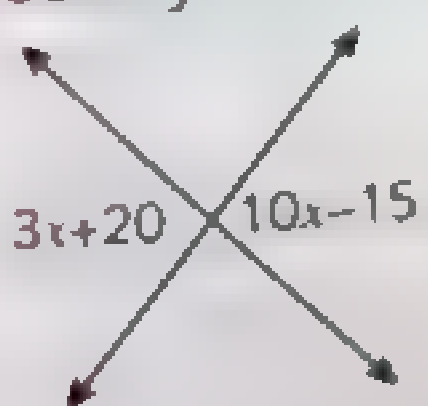
Solution

We know that sum of supplementary angles is 180°

$$\text{Hence } (3x+5)+(9x-5) = 180^\circ$$
$$3x+9x+5-5 = 180^\circ$$
$$12x = 180^\circ$$
$$x = \frac{180^\circ}{12}$$
$$x = 15^\circ$$

Guided Practice

(i) Solve for x



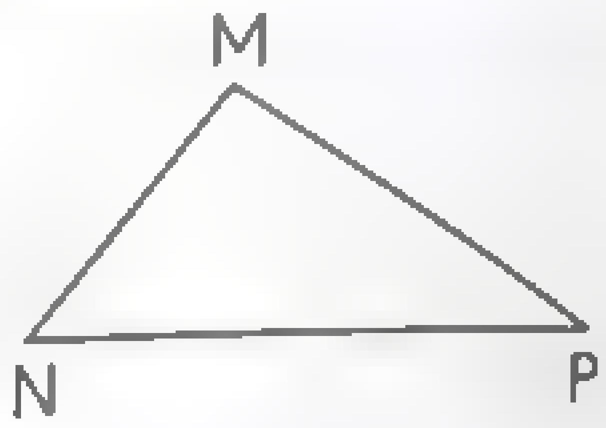
Two vertical angles measure 100° and $5x$. How many degrees are there in x?

10.6 Calculate The Measure Of Unknown Angles Of A Triangle

Interior Angles of a Triangle

The sum of the measures of the interior angles of any triangle is 180°

In ΔMNP, $m\angle M + m\angle N + m\angle P = 180°$.



Example 7

In ΔABC, $m\angle A = 42°$ and $m\angle C = 63°$. What is the measure of $\angle B$?

Let $m\angle B = x$.

Since the sum of measure of all the three angles is equal to 180°

Thus

$$m\angle B + 42° + 63° = 180°$$
$$m\angle B + 105° = 180°$$
$$m\angle B = 75°$$
$$\text{So } m\angle B = 75°$$

Example 8

The angles of a triangle are in the ratio of 1:2:3. Find the measure of the smallest angle of the triangle.

Let measure of the smallest angle $= x$ measure of the second angle $= 2x$ and measure of the largest angle $= 3$

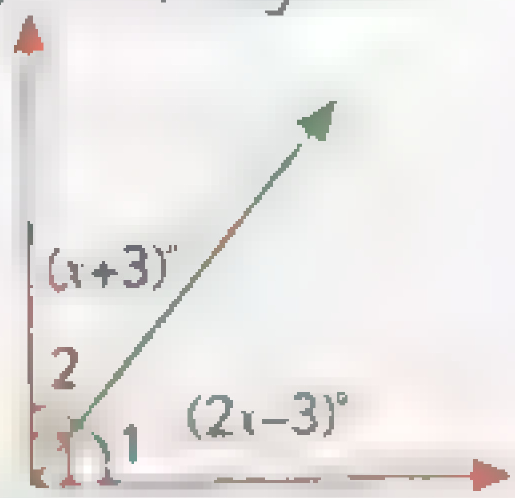
Then

Sum of the measures $= x + 2x + 3x = 180°$

$$\text{i.e. } 6x = 180°$$
$$\text{or } x = 30°$$

Exercise 10.1

1. Two complementary angles measure x and $65°$. How many degrees are there in x?
2. Two vertical angles measure x and $45°$ How many degrees are there in x?
3. Two supplementary angles measure x and $75°$ How many degrees are there in x?
4. Two vertical angles measure $2x$ and $80°$ How many degrees are there in x?
5. Two complementary angles measure $(2x + 10)$ and $(x + 20)$ degrees What is the value of x?
6. Two supplementary angles measure $(5x - 30)$ and $(x + 90)$ degrees What is the value of x?
7. Solve for x.



10.6.1 Congruent Figures

Two objects are said to be congruent if they are same in the size and shape. This phenomenon is known as congruency If one of the two objects has the same shape and size as mirror image of the other then these shape are said to be congruent. For example·

In the above figure (i) and (ii) are congruent while figures (iii) and (iv) are congruent if their corresponding angles are equal

Symbol of Congruency

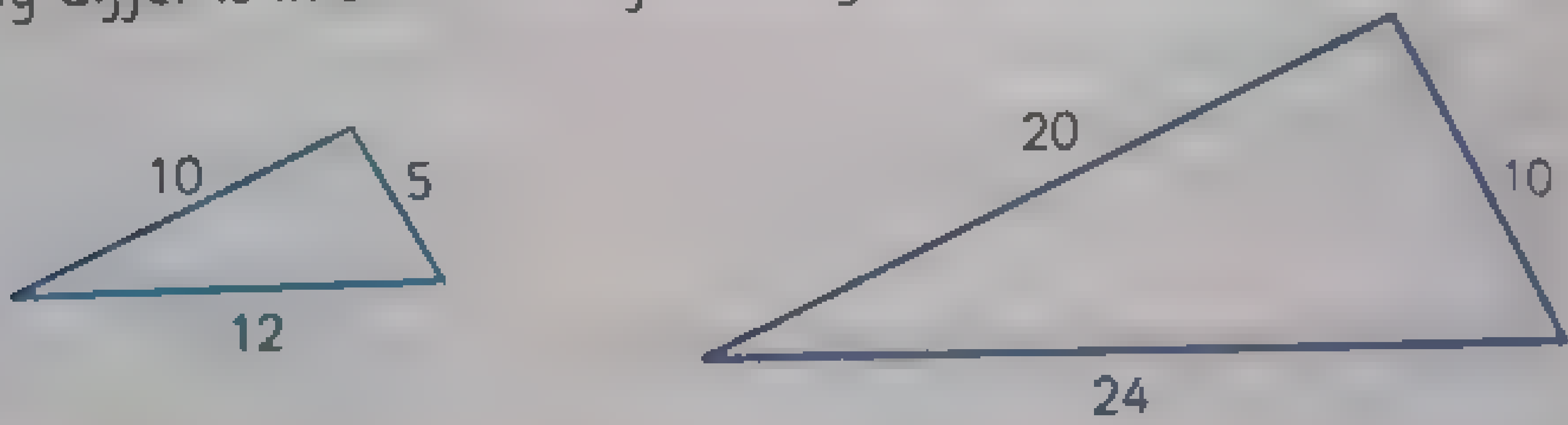
congruent is denoted by "≅".
"=" means the same in size and "∪" means same in shape

Congruent angles

Two angles are said to be congruent if they have the same measure for example

Similar Figures

Two shapes are said to be similar when the shape is same but they only differ is in size In the following the two triangles are similar



Following is a comparison of similar and congruent figures

	Congruent	Similar
Corresponding Angles	Corresponding angles are the same	Corresponding angles are the same
Corresponding Sides	Corresponding sides are the same	Corresponding sides are proportional

Exercise 10.2

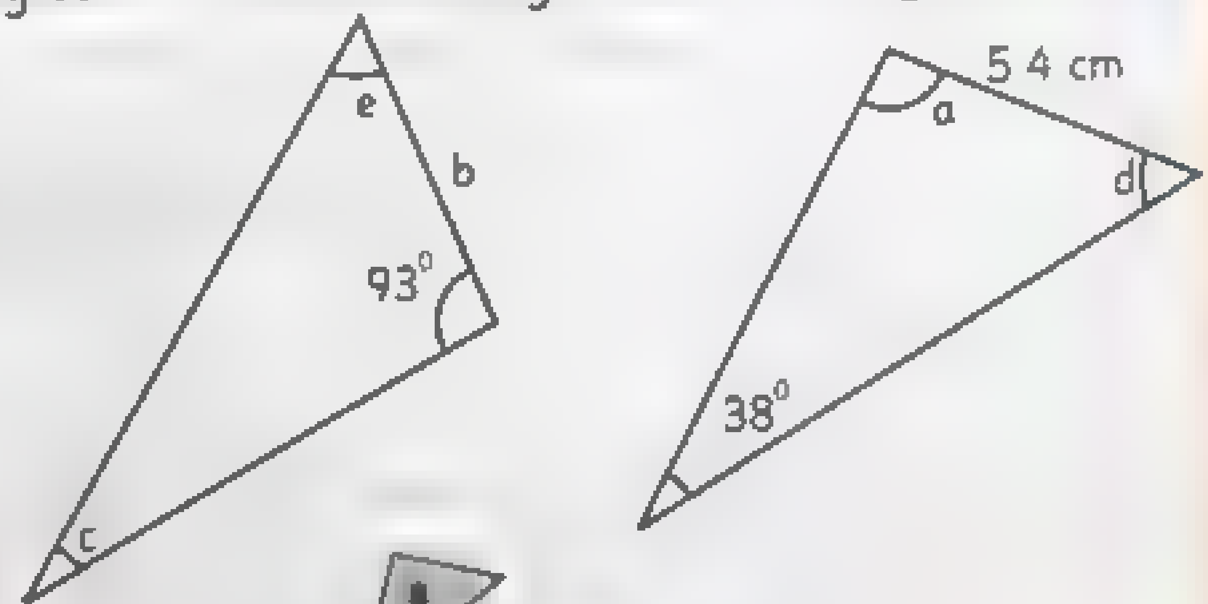
Answer the following question regarding the figures:

1. Is this an example of similar figures?

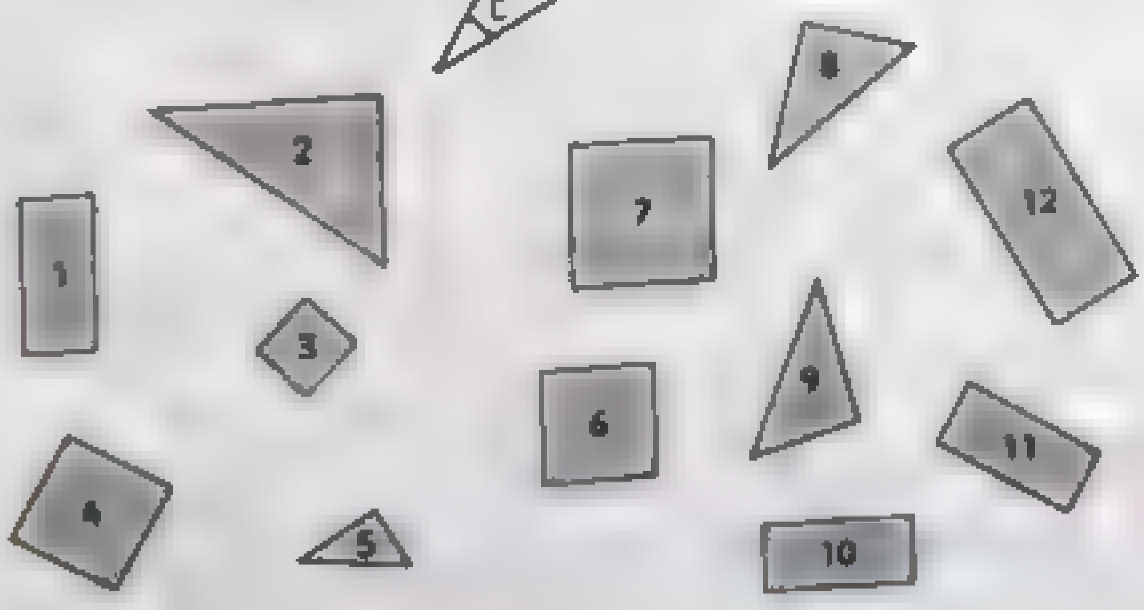
   a. Yes b. No

2. These are two congruent triangles. Fill in the missing values in the given blanks

   i. a = ______ °
   ii. b = ______ cm
   iii. c = ______ °
   iv. d = ______ °
   v. e = ______ °



3.



Look at the shapes above and complete the following statements

a. Shape 1 is congruent or similar to shape ______, ______ and ______.

b. Shape 6 is congruent to shape ______.

c. Shape 11 is ______________ to shape 12.

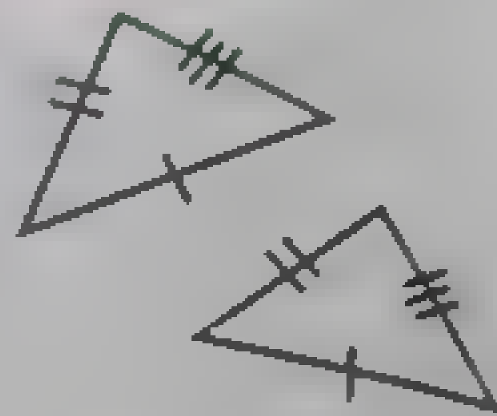
d. Shape 8 and 9 are ______________ to each other.

e. Shape 5 is similar to shapes ______, ______ and ______.

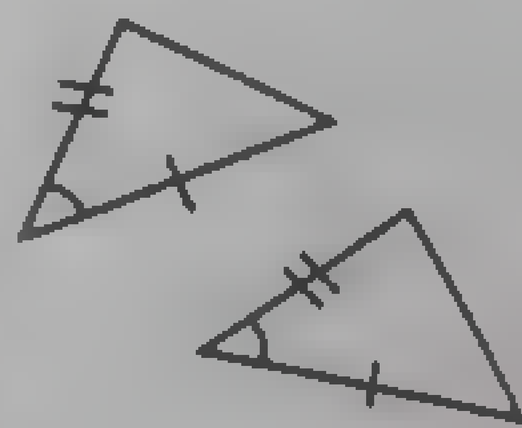
10.7 Application of the following properties for two figures to be congruent or similar

[illegible]) triangles to be congruent

SSS≅SSS — If all the three sides of a triangle are congruent to the three sides of another triangle, the triangles are congruent

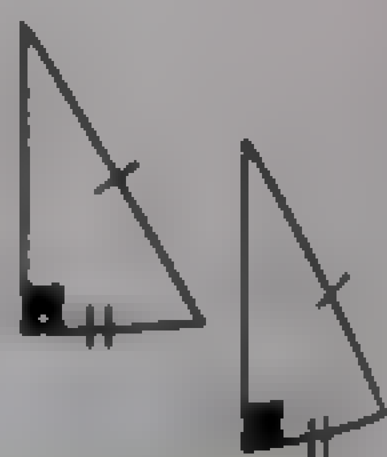


SAS≅SAS — If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent (The included angle is the angle formed by the sides being used.)



ASA≅ASA — If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.

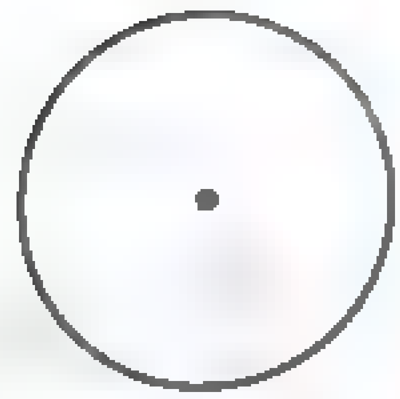
HS≅HS — If the hypotenuse and base of one right angled triangle are congruent to the corresponding parts of another right angled triangle, the right angled triangles are congruent (Either base of the right angled triangle may be used as)



HS ≅ HS is only true for right triangle.

10.8 Circle

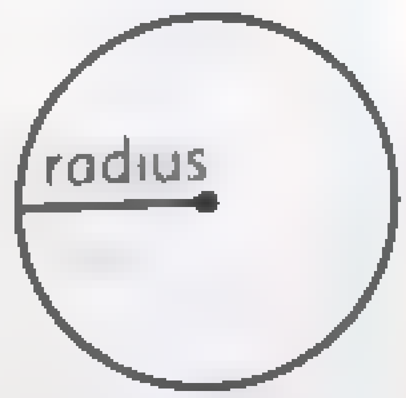
Here we recall various notions of a circle



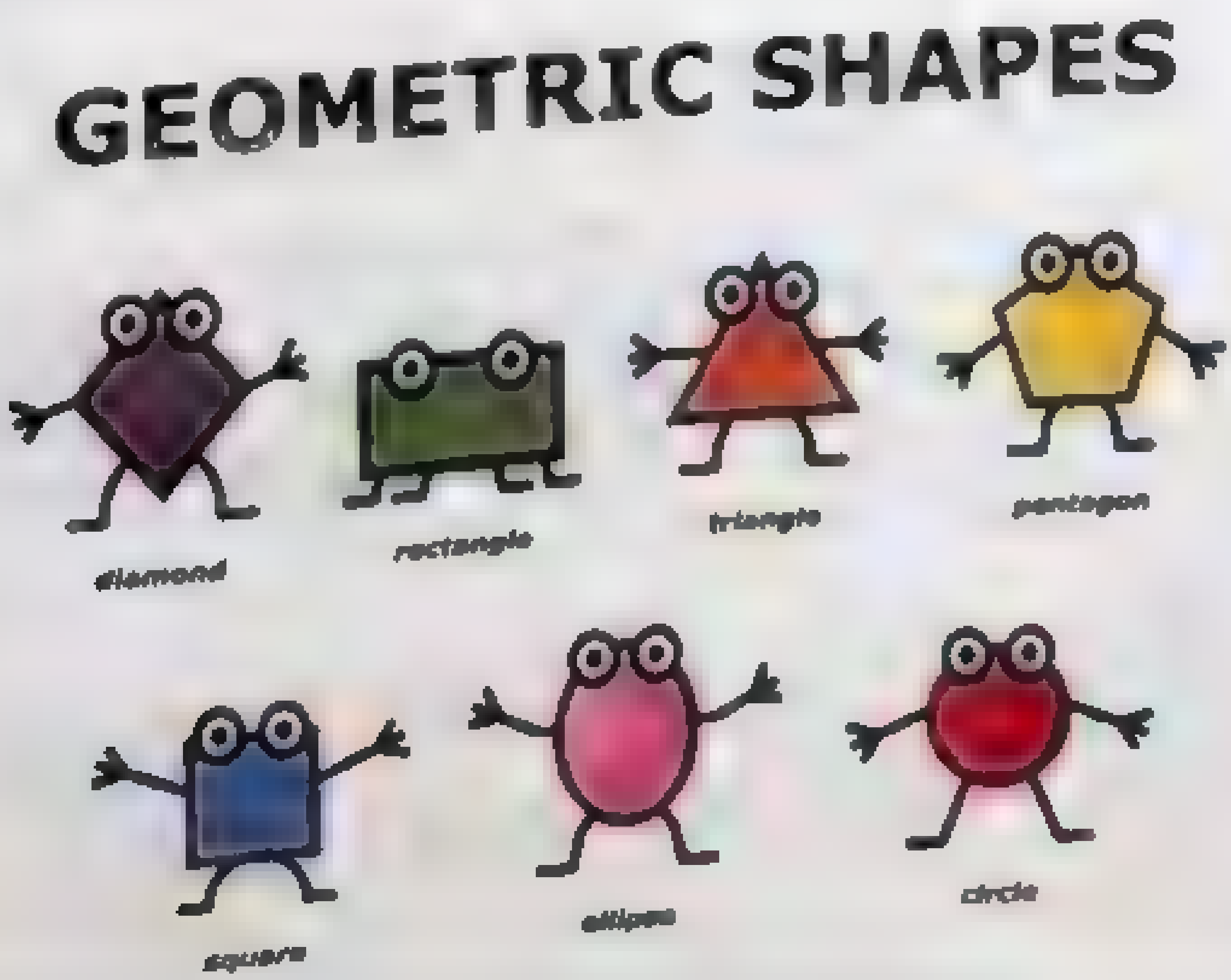
10.8.1 Definition of Circle

A **circle** is a shape in a plane with all its points at the same distance from a fixed point known as centre of the circle. In figure point 'A' is the centre of the circle.

10.8.2 Definition of Radius



The **radius** of a circle is the distance from the center of a circle to any point on the circle, it is denoted by **r**.
It is clear that: $2r = d$

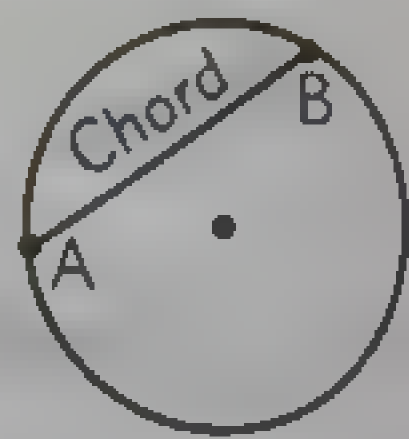


10.8.3 Definition of Diameter

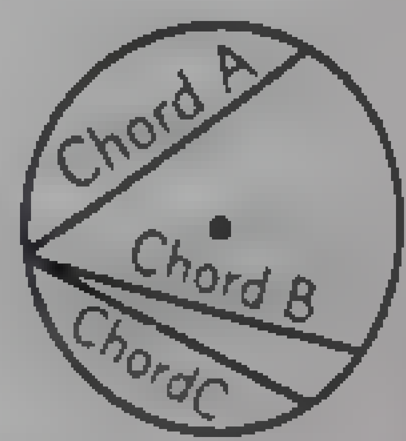
The length of the line joining two points of a circle through the center is called the **diameter**, it is denoted by **d**.

Remember A diameter of a circle is twice of its radius

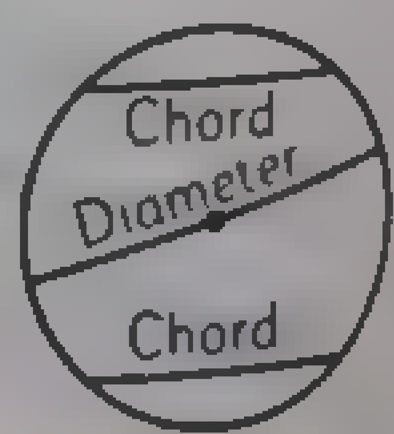
10.8.4 Definition of Chord



A chord is a line segment joining two points on a circle.



A circle has many different chords. Some chords pass through the center and some do not. A chord that passes through the center is called a **diameter**.



It turns out that a diameter of a circle is the longest chord of that circle since it passes through the center. A diameter satisfies the definition of a chord, however, a chord is not necessarily a diameter. Thus, it can be stated, that **every**.

Remember

Diameter is a chord, but every chord is not necessarily a diameter.

10.8.5 Arc of a Circle

Minor & Major arcs

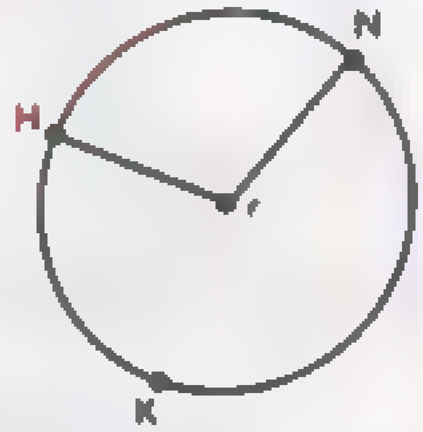
As the picture shows, an arc is a part or a portion of the circumference of a circle.

Arcs are grouped into two descriptive categories:

(i). Minor arc

(ii). Major arc

In the given circle, there are both major arc and minor arcs. Look at the circle and try to figure out how you would divide it into a portion that is 'major' and a portion that is 'minor'

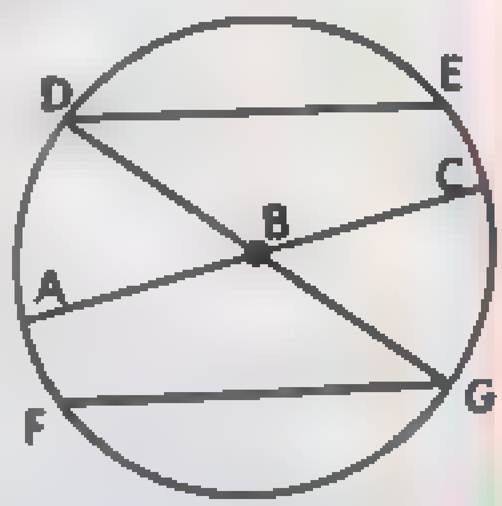


NKH is the major arc

HN is the minor arc

Example 9

Name the two chords on this circle that are not diameters



Solution

$\overline{DE}$ and $\overline{FG}$ are the two chords

Example 10

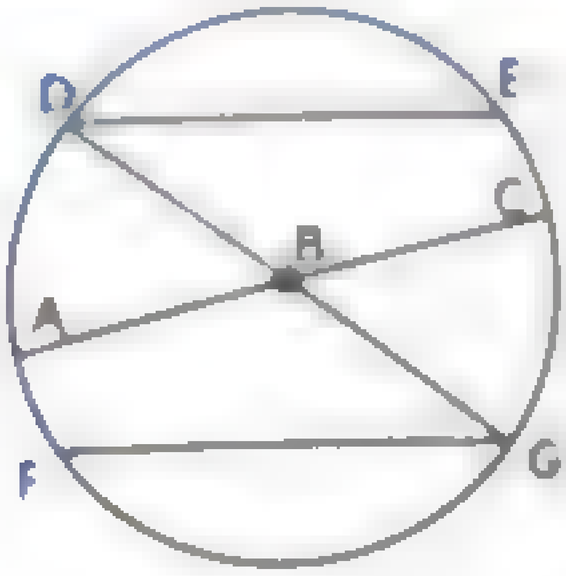
Name all radii of the given circle.

Solution

$\overline{BA}$, $\overline{BC}$, $\overline{BD}$ and $\overline{BG}$ are all the radii of the circle.

Example 11

What are $\overline{AC}$ and $\overline{DG}$?

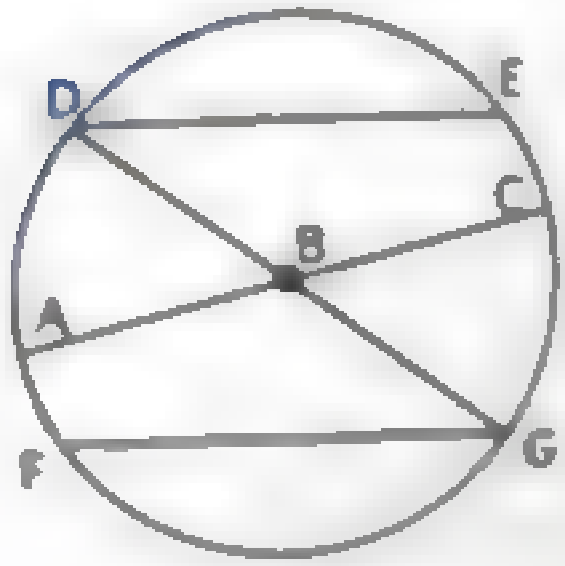


Solution

$\overline{AC}$ and $\overline{DG}$ are diameters

Example 12

If $\overline{DG}$ is 5 inches long, then how long is $\overline{DB}$?



Solution

Here $\overline{DG}$ is diameter while $\overline{DB}$ is the radius

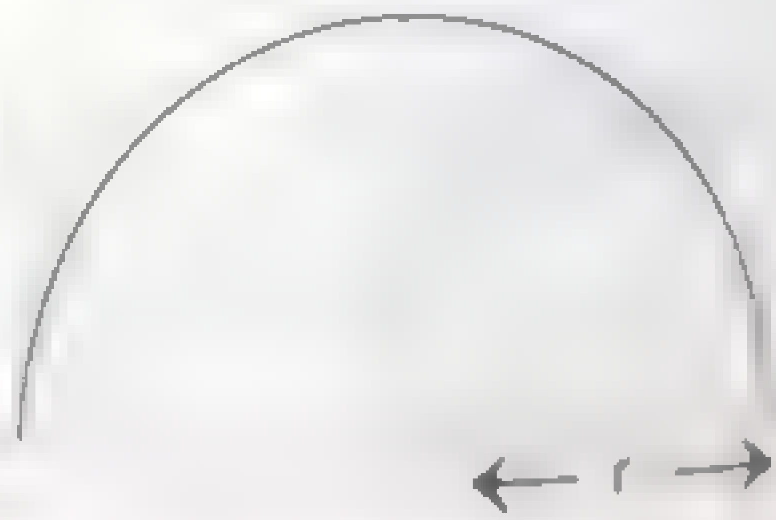
Diameter $\overline{DG} = 5$ inches

Radius $\overline{DB} = ?$

As the diameter of a circle is twice as long as its radius. Or radius is half of the diameter

Hence radius $= 5$ inches $\div 2 = 2.5$ inches

The length of $\overline{DB}$ is 2.5 inches



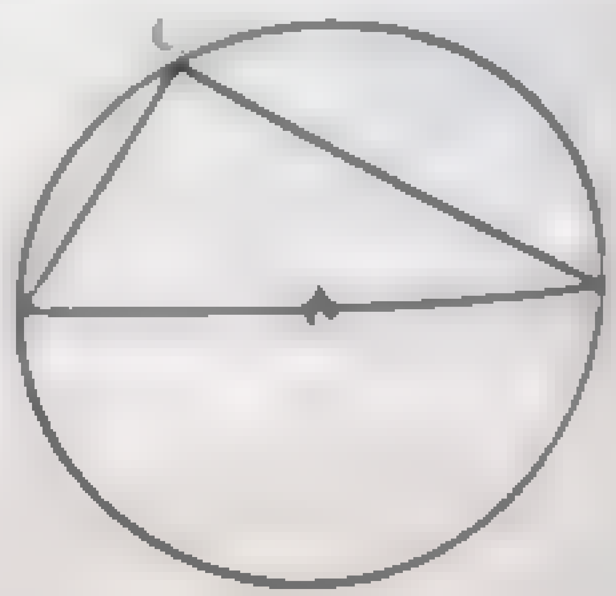
Semicircle

- Half of a circle is called semi circle. Every circle can be divided into two semi circle

10.8.6 Angle in a Semi Circle

Angles formed by drawing lines from the ends of the diameter of a circle to its circumference form a right angle.

So, $\angle C$ is a right angle

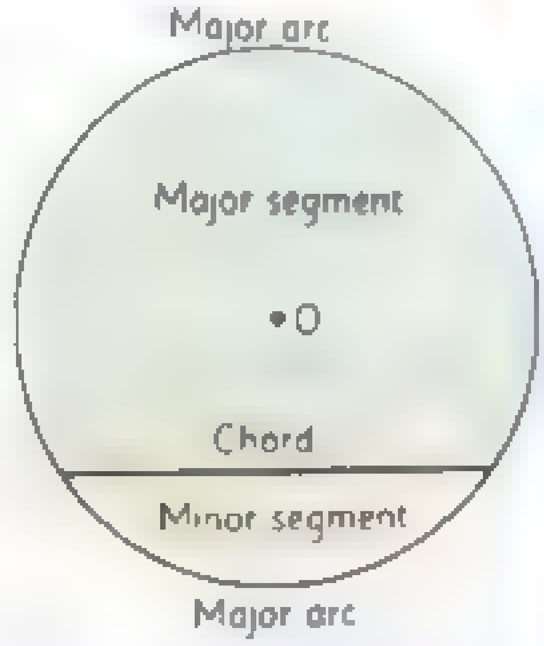


10.8.7 Segments of a Circle

A chord of a circle divides the circle into two regions, which are called the segments of the circle

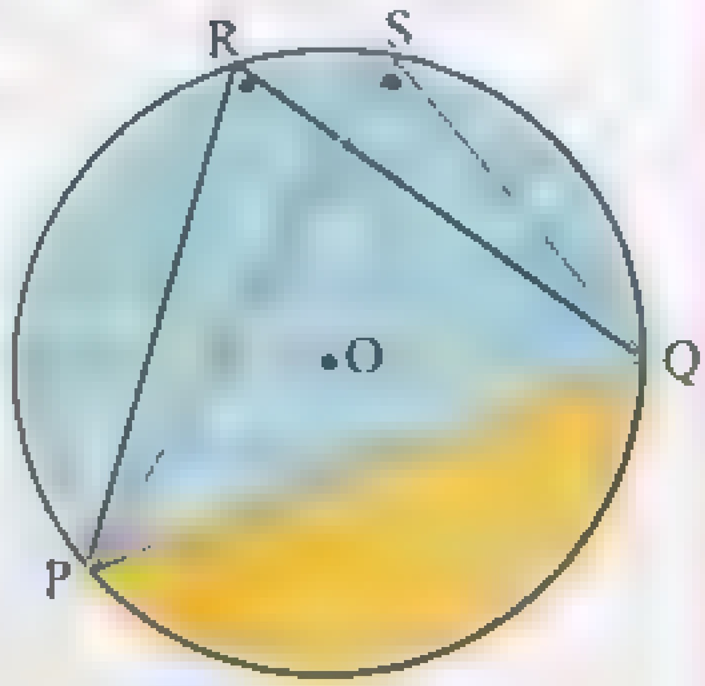
The **minor segment** is the region bounded by the chord and the minor arc

The **major segment** is the region bounded by the chord and the major arc.

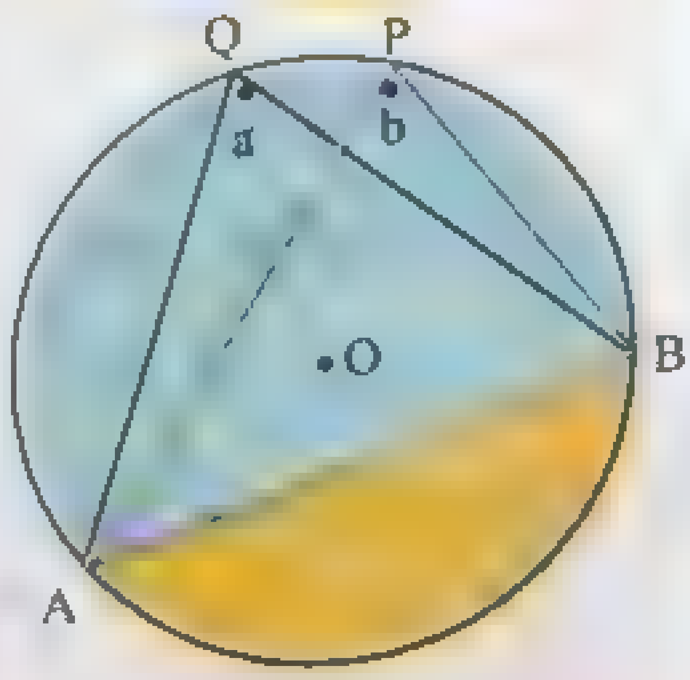


10.8.8 The angles in the same segment of circle are equal

In the following figure, the two angles PRQ and PSQ are in the major segment of the circle So we say these angles are in the same segment. Note that the chord PQ divides the circle in two segments



We observe that the angles subtended (made) by the same arc at the circumference are equal That is, a = b Try to measure these angles



Exercise 10.3

1. Draw circles with radii.

(i) 3.5cm (ii) 4.4cm

2. Draw circles with diameters

(i) 6.8cm (ii) 8.6cm

3. Find the length of the diameter of the circle whose radius is

(i) 5cm (ii) 8.6cm

4. Find the length of the radius of the circle whose diameter is

(i) 11cm (ii) 14mm

5. Draw a circle with centre O and any radius. Draw any chord not passing through the centre. Also draw a diameter of the circle. Name the chord and the diameter.

6. Draw a circle with cetre O. Draw any four diameters. How many diameters can you draw in this circle?

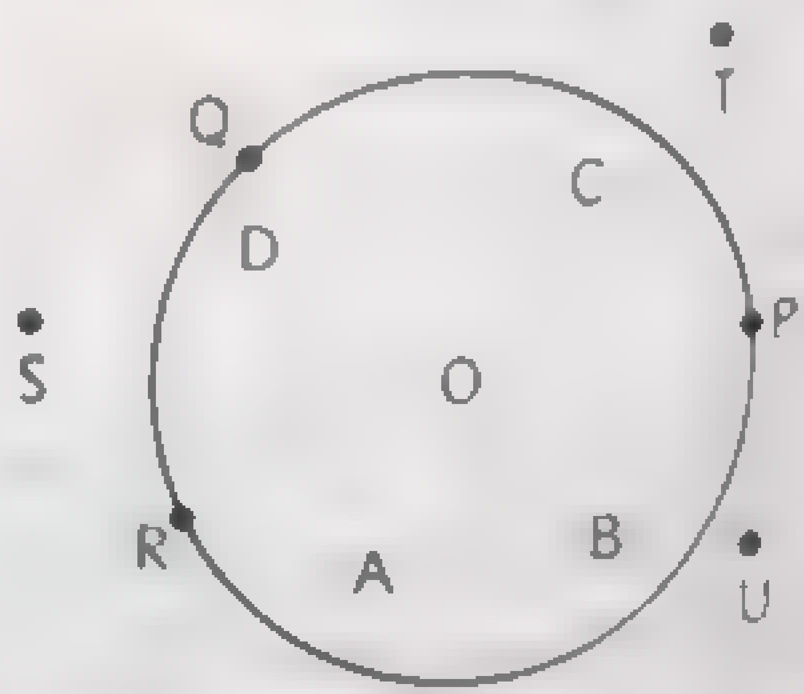
7. Draw a circle with centre O and radius 3 cm. In it draw any $\widehat{AB}$

8. In fig, name the points which are

(i) in its exterior

(ii) in its interior

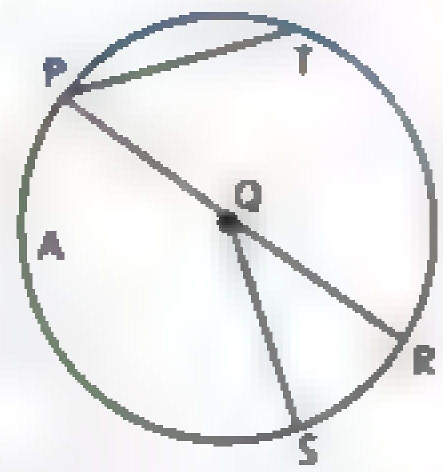
(iii) on the circle



9. Two points A and B are given. Draw any circle whose centre is A and which contains B in its interior.

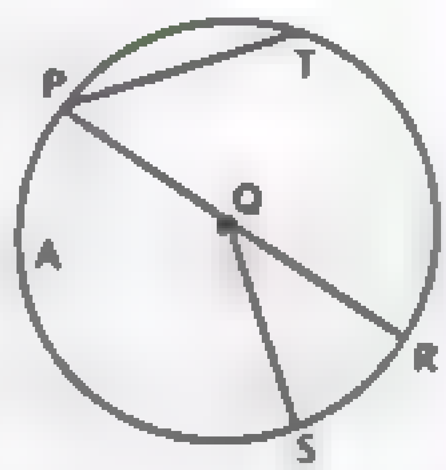
REVIEW EXERCISE 10

1. Encircle the correct choice



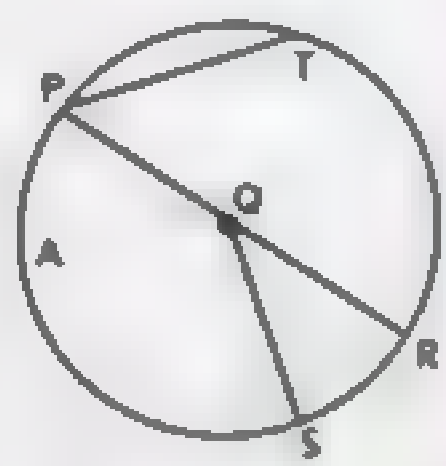
(i) Which of the following is a chord, but not a diameter?

a. $\overline{PR}$
b. $\overline{QS}$
c. $\overline{PT}$
d. None of the above



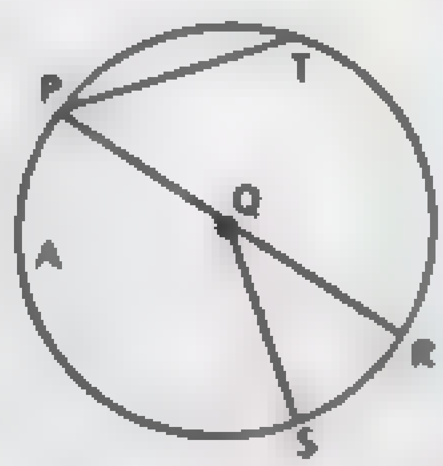
(ii) Which of the following is a radius?

a. $\overline{PQ}$
b. $\overline{QR}$
c. $\overline{QS}$
d. All of the above



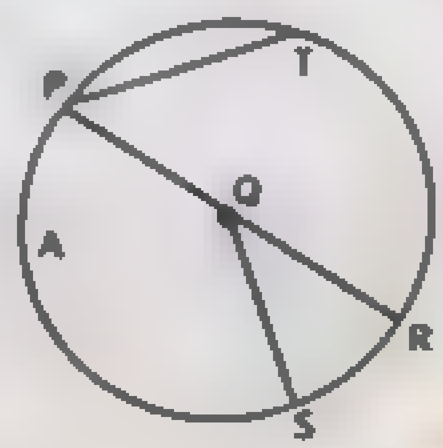
(iii) Name the center of this circle.

a. Point Q
b. Point R
c. Point P
d. None of the above



(iv) What is $\overline{PR}$ (or PQR)?

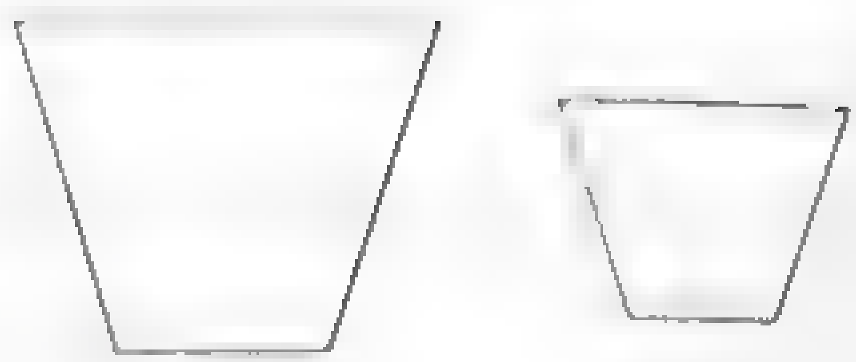
a. Diameter
b. Radius
c. Center
d. None of the above



(v) If $\overline{PQ}$ is 3 cm long, then how long is $\overline{PR}$?

a 1.5cm
b 12cm
c. 6cm
d. None of the above

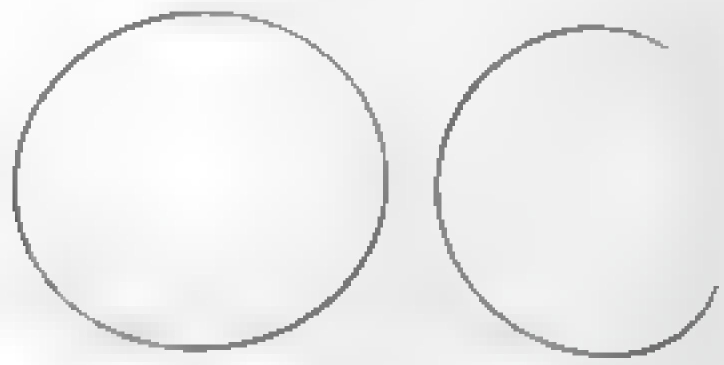
2. Fill in the blanks



The two figures are

________ not similar

________ not congruent



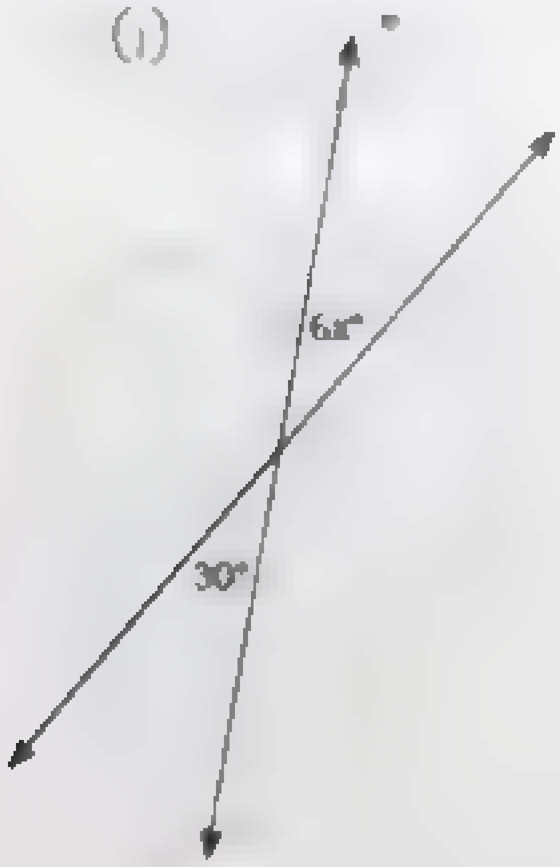
The two figures are

_______ similar and congruent

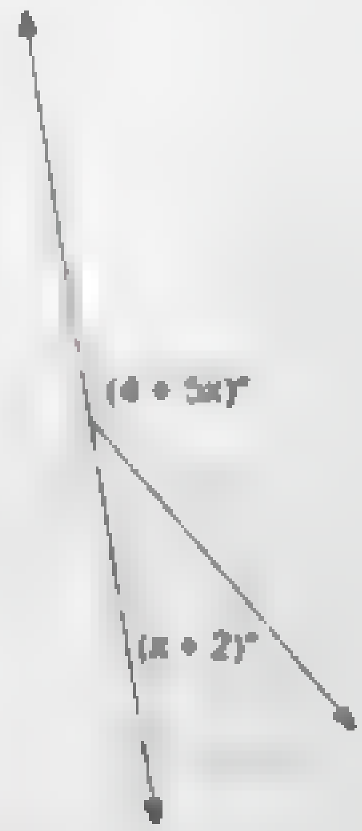
_______ not similar and not congruent

3. Write and solve an equation to find the missing angle measures.

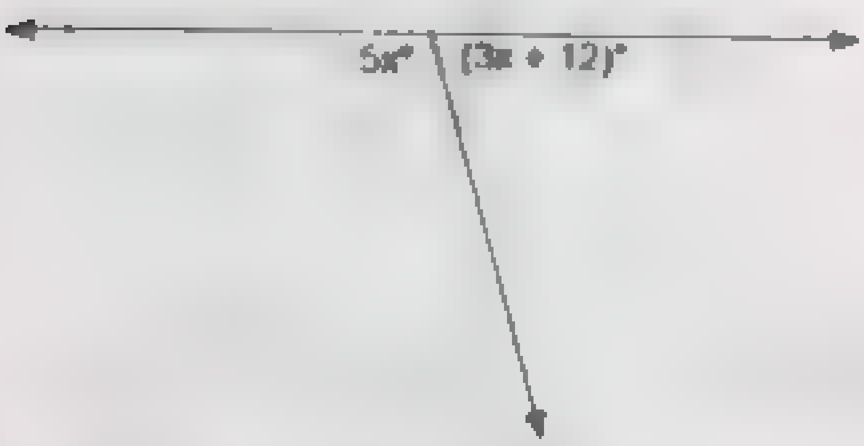
(i)



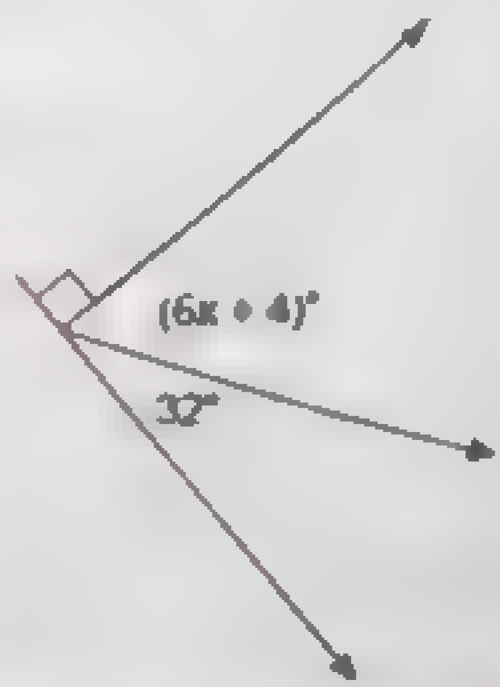
(ii)



(iii)



(iv)



4. What is the measure of an angle, if three is subtracted from twice the supplement and the result is 297 degrees?

Glossary

- ...jacent angles: Two angles are adjacent if they have a common side and a common vertex (corner point) and their intersection is null set
- A pair of angles whose sum is 90°
- A pair of angles whose sign is 180°
- are two angles whose sides form two pairs of opposite rays (straight lines)
- Two objects are said to be congruent if they are same in the size and shape.
- Similar angles: Two shapes are said to be similar when the shape is same but they only differ is in size.
- A **circle** is a shape with all its points at the same distance from a fixed point known as centre of the circle.
- The length of the line joining two points of a circle through the center is called the **diameter.**
- The **radius** of a circle is the distance from the center of a circle to any point on the circle.
- A chord is a line segment joining two points on a circle.
- An arc is a part or a portion of the circumference of a circle.
- Half of a circle is called semi circle. Every circle can be divided into two semi-circle.
- Segments: A chord of a circle divides the circle into two regions, which are called the segments of the circle.
- Minor segment: The **minor segment** is the region bounded by the chord and the minor arc.
- Major segment: The **major segment** is the region bounded by the chord and the major arc.

Unit 11 Practical Geometry

What You'll Learn

- Divide a line segment into the given number of equal segments
- Divide a line segment internally in the given ratio
- Construct a triangle when its perimeter and the ratio among the lengths of its sides are given.
- Construct an equilateral triangle.
  - Base is given
  - Altitude is given.

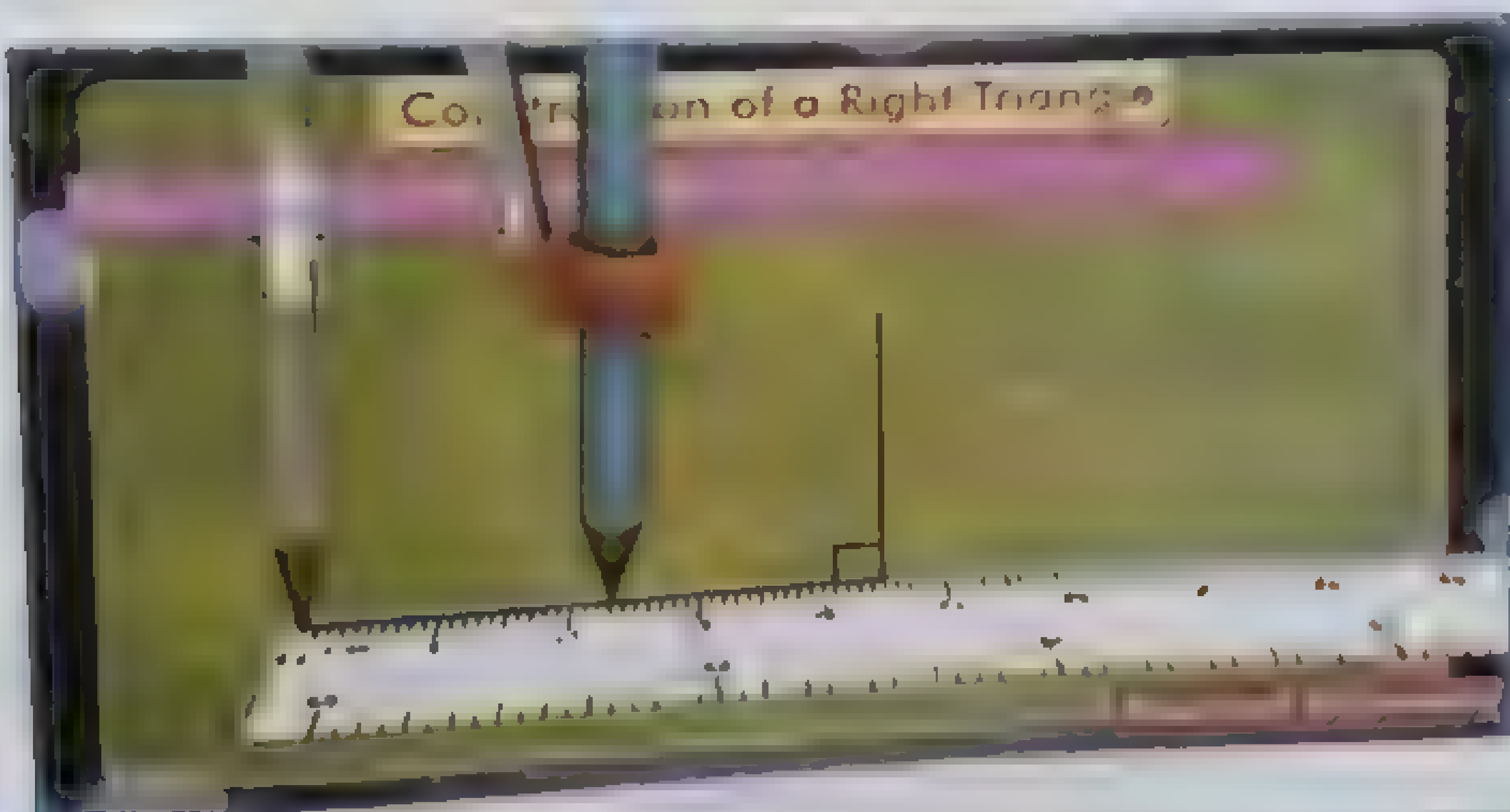
- Construct an isosceles triangle when
  - Base and the base angle are given
  - Vertical angle and the altitude are given
  - Altitude and base angle are given
- Construct a parallelogram when.
  - Two adjacent sides and their included angle are given
  - Two adjacent sides and a diagonal are given
- Verify practically that the sum of.
  - Measures of the angles of a triangle is 180°
  - Measures of angles of a quadrilateral is 360°

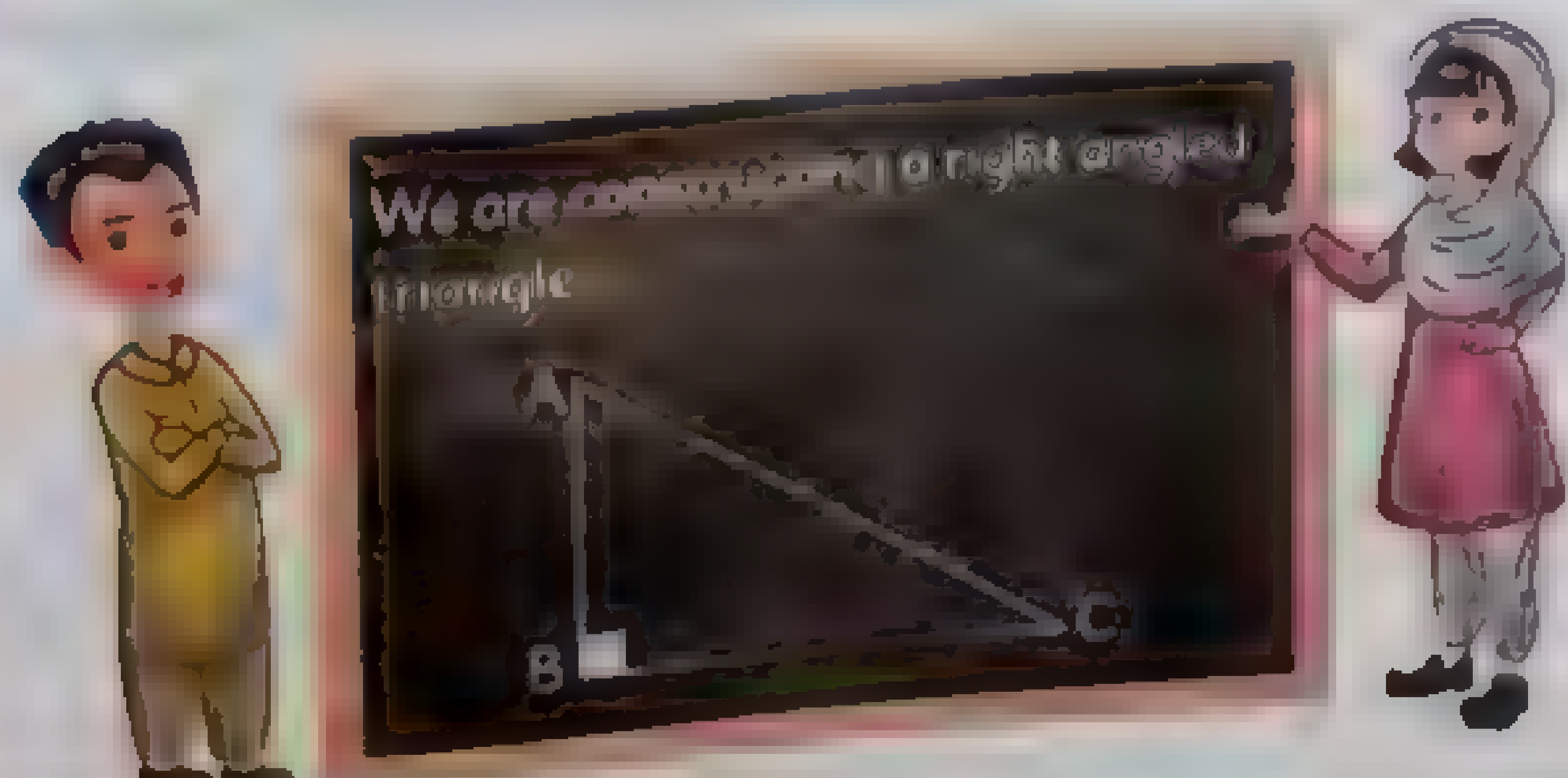
Tidbit

Geometry is all about shapes and their properties. If you like playing with objects, or like drawing, then geometry is for you.

It's Important

t is a very famous saying that "when I hear I forget, when I see I remember and when I do, I understand". Keeping this in view, the importance of practical work in geometry can be judged easily. Practical geometry not only providesbase for architectural engineering, general engineering designs, medical science and many more fields but also provides an opportunity to the students to explore many more ideas.



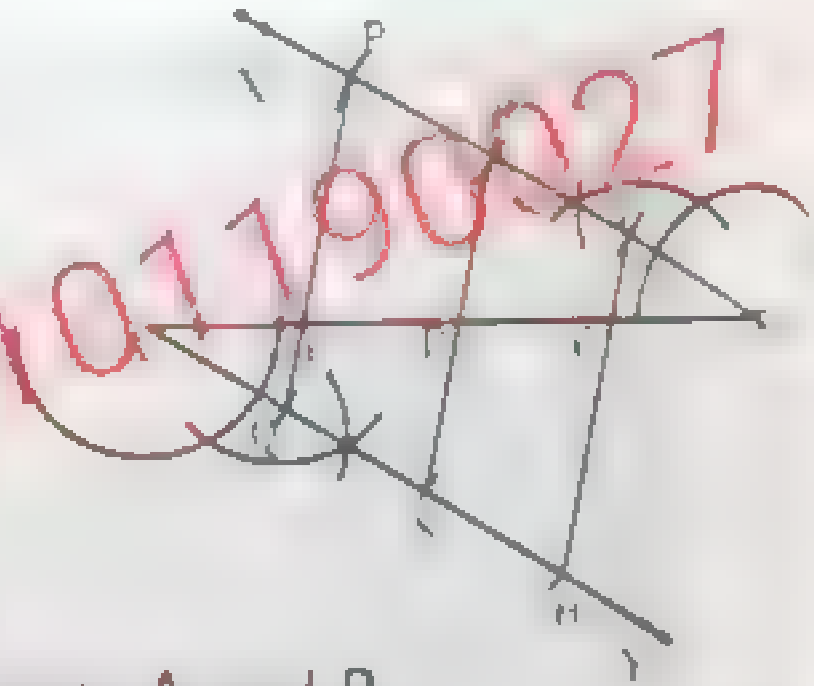


11.1 Dividing a line segment into the given number of equal segments

The following example will make it clear how to divide a line segment into equal segment. This method will enable us to divide any distance into required equal segments or parts.

Example 1 Divide a 10 cm long line segment into four equal parts

Solution



Steps of Construction

1. Draw a line segment $\overline{AB}$ of measure 10 cm
2. Draw arcs of equal radius on its two ends, but on the opposite side of $\overline{AB}$
3. Construct equal angles $\angle ABX$ and $\angle BAY$ of any measure with a pair of compasses at points A and B
4. Draw three arcs (i.e. one less than the required parts) of the same radius on $\overrightarrow{BX}$ and $\overrightarrow{AY}$. We get six congruent segments $\overline{BT}$, $\overline{TR}$ and $\overline{RP}$ on and $\overline{AQ}$, $\overline{QS}$ and $\overline{SH}$ on $\overline{AY}$.
5. Now Join (T to H), (R to S) and P to Q
6. The lines segments so produced intersect $\overline{AB}$ at E, D and C

Hence $\overline{BC}$, $\overline{CD}$, $\overline{DE}$ and $\overline{EA}$ are the required equal segments

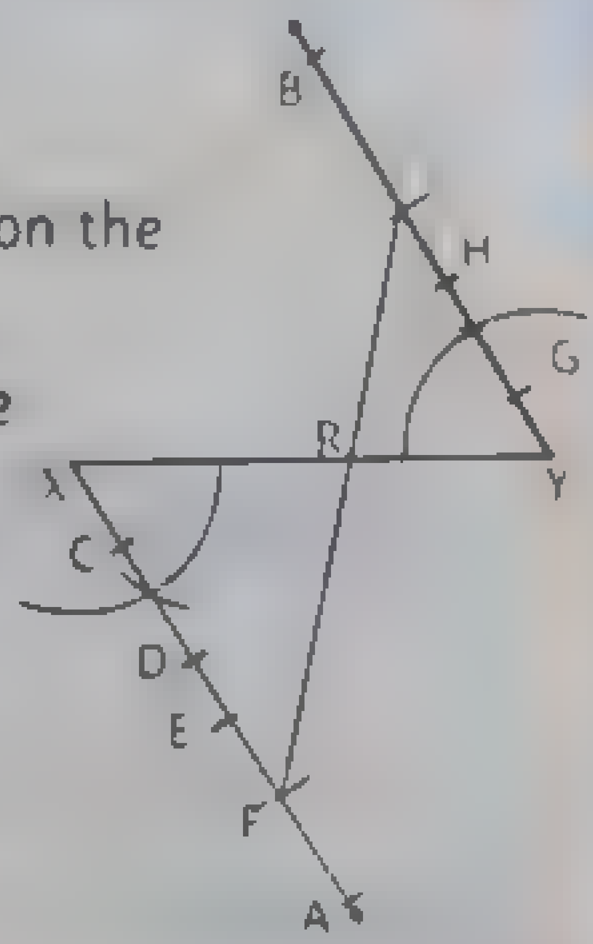
11.2 Division of a line segment internally in the given ratio

The procedure for division of a line segment in a given ratio is more or less the same as in 11.1. However, it differs slightly as the sum of the ratio is determined first. The length of each part is then determined from the overall length of the line segment.

The following example will make it clear

Example (2) Divide a line segment of 7 cm in the ratio 4 3

Steps of Construction



1. Draw line segment XY of measure 7 cm
2. Construct angles of equal measure on both ends on the opposite sides of XY
3. Draw 4 arcs on XA and 3 arcs on YB of the same radius at points X and Y respectively
4. Join the last point on XA i.e. F to the last point on YB i.e. I.
5. FI intersects XY each other at point R

The point R divides XY in two segments XB and YR in the ratio 4 : 3.

11.3 Construction of a triangle when its perimeter and the ratio among the lengths of its sides are given.

Example (3)

Construct a triangle ABC, whose perimeter is 12 cm and the ratio among its sides is 5 : 4 : 3.

Solution To construct the required triangle first we have to find the original lengths of the three sides with the help of the given ratios

Perimeter $= 12cm$

Sum of the ratios $= 5 + 4 + 3 = 12$

Length of the first side $= \frac{5}{\cancel{12}} \times \cancel{12}$

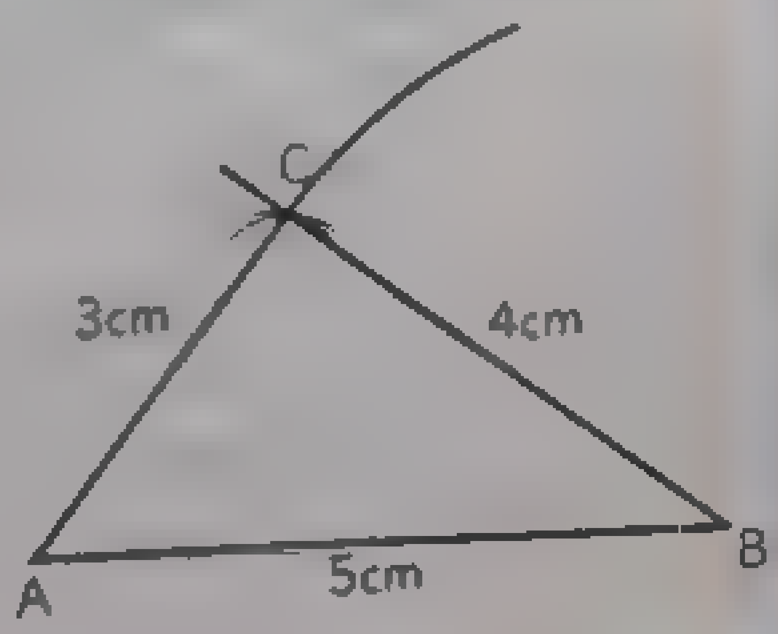
$= 5cm$

Length of the second side $= \frac{4}{\cancel{12}} \times \cancel{12}$

$= 4cm$

Length of the 3rd side $= \frac{3}{12} \times \cancel{12}$

$= 3cm$



Steps of Construction

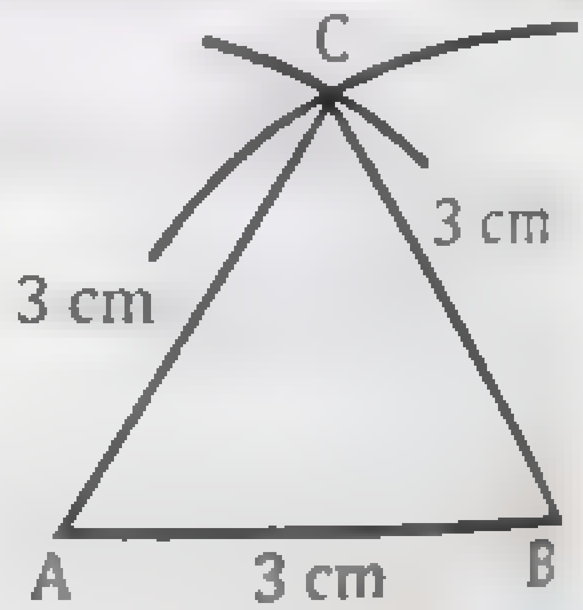
1. Draw $\overline{AB}$ of the measure 5 cm (It is recommended that the longest side be taken as the base).
2. Draw an arc of radius 4 cm with B as centre and another arc of radius 3cm by taking A as centre. Both the arcs intersect at C.
3. Join C to A and B. Thus ABC is the required triangle.

11.3.1 Constructing an equilateral triangle when the base is given

An equilateral triangle is the one whose all three sides are of equal length and each angle measure 60°.

Example 4 Construct an equilateral triangle ABC whose base is 3 cm

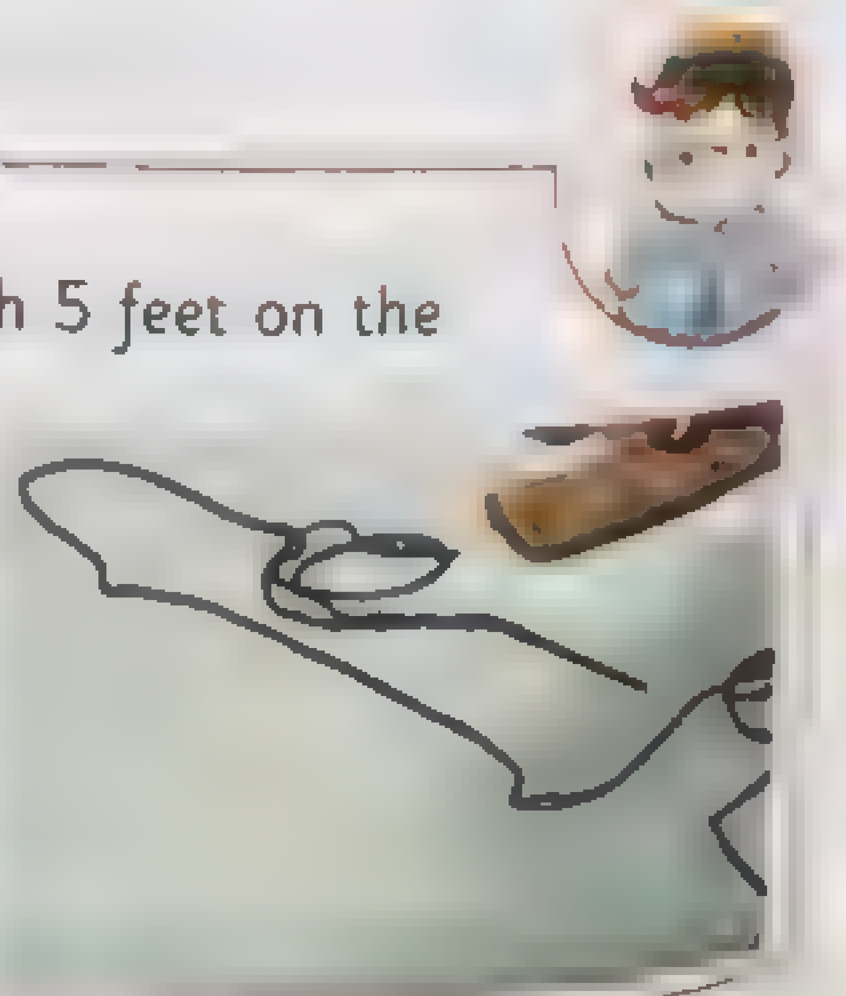
Steps of Construction



1. Draw $\overline{AB}$ 3 cm long.
   Draw two arcs of radius 3 cm at each
2. A and B both above $\overline{AB}$.
3. The arcs intersect at C.
4. Join C to A and B.
   Thus ABC is the required equilateral triangle.

Activity

Construct an equilateral triangle of side length 5 feet on the floor of your classroom or on the ground.



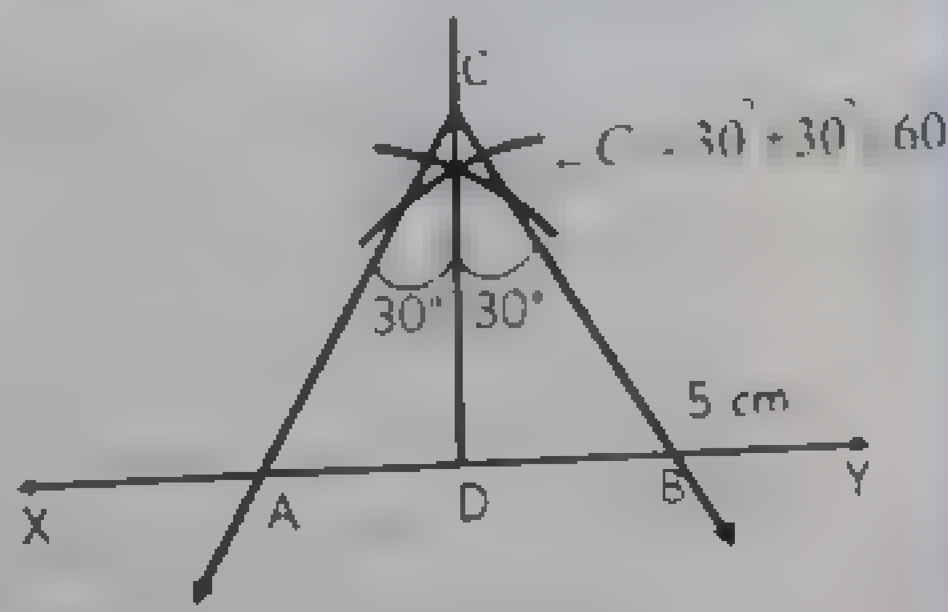
Draw a line AB of length of 5 ft as base. We cannot use a compass, so we use a nail and thread Measure a thread now of required length of 5 ft and fasten it to a nail.This can be now used as a compass

iii. Repeat the process as in 11.4.1

11.3.2 Constructing an equilateral triangle when its altitude is given.

The height or altitude, of a triangle is a line segment that starts from any of its vertex and is perpendicular to the opposite side of the triangle.

Example 5 Construct an equilateral triangle whose altitude is 5 cm



Steps of Construction

1. Draw a line $\overleftrightarrow{XY}$ of any length.
2. Draw a perpendicular $\overleftrightarrow{DC}$ on $\overleftrightarrow{XY}$
3. Cut off $\overline{DC}$ of the required length i.e. 5cm.
4. Draw two angles of 30° at C on the both sides of altitude $\overline{CD}$
5. Extend the two rays to intersect $\overleftrightarrow{XY}$ at A and B.

Thus ΔABC is the required triangle in which CD is an altitude of 5 cm long.

Guided Practice

Construct an equilateral triangle whose altitude is,

i. 6cm ii. 8cm

Activity

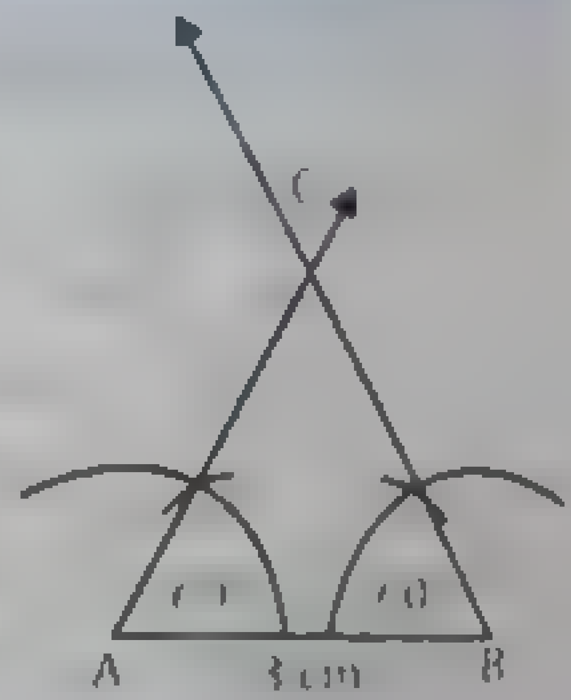
Draw an equilateral triangle of altitude 5 feet on the floor of your classroom or on the ground.

11.4 Constructing an isosceles triangle

An isosceles triangle is one in which the two sides are congruent. Angles opposite to each side are also equal however the third angle is different.

11.4.1 Constructing an isosceles triangle when the measure of its base and base angles are given

Example 6



Construct an isosceles triangle when the base is 3 cm and a base angle is 60°

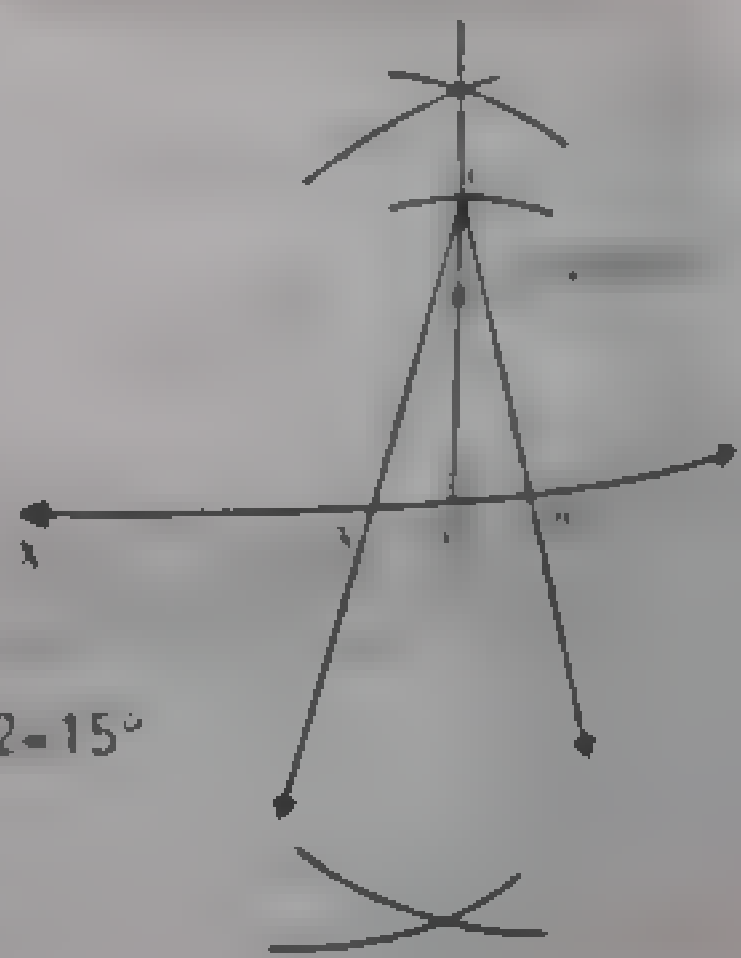
Steps of Construction

1. Draw AB 3cm long
2. Construct 60° angle at both the points A and B
3. Extend the ray to meet each other at C

Thus triangle ABC is the required triangle

11.4.2 Constructing an isosceles triangle when the vertical angle and altitude are given.

Example 7 Construct an isosceles triangle in which the altitude is 3 cm and the vertical angle is 30°



Steps of construction

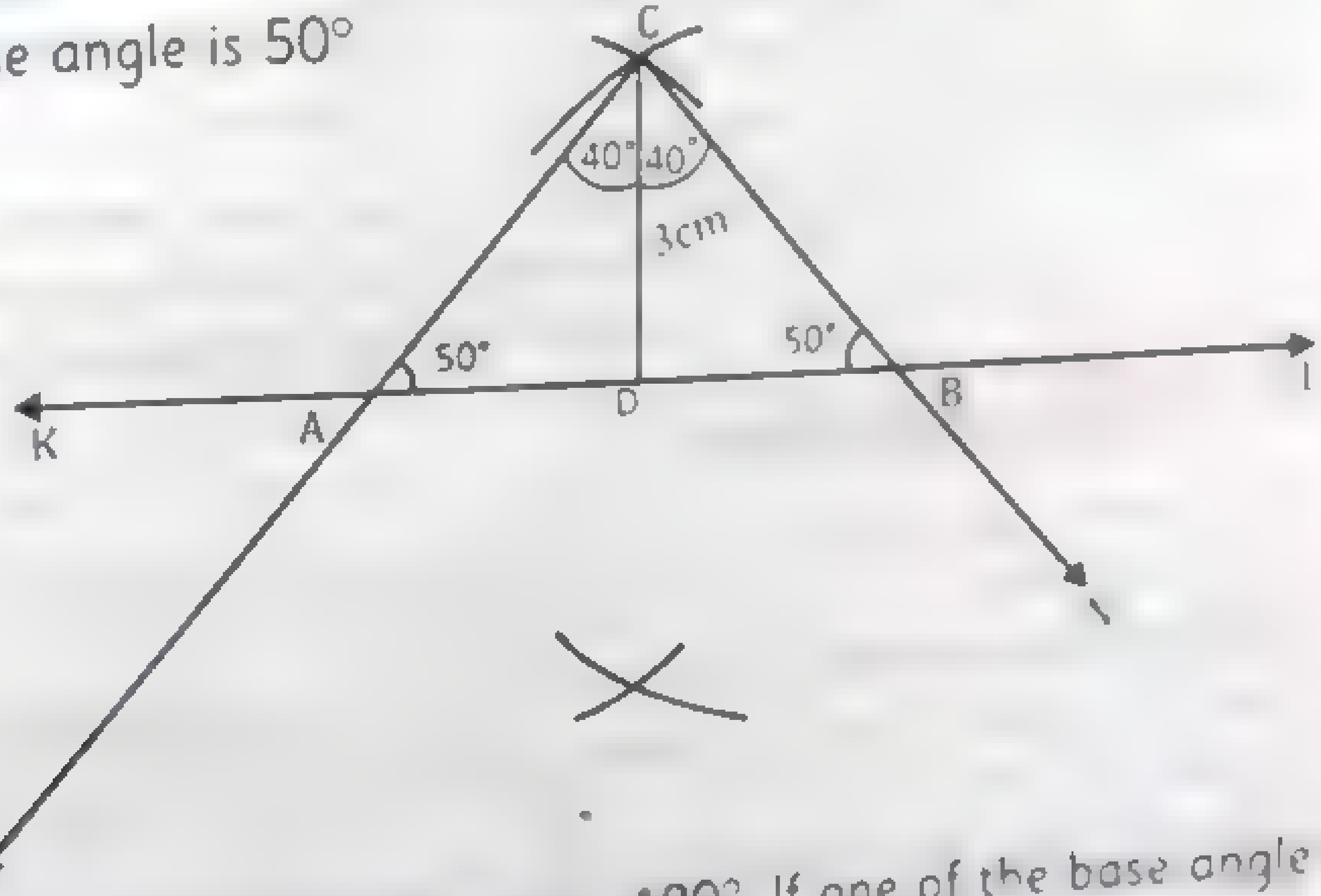
1. Draw a line $\overleftrightarrow{XY}$
2. Draw a perpendicular $\overleftrightarrow{CD}$ on $\overleftrightarrow{XY}$
3. Join the intersection of two arcs by a ruler and draw line up to $\overleftrightarrow{XY}$
4. Cut mCD = 3cm This is the required altitude
5. Construct half of the vertical angle (i.e. 30° ÷ 2=15°) on both sides of altitude.
6. Extend the rays to intersect $\overleftrightarrow{XY}$ at A and B

Hence ABC is the required triangle

11.4.3 Constructing an isosceles triangle when the altitude and a base angle are given

In isosceles triangle the two base angles are of the same magnitude

Example 8 Construct an isosceles triangle ABC in which altitude is 3 cm and the base angle is 50°



Solution

In a triangle the sum of three angles is 180°. If one of the base angle is 50° then the second base angle is also 50°. So,

$$\text{Vertical angle} = 180° - (\text{sum of base angles})$$
$$180° - (50° + 50°)$$
$$= 180° - 100°$$
$$= 80°$$
$$80° = 40° + 40°$$

Steps of Construction

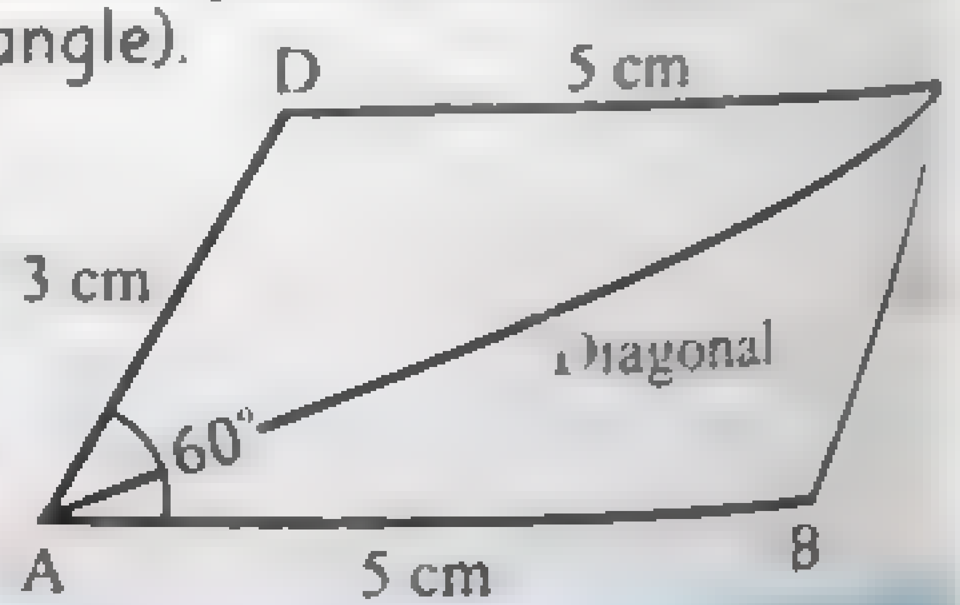
1. Draw a line KL
2. Draw an altitude DC 3 cm long
3. Construct angles of 40° at C on both sides of CD
   40° + 40° = 80°
4. Rays of the angles cut KL at points A and B
   Thus ABC is the required triangle

Exercise 11.1

1. Construct the following
   - A triangle ABC whose perimeter is 18 cm and the ratio among the three sides ...
   - A triangle PQR whose perimeter is16cm and the ratio among the sides ...
2. Construct the required equilateral triangles
   - i. XYZ whose base is 3 cm
   - ii. KLM whose altitude is 4 cm
3. Construct isosceles triangles
   - ABC whose base is 5cm and base angle is 45°
   - RST with an altitude 7 cm and vertical angle of 30°
   - XYZ with 5cm altitude and 30°base angle.

11.5 Parallelogram

The quadrilateral figure (four sided) with two facing sides of equal length and parallel but no angle of 90°(right angle). Below is given a parallelogram.

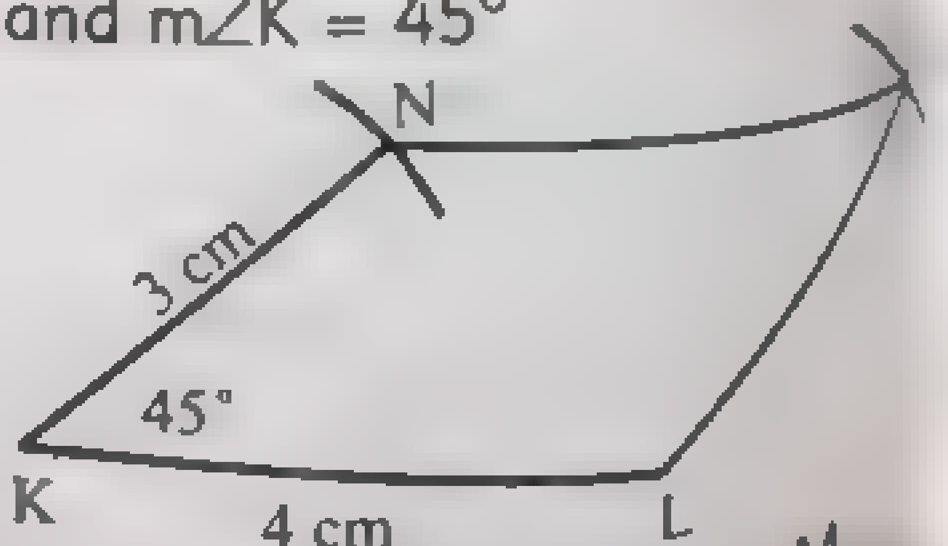


It can be seen that the two facing sides AD and BC, AB and DC are of equal length i.e 3 cm and 5cm respectively. AC is a diagonal and measure of angel DAB is 60°.

11.5.1 Constructing a parallelogram when the measure of two adjacent sides and their included angle are given.

Example 9 Construct a parallelogram KLMN with the following measurements mKL = 4cm, mKN = 3cm and m∠K = 45°

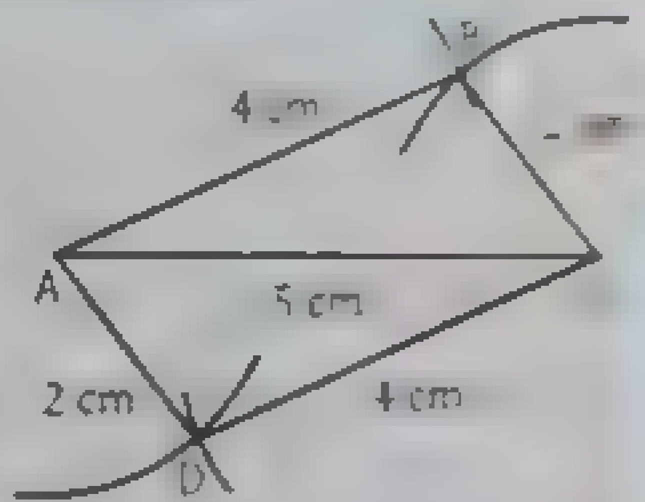
Steps of Construction



1. Draw a line segment KL = 4cm
2. Construct an angle of 45° at K.
3. Cut off KN measuring 3 cm.
4. Draw an arc of radius 3cm from L and another arc of 4 cm from N, which meet each other at M
5. Join N to M, and M to L. Hence KLMN is the required parallelogram

11.5.2 Constructing a parallelogram when the measure of two adjacent sides and a diagonal are given

In the construction of such parallelogram first we draw the diagonal and the four sides are constructed according to the requirement



Example 10

Construct a parallelogram ABCD such that m$\overline{AB}$ = 4cm, m$\overline{BC}$ = 2cm and m$\overline{AC}$ = 5cm

Steps of construction

1. Draw a line segment AC 5cm long
2. Draw an arc of radius 2 cm above at C and below AC at A
3. Draw an are of radius 4 cm above AC at A and below AC at C to intersect the other arc at B and D respectively
4. Join the intersecting points with A and C

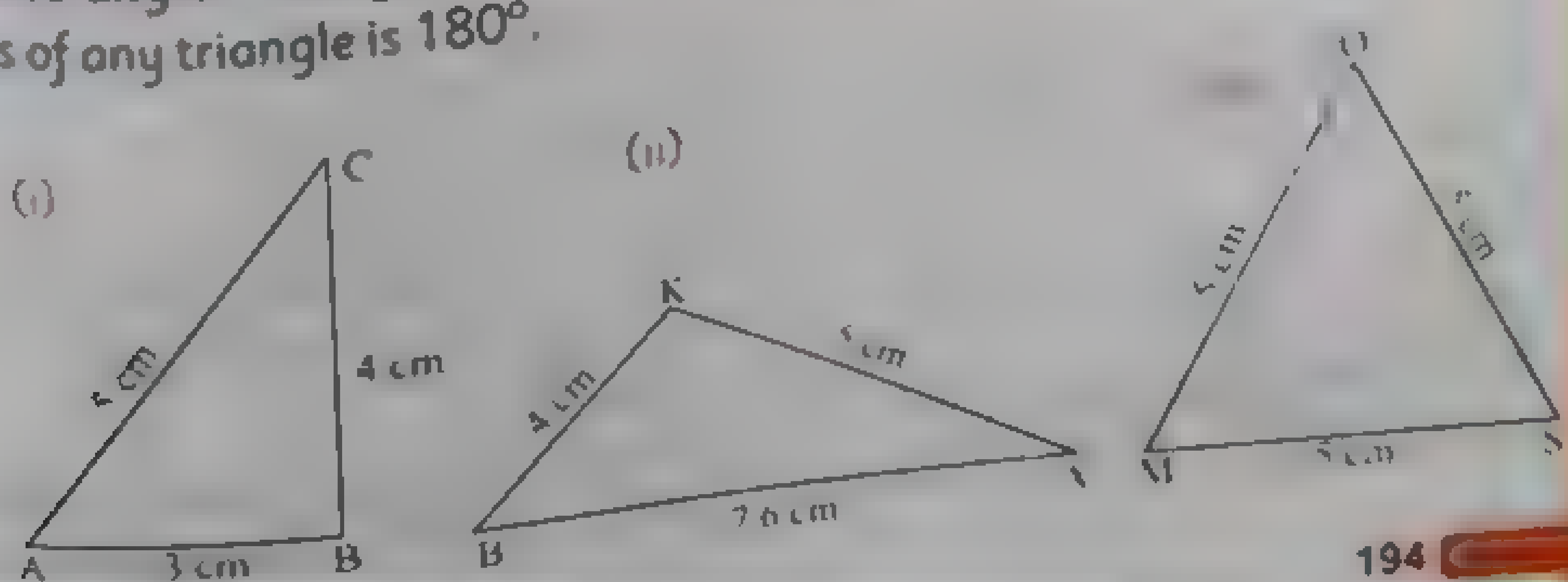
Hence ABCD is the required parallelogram

11.6 Practical Verification of the sum of all the angles of a triangle and of a quadrilateral

11.6.1 A triangle no matter what be the length of its three sides sum of its three angles is always 180°.

Example 11

Copy the given triangles in your note book and measure the three angles using protractor to verify that the sum of the three interior angles of any triangle is 180°.



Solution(i)

Put protractor on point A and measure angle m∠BAC = 37°

Now measuring angle m∠BCA = 53°

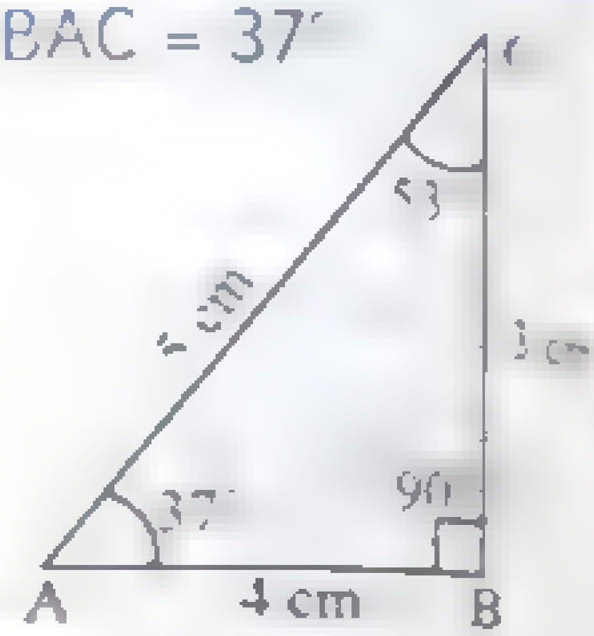
Measuring angle m∠ABC = 90°

Adding the measures of the three angles

$m\angle BAC + m\angle BCA + m\angle ABC$

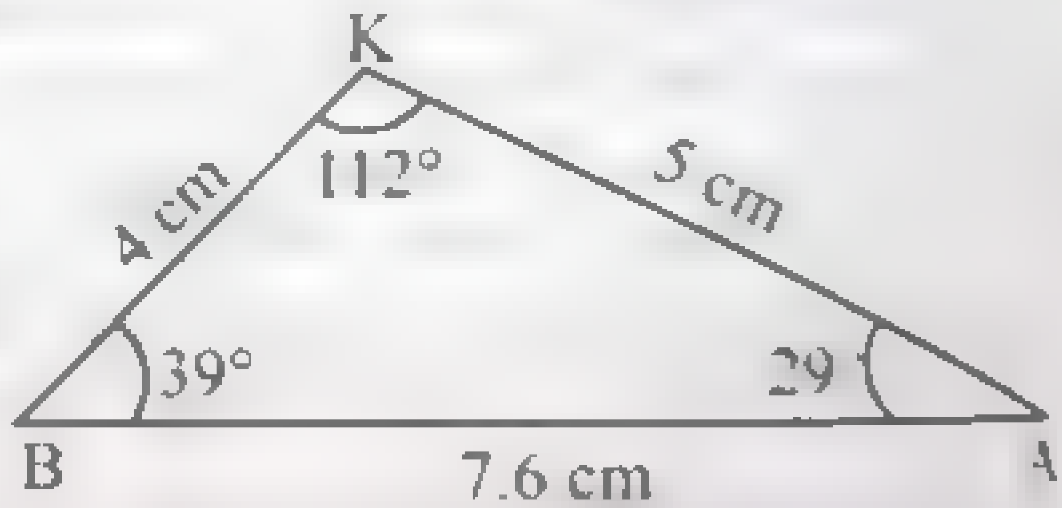
$= 37^\circ + 53^\circ + 90^\circ$

$= 180^\circ$



Hence it is practically verified that the sum of three interior angles of a triangle is 180°

Solution(ii)



Measuring angle B, $m\angle B = 39^\circ$

Measuring angle A, $m\angle A = 29^\circ$

Measuring angle K, $m\angle K = 112^\circ$

Adding the three angles

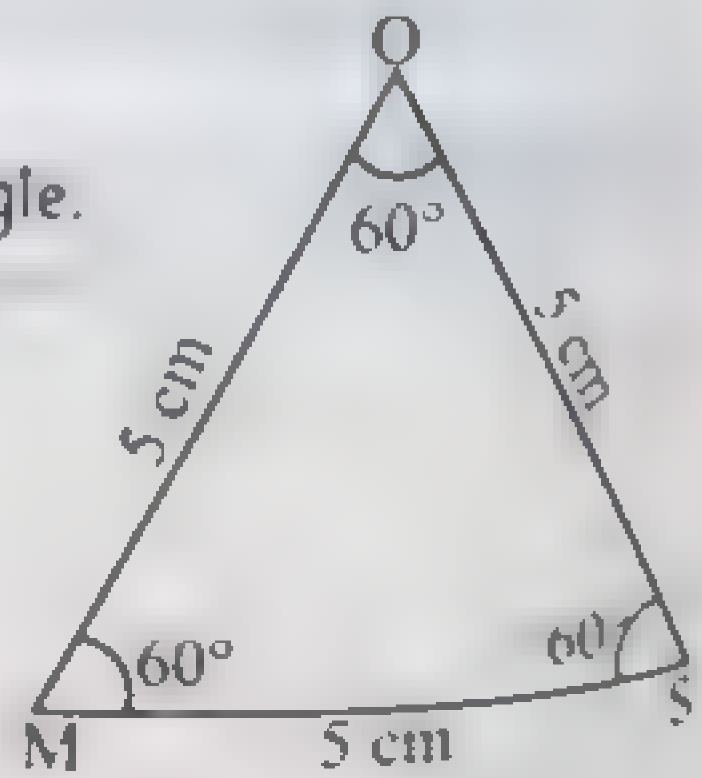
$m\angle B + m\angle A + m\angle K$

$= 39^\circ + 29^\circ + 112^\circ$

$= 180^\circ$

Hence the sum of the interior angels is 180°

Solution(iii)



Put the protractor on M and measure the angle.

The same procedure is repeated for S and O

$m\angle M = 60^\circ$

$m\angle S = 60^\circ$

$m\angle O = 60^\circ$

The sum of three angles

$m\angle M + m\angle S + m\angle O$

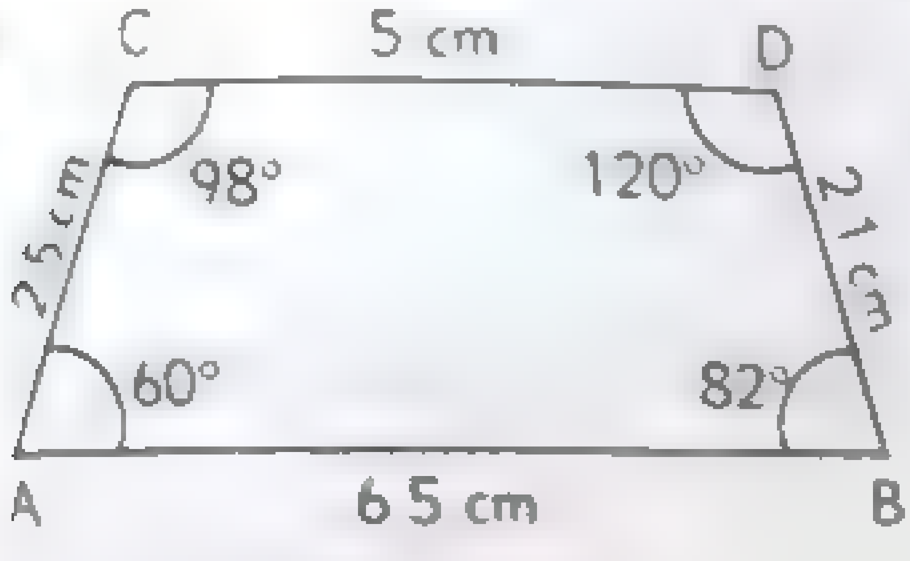
$= 60^\circ + 60^\circ + 60^\circ$

$= 180^\circ$

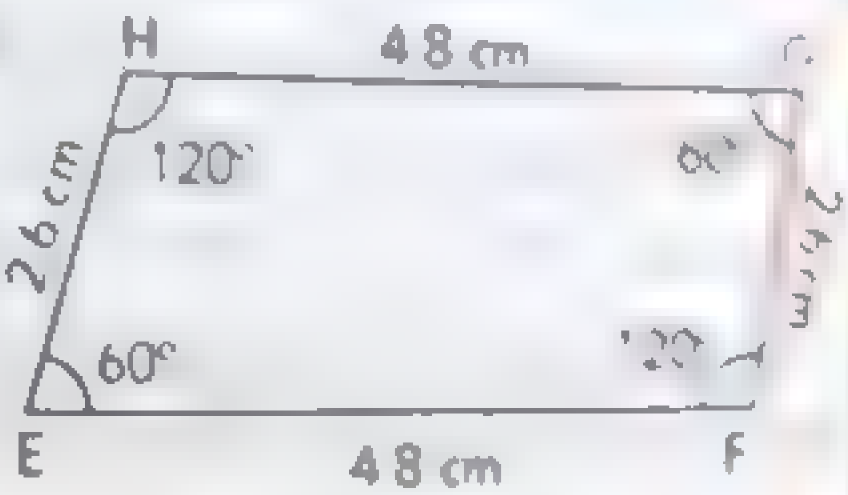
A geometrical figure bounded by four sides is called quadrilateral Trapezoid parallelogram, rectangle rhombus and a square are all quadrilaterals

[illegible] 12

Measure the angels of the following figures and prove that the sum of the four interior angles of each is 360°

(i) 

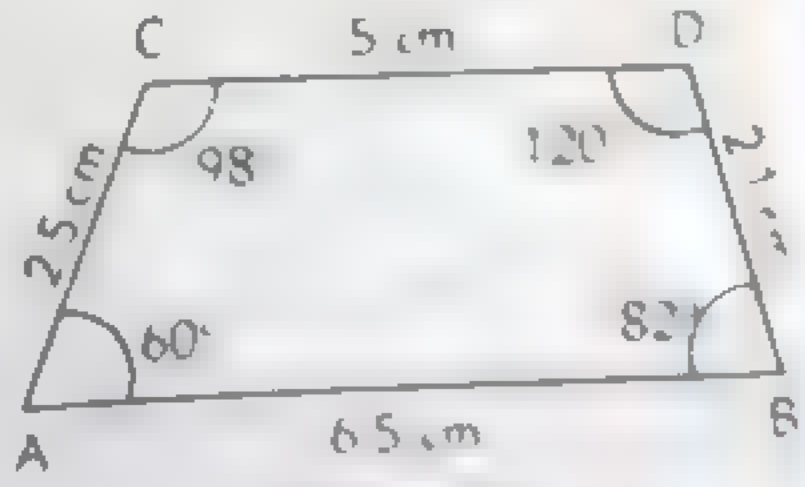
(Trapezoid)

(ii) 

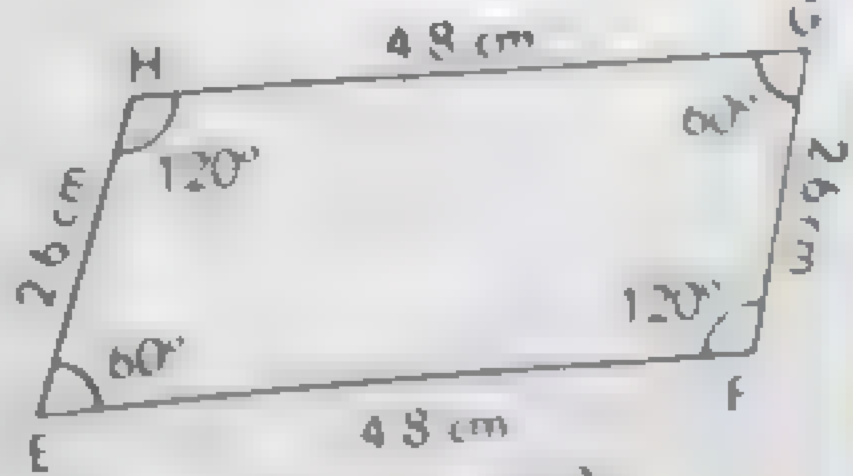
(Parallelogram)

Solution

(i) Measure angle A, $m\angle A = 60°$
Measure angle B, $m\angle B = 82°$
Measure angle C, $m\angle C = 98°$
Measure angle D, $m\angle D = 120°$
Adding the four angles
$m\angle A + m\angle B + m\angle C + m\angle D$
$= 60° + 82° + 98° + 120°$
$= 360°$



(ii) i Measure angle E, $m\angle E = 60°$
ii Measure angle F, $m\angle F = 120°$
iii Measure angle G, $m\angle G = 60°$
iv Measure angle H, $m\angle H = 120°$
Adding the four angles
$m\angle E + m\angle F + m\angle F + m\angle H$
$= 60° + 120° + 60° + 120°$
$= 360°$

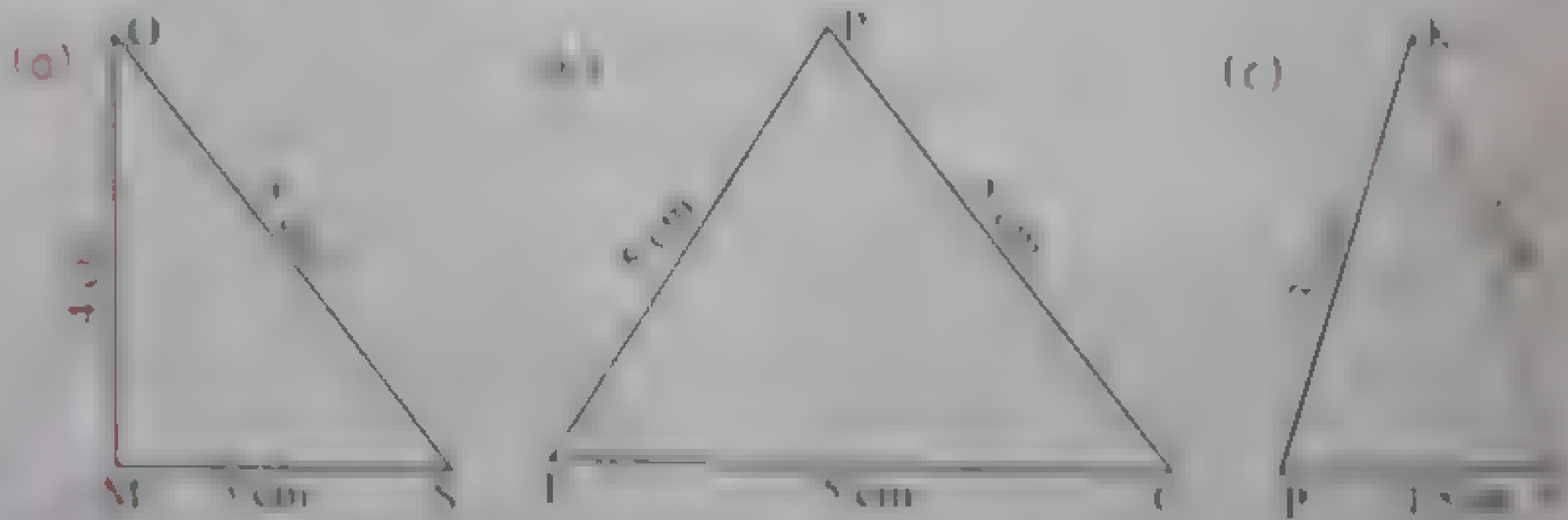


(Parallelogram)

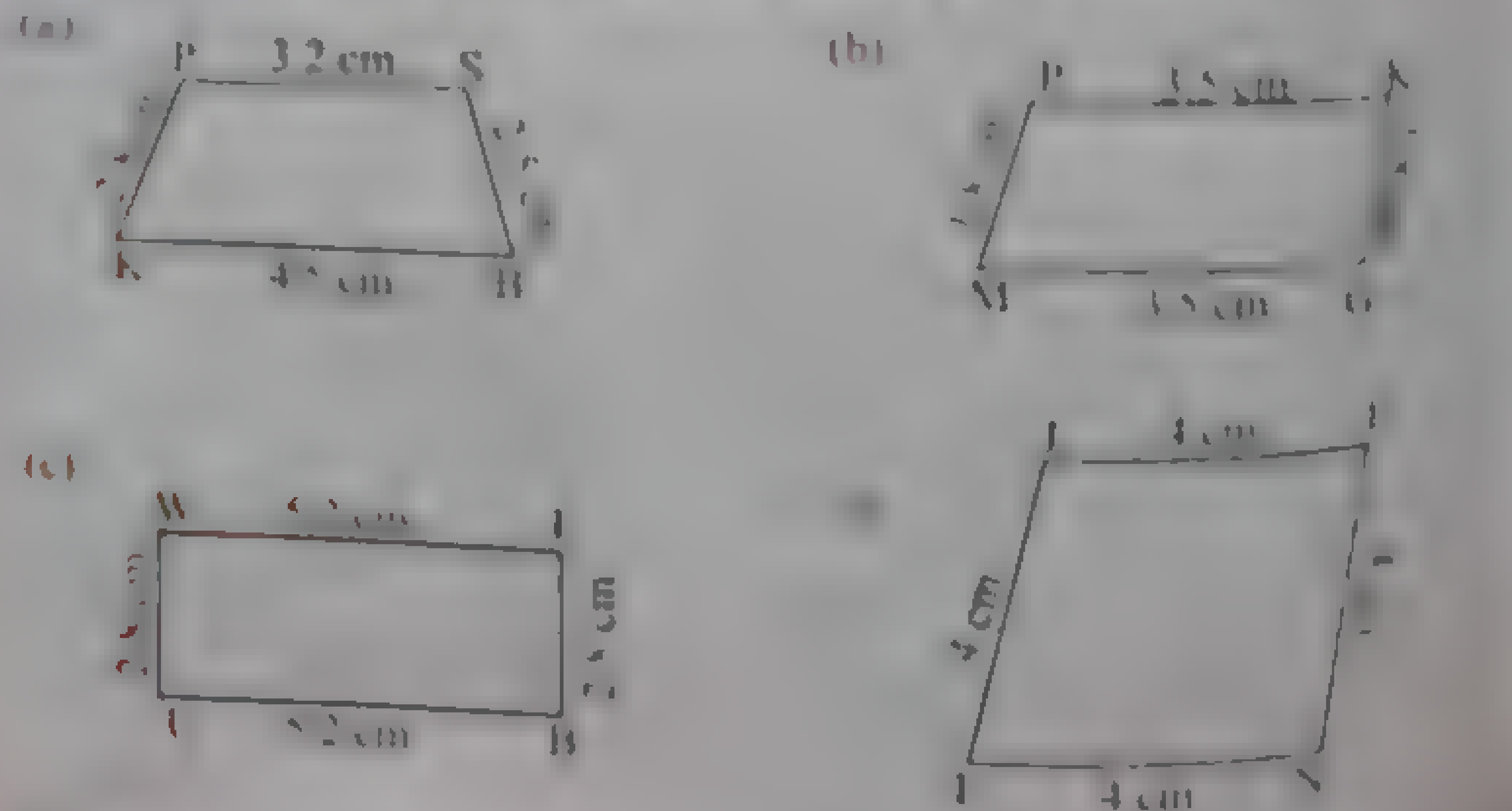
Exercise 11.2

1. Construct the following parallelograms.
   (i) PQRS in which $\overline{PQ}$ = [illegible] cm, $\overline{QR}$ = [illegible] cm and m∠Q = [illegible]
   (ii) ABCD in which $\overline{AB}$ = [illegible] cm, $\overline{BC}$ = [illegible] cm and m∠B = [illegible]
   (iii) [illegible] in which $\overline{[illegible]}$ = [illegible] cm, $\overline{[illegible]}$ = [illegible] cm and m∠[illegible] = [illegible]
   (iv) [illegible] in which $\overline{[illegible]}$ = [illegible] cm, m∠B = [illegible] and $\overline{MC}$ = [illegible] cm

2. Copy the following triangles in your note book and verify that the sum of three angles of each triangle is 180°.



3. Copy the following quadrilateral in your note book. Measure the four angles and prove that the sum of four angles of each figure is 360°.



REVIEW EXERCISE 11

1. Divide a 10 cm line segment in 5 equal parts
2. Divide 5 cm line segments in the ratio of 3:2
3. Construct a isosceles $\triangle$ PQR whose altitude is 5cm and a base angle is 45°
4. Construct a parallelogram SACT with two adjacent sides measuring [illegible] and 3 cm at 60° with each other
5. Construct a parallelogram ABCD such that $m\overline{BC} = 4.5cm$, $m\overline{AB} = 3cm$ and $m\angle B = 45^\circ$

Project

(It might be somewhat tricky) Using the procedure in Q 1 above divide the floor of your classroom (or your room) in 3 equal parts After that divide it in the ratio 3 and 2 and ask you teacher (parents) for verification (Use only chalk sticks for marking)

Glossary

- Triangle: A closed geometrical figure bounded by three sides
- Acute triangle: A triangle in which all the three angles are less than 90°
- Obtuse triangle: A triangle having one angle greater than 90°
- Equilateral triangle: A triangle whose all sides are congruent
- Isosceles triangle: A triangle whose two sides are congruent
- Quadrilateral: A closed figure having four sides
- Trapezium: A quadrilateral having one pair of parallel lines
- Parallelogram: A quadrilateral having opposite sides parallel and equal but no angle of 90°.
- Rectangle: A quadrilateral having facing sides equal with all angles of 90°
- Rhombus: A quadrilateral having four congruent sides with no angle of 90°
- Square: A quadrilateral with four congruent sides and all angles of 90

Unit 12 Circumference, Area and Volume

You will Learn

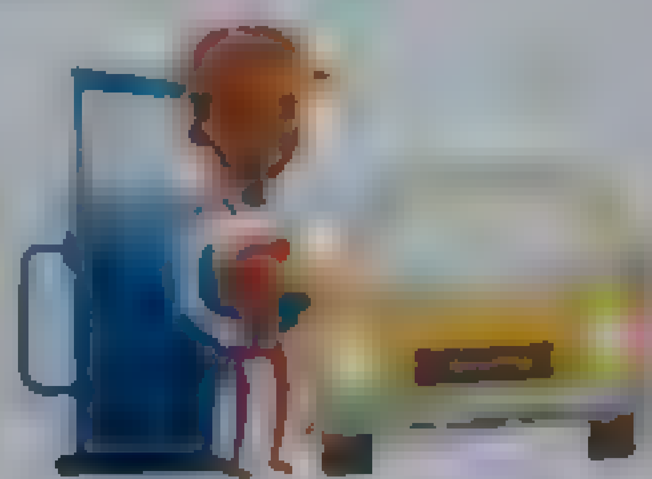
- Express the ratio between the circumference and diameter of a circle
- Find the circumference of a circle using formula
- Find the area of a circular region using formula
- Find the surface area of a cylinder using formula
- Find the volume of cylindrical region using formula
- Solve the real life problems involving the circumference and area of a circle, surface area and volume of a cylinder

Why It's Important

If you want to know how far a wheel will travel with each rotation, or how many rotations a wheel will make when it travels a given distance, you need to be able to calculate circumference.

In construction, surface area affects planning (how much to buy) and costs (how much to charge).

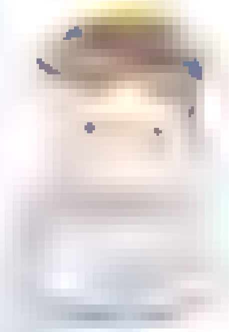
Whether you are measuring out ingredients for a recipe, filling up a car's gas tank or just adding detergent to the washing machine, you are using volume.



12.1 Pi (π)

Consider the following activity

Activity



Coins, paper plates, cookies, and CDs are all examples of objects that are circular in shape.

- Collect three different-sized circular objects. Then copy the table shown.

- Using a tape measure, measure each distance below to the nearest millimeter. Record your results.
  - The distance across the circular object through its center (d)
  - The distance around each circular object (C)

  For each object, find the ratio. Record the results in the table.

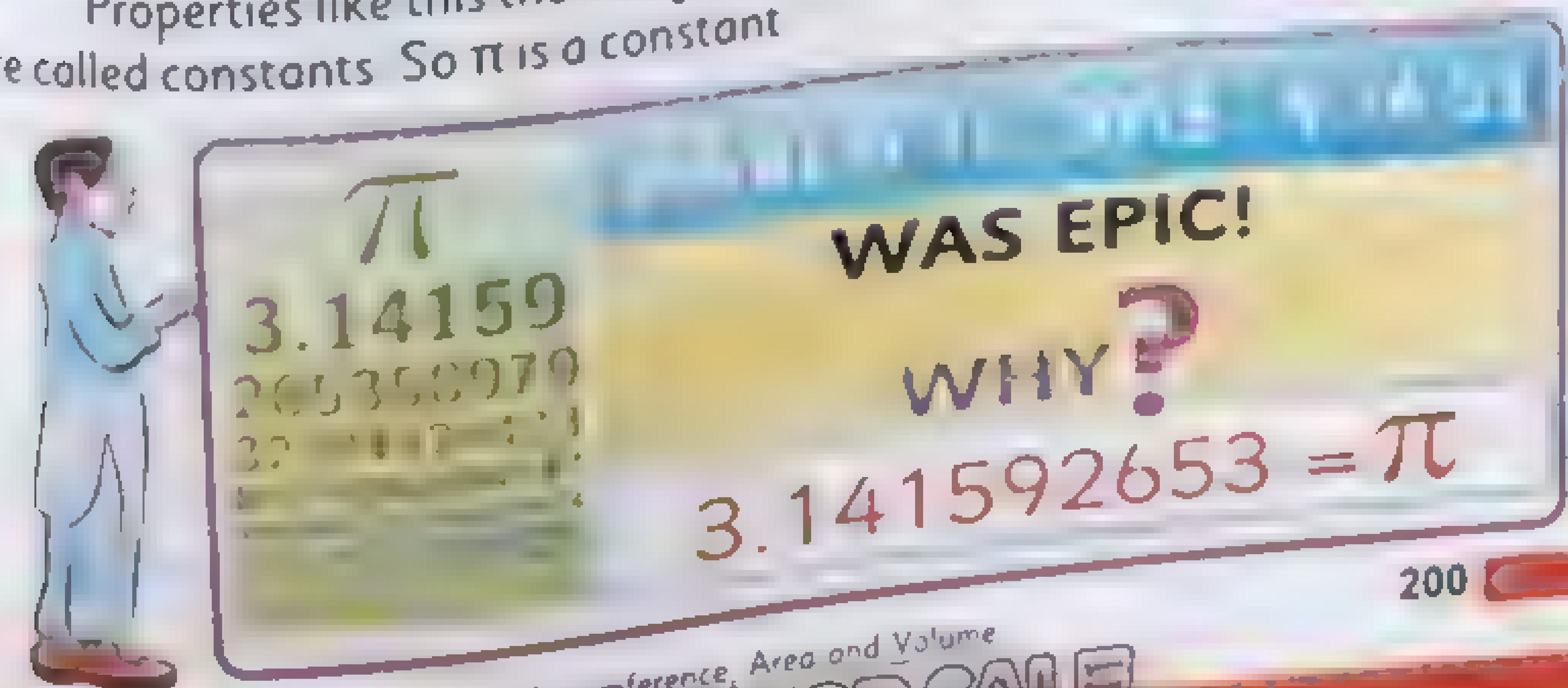
Note

In the above activity the ratio $\frac{C}{d}$ is called Pi (π).

Pi (π) is the ratio of the circumference of a circle to its diameter.

It doesn't matter how big or small the circle is - the ratio stays the same.

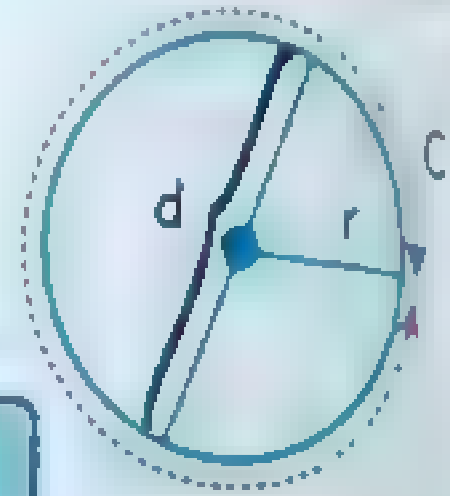
Properties like this that stay the same when you change other attributes are called constants. So π is a constant.



12.2 Circumference of circle

Key Concept

Model: Circumference

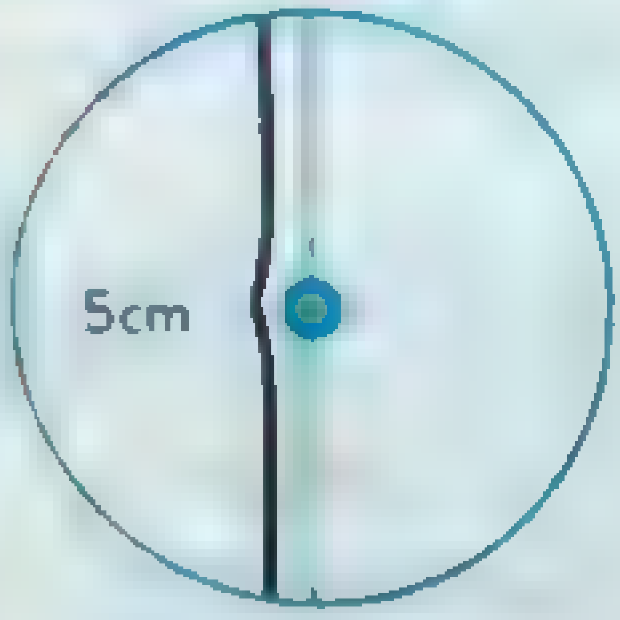


The circumference of a circle is equal to its diameter times π, or 2 times its radius times π.

Symbols: $C = \pi d \Rightarrow C = 2\pi r$

Note $d = 2r$

Example 1 Find the circumference of each circle to the nearest tenth



$C = \pi d$	Circumference of a circle
$C = \pi 5$	Replace d with 5.
$C = 5\pi$	Simplify

$C = 2\pi r$	Circumference of a circle
$C = 2 \cdot \pi \cdot 3.2$	Replace r with 3.2.
$C = 6.4\pi$	Simplify.

The circumference is about 6.4π feet.

Math fun

Geometry keeps you in shape

Example 2

The diameter of a bicycle wheel is 35 cm. What will be the length of the marks on the dust if the wheel completes one rotation?

Solution

Diameter of the wheel = D = 35 cm

Length of the marks (circumference) = C = ?

$$\pi \text{ (pi)} = \frac{22}{7}$$

As $\qquad C = \pi D$

$$C = \frac{22}{\cancel{7}_1} \times \cancel{35}^5 = 22 \times 5 = 110 \text{ cm}$$

So the length of the marks on the road will be 110 cm.

Guided Practice

If a bicycle tire has a diameter of 27 inches, what is the distance the bicycle will travel in 10 rotations of the tire?

Exercise 12.1

1. Radius of a circle is 35 cm. Find the circumference of the circle.

2. Radius of a horse cart (tonga) wheel is 72 cm. How much iron strip will be needed to circle around the wheel $\left(\pi = \frac{22}{7} \text{ or } 3.14\right)$

3. The diameter of a bicycle tyre is 42 cm. If it completes two revolutions in 1 sec, how much distance will it cover in [illegible] seconds?

4. The second needle of a clock is 7 cm. How much distance is covered by its outer edge in 24 hours?
(Hint: there are 1440 minutes in a day and night

12.3 Area of a Circle

To derive a formula for the area of a circle we divide the circle in very small equal parts as shown in the diagram. Now we rearrange these segments to form a parallelogram. The more the segments the more accurate will be the parallelogram. We know that area of a parallelogram can be determined by

Area of a parallelogram = base × height

$= \frac{1}{2}$ C.r where C is the circumference of the circle.

$= \frac{1}{\cancel{2}} \times \cancel{2}\pi r.r$ $(\because C = 2\pi r)$

Area of a circle $= \pi r^2$

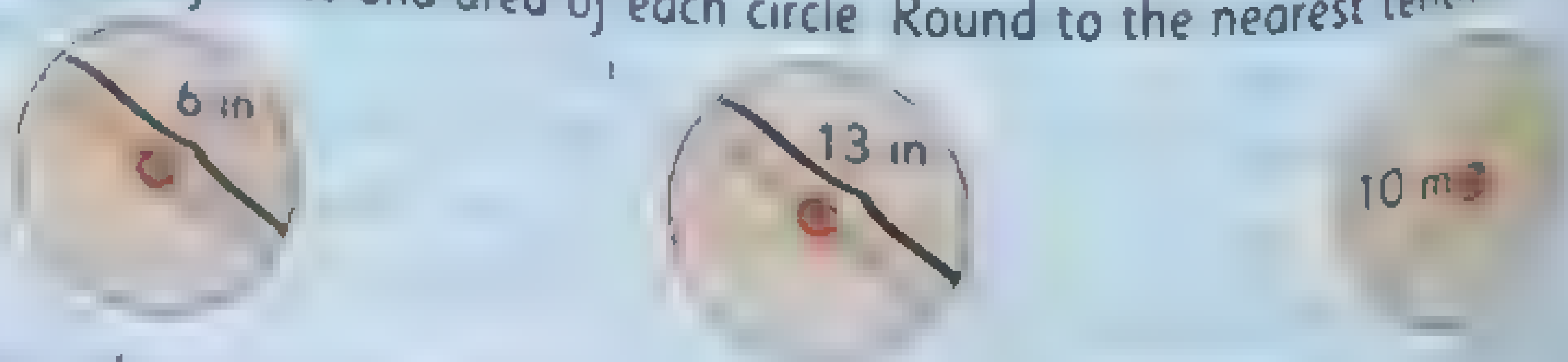
Key Concept — **Area of a Circle**

The area of a circle is equal to π times the square of its radius.

$A = \pi r^2$

Guided Practice

Find the circumference and area of each circle. Round to the nearest tenth.



iv. The radius is 4.5 meters.

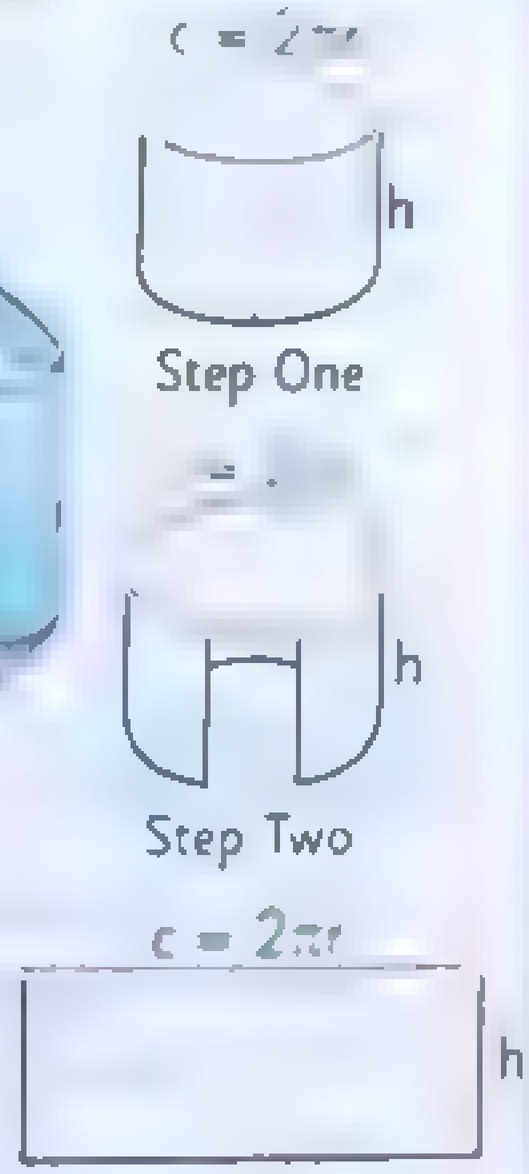
v. The diameter is 7.3 centimeters.

12.4 Surface area of a cylinder

A cylinder is a solid or hollow tube with long straight sides and two equal sized circular ends. A cylinder can be formed by rolling a sheet as shown below

From the figure it can be seen that a cylinder has two circular ends Lid A and Base B. Width (w) is the circumference of each circular face A and B of the cylinder. The lateral area can easily be converted to a rectangle as shown the figure with length $2\pi r$ and height h. We know that area of a sheet can be determined by formula

Lateral Area of a Cylinder

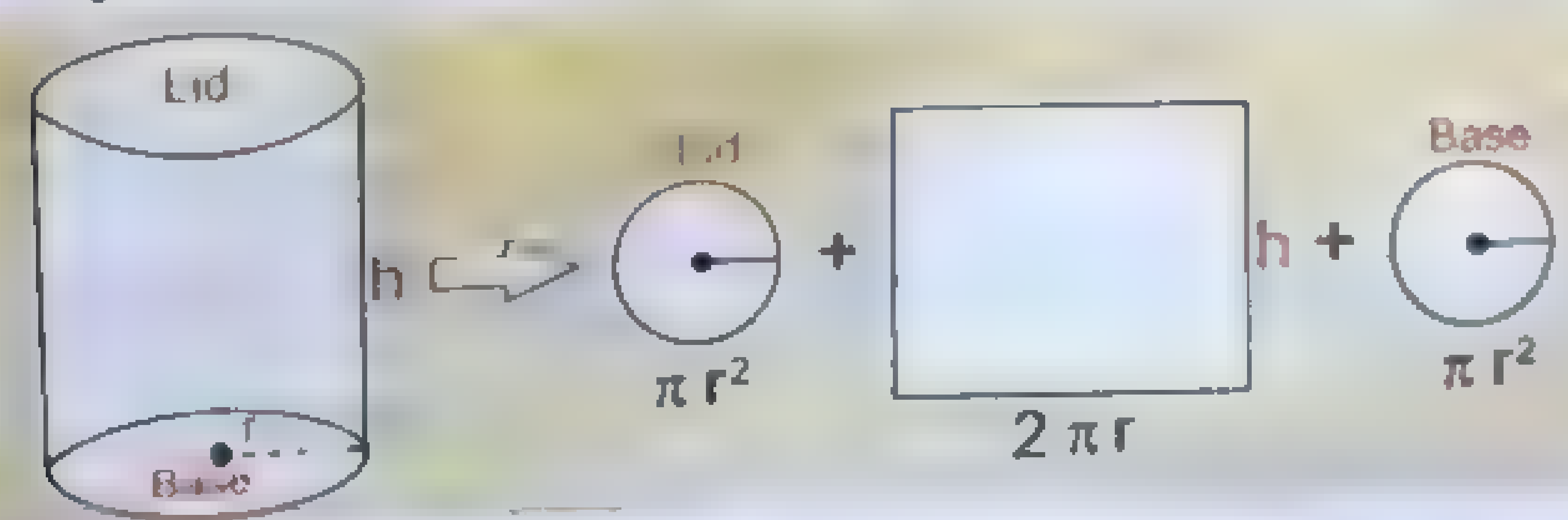


Area = length (h) × width (w)

$A = h \times w \quad$ as $w = 2\pi r$

$A = h \times 2\pi r$

Area of each circle $r\pi^2$



Area of two equal circles =

So the whole Area of a cylinder will be

$$= 2\pi rh + 2\pi r^2$$
$$= 2\pi r[h + r]$$

The surface area of a cylinder = $2\pi r(h + r)$

Example 3

Find the surface area of a cylinder whose diameter is 42 cm and its height is 150 cm

Solution

Diameter = D = 42cm As $r = \dfrac{D}{2}$

Radius = r = 21cm

Height = h = 150cm

The surface area of the cylinder = ? Formula for surface area = $2\pi r(h + r)$

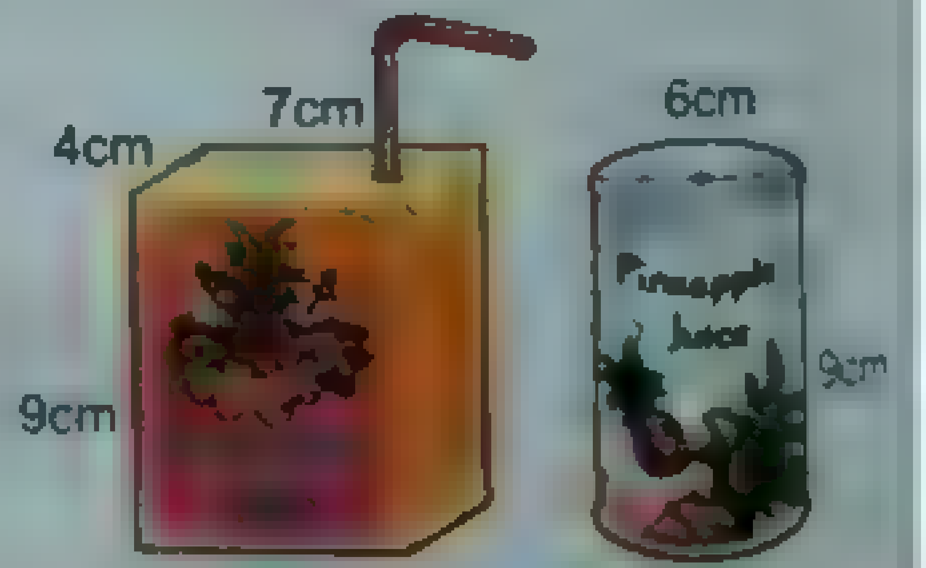
$= 2 \times \frac{22}{\cancel{7}} \times \cancel{21}^{3}(150 + 21)$ (putting the values)

$= 2 \times 22 \times 3(171) \quad = 22572\ cm^2$

Thus surface area of cylinder is $22572\ cm^2$.

Example 4

Both containers hold about the same amount of pineapples juice. Does the box or the can have a greater surface area?



Surface area of box

$$S = \underbrace{2lw}_{\text{top, bottom}} + \underbrace{2lh}_{\text{sides}} + \underbrace{2wh}_{\text{front, back}}$$

$= 2(1 \times 7) + 2(1 \times 9) + 3(7 \times 9)$

$= 254cm^2$

Surface area of Can

$$\underbrace{\quad}_{\text{top, bottom}} \quad \underbrace{\quad}_{\text{top, bottom}}$$

$S = 2\pi r(r + l)$

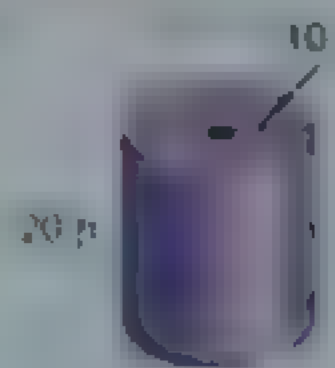
$= \dfrac{2 \times 22 \times 3(3+9)}{7} = 226.3cm^2$

Since $254\ cm^2 > 226.3\ cm^2$, the box has a greater surface area

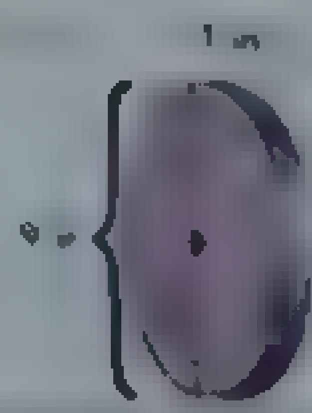
Guided Practice

Find the surface area of the following cylinders.

13



14

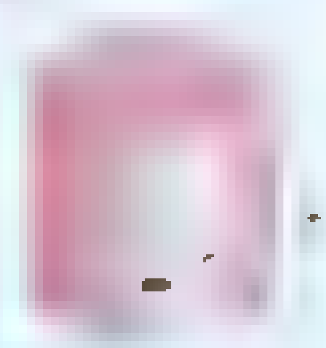


15

Key Concept — Volume of a Cylinder

The volume V of a cylinder with radius r is the area of the base B times the height h.

$V = Bh$ or $V = \pi r^2 h$, where $B = \pi r^2$



Example 5 Volume of a Cylinder

Find the volume of each cylinder. Round to the nearest tenth.

a.

$V = \pi r^2 h$ Formula volume of a cylinder

$V = \pi \cdot 5^2 \cdot 15$ Replace r with 5 and h with 15

$V = 1178.1$ Simplify

The volume is about 1178.1 cubic feet

b. diameter of base 16.4 mm, height 20 mm

Since the diameter is 16.4 mm, the radius is 8.2 mm

$V = \pi r^2 h$ Formula volume of a cylinder

$V = \pi \cdot (8.2)^2 \cdot 20$ Replace r with 8.2 and h with 20

$V = 4224.8$ Simplify.

The volume is about 4224.8 cubic feet

Project Complete the given table and insert a column for surface area of cylinders. Answer to 1d.p.

Radius (cm)	Diameter (cm)	Height (cm)	Volume (cm³)
		12	
3		18	
	11		360
7		20	400

Example 6 Volume of a Cylinder

A water heating geyser is 120 cm tall and has the inner radius of 30 cm What is its capacity to store water?

Solution

Height of the cylindrical geyser (h) = 120 cm

Radius of the geyser $= r = 30$ cm

Volume of the geyser $= V = ?$ $\qquad V = \pi r^2 h$

$= \frac{22}{7} \times (30)^2 \times 120 \qquad = \frac{22 \times 900 \times 120}{7} \qquad = 339428.57\ cm^3$ (ml)

$= \frac{339428.57}{1000} = 339.428$ liter. The capacity of the geyser = 339.428 liter

Exercise 12.2

1. The height and radius of a cylinder are 3 cm and 1.5 cm respectively. Find the surface area of the cylinder.
2. A ball point pen has inner radius of 2mm and length of 90 mm. How many mm^3 of ink will be needed to fill it?
3. Radius of circular play ground is 35m. Find its area.
4. The circumference of a circle is 176 cm. Find its area.
5. A car engine has three cylinders if each cylinder has a radius 4 cm and length 6.6 cm, find the total volume of all the three cylinders.
6. A tube light is 100 cm long and has a diameter of 3 cm. Find the surface area of the tube light.
7. A godown for grain has cylinder for storing grain. If the height and radius of cylinder is 15 m and 3 m respectively. Find the volume of cylinder.
8. A honey bottle is cylindrical. If its volume is 275 cm^3 (ml), find the height of the bottle if its top has the radius of 3.5 cm.
9. The circumference of Earth is about 25,000 miles. What is the distance to the center of Earth?

REVIEW EXERCISE 12

1. Choose the Correct Answer

i. Pi (π) is the ratio between

(a) Radius and circumference (b) Circumference and diameter

(c) Diameter and radius (d) Diameter and circumference

ii. The value of Pi (π) is approximately equal to

(a) $\frac{22}{7}$ (b) $\frac{7}{22}$ (c) $\frac{27}{7}$ (d) $\frac{23}{7}$

iii. Formula for the circumference of a circle is

(a) πr^2 (b) $2\pi r^2$ (c) $\frac{44}{7}r$ (d) None of these

iv. Formula for the area of a circle is

(a) $\pi^2 r^2$ (b) $\pi^2 r$ (c) $2\pi r^2$ (d) πr^2

v. The surface area of a cylinder is determined by

(a) $2\pi r + h$ (b) $2\pi r^2 + h^2$ (c) $2\pi r(h+r)$ (d) $2\pi h(h+r)$

vi. The volume of a cylinder is found by

(a) $2\pi r^2 h$ (b) $\pi^2 rh$ (c) $\pi r^2 h$ (d) $2\pi rh^2$

vii. The constant ratio between the circumference and the diameter of a circle is called

(a) Pi (b) Phi (c) Si (d) None of these

viii. The ratio between a circumference and a diameter of all sizes of circle is

(a) $\frac{22}{7}$ (b) $\frac{7}{22}$ (c) $\frac{27}{7}$ (d) none of these

ix. If the radius of a circle is 14 cm, its circumference will be

(a) 88 cm (b) 88 mm (c) $\frac{44}{7}$ cm (d) $\frac{44}{7}$ mm

x. The diameter of a circle is 8 units. What is the area of the circle if the diameter is doubled?

(a) 50.3 units^2 (b) 100.5 units^2 (c) 201.1 units^2 (d) 804.2 units^2

2 A compass is opened 14 cm on a ruler. Find the length of the circle drawn by it.

3 A cylindrical milk container needs to be painted. If the cylinder is 7m long with radius of 2m. Find the cost of painting it, if per m^2 cost is 27 rupees.

4 A cold drink tin has the radius of 3 cm and is 10 cm in height. In a junkyard its height is compressed to 1.5 cm. Find its volume before and after the compression.

5 Find the area of the head of a screw if its diameter is 14 mm.

6 A well is 50 m deep and its diameter is 4 m. How many tiles will be required to cover all its inner surface if the area of one tile is 0.9 m^2?

Glossary

Pi (π): The Greek letter is a ratio between the circumference of a circle and its diameter.

Circumference: The length of a line bounding a circle is called circumference of a circle.

Diameter(d). A line segment joining the two sides of a circle passing through the centre.

Radius (r): Half of the diameter. The distance from the centre of a circle to the edge.

Area (A): It is the measure of the covered surface. Area of a circle is πr^2

Volume (V): The space required to keep an object. The product of area of circle and height of a cylinder is equal to the volume of a cylinder.

Cylinder. A geometrical shape having two circular faces of equal area and the same length e.g. coins, tins etc.

Unit 13 Information Handling

What You'll Learn

- Demonstrate data presentation.
- Define frequency distribution (i.e. frequency, lower class limit, upperclass limit, class interval).
- Interpret and draw a pie graph.

Why It's Important

In the world around us, there are a lot of questions and situations that we want to understand, describe, explore and access. For example, How many hospitals are there in different cities of Pakistan? How many children were born during the last 10 years? How many doctors will be required in the next 5 years? To know about such things, we collect information and present it in a manageable way so that useful conclusions can be drawn. The branch of statistics that deals with this process is called information handling

"Tabber" monument at the Pakistan Institute of Medical Sciences in Islamabad.

13.1 Frequency Distribution

13.1.1 Data

Data means facts or groups of information that are normally the results of measurements, observations and experiments.

Any information collected for the first time develops the raw data. Obtaining appropriate information is essential for conducting problems in uncertainties. There are many instances in which data are needed. For example, the government of a state prepares its budgets and development plans on the basis of a collected data about the resources and population.

13.1.2 Presentation of Data

After the collection of a data, the most important step is its presentation that provides basis to draw conclusions. Data can be represented in the form of tables and different kinds of graphs. There are two types of data.

i. Ungrouped Data: We know that data are collected in raw form and it provides us information about individuals. Data in such form is called ungrouped data.

ii. Grouped Data: After arranging the data for desired information, it is called grouped data. Now consider the following example.

Definitions of some terms

Frequency Distribution

The conversion of ungrouped data into grouped data so that the frequencies of different groups can be visualized is called frequency distribution.

Frequency table

The table which shows the frequencies of class intervals is called the frequency table.

13.2 Frequency

The number of values that occurs in a group of a data is called its frequency. e.g. in the above given example,

- The frequency of (21–25) is 4
- The frequency of (26–30) is 7
- The frequency of (31–35) is 4

> **Tidbit**
>
> Frequency tables are sometimes called tally charts.

Class Limits: Upper Class Limit: The greatest value of a class interval is called the upper class limit, e.g. in the class interval (21–25), 25 is the upper class limit.

Lower Class Limit: The smallest value of a class interval is called the lower class limit, e.g. in the class interval (21–25), 21 is the lower class limit.

Class Intervals: Each group of a data is also known as the class interval. For example, (21–25), (26–30) and (31–100) are class intervals. Each interval represents all the values of a group.

Size of the Class Interval: The number of values in a class interval is called its size or length. For example, the size or length of class interval (21–25) is 5 that can be checked by counting. The class interval is also calculated by using the formula.

$$\text{Class interval } (h) = \frac{\text{Largest value} - \text{Smallest value}}{\text{Number of classes}}$$

Smallest value = 21

Largest value = 50

Now use the formula to calculate the size

$$\text{Class-interval } (h) = \frac{\text{Largest value} - \text{Smallest value}}{\text{Number of classes}}$$

$$= \frac{50 - 21}{6} = \frac{29}{6} = 4.83 \text{ or } 5 \text{ (Round off the answer)}$$

Example 1

Find a frequency distribution table from the marks of the students in a monthly test: 25, 30, 40, 21, 24, 25, 36, 30, 45, 50, 22, 25, 36, 46, 35, 38, 40, 28, 34, 45, 42, 46, 38, 48, 28, 29, 31, 33, 30, 26.

Solution

25, 30, 40, 21, 24, 25, 36, 30, 45, 50, 22, 25, 36, 46, 35, 38, 40, 28, 34, 45, 42, 46, 38, 48, 28, 29, 31, 33, 30, 26

This is an ungrouped data.

Now if we arrange it to represent information into groups, then it is called grouped data.

- Number of students scored from 21 to 25 = 6
- Number of students scored from 26 to 30 = 7
- Number of students scored from 31 to 35 = 4
- Number of students scored from 36 to 40 = 6
- Number of students scored from 41 to 34 = 3
- Number of students scored from 46 to 50 = 4

It can be seen that it is easier to visualize the given information if data is presented in grouped form. We can also represent a grouped data using a table.

Group Score	Tally Marks	Marks of the students
21 – 25	𝍸 I	25, 21, 24, 25, 22, 25
26 – 30	𝍸 II	30, 30, 28, 28, 29, 30, 26
31 – 35	IIII	36, 36, 35, 34
36 – 40	𝍸 I	40, 36, 36, 38, 40, 38
41 – 45	III	45, 45, 42
46 – 50	IIII	50, 46, 46, 48

The method that we used to record the results in the table is called tallying in which we draw tally marks according to the number of individuals of a group. We make the set of fives by crossing the four marks with the fifth mark. This makes easy to count the tally marks. For example, to show 6 individuals of a group we draw tally marks 卌 I.
Thus the required frequency distribution is

Class-Limits	Tally Marks	Frequency
21 – 25	卌 I	6
26 – 30	卌 II	7
31 – 35	IIII	4
36 – 40	卌 I	6
41 – 45	III	3
46 – 50	IIII	4

Example 2

A sample of forces (in lbs) used in breaking a certain gauge of wire is: 29, 44, 12, 53, 21, 34, 39, 25, 48, 23, 17, 24, 27, 32, 34, 15, 42, 21, 28, 37

Organize the raw data set in frequency distribution.

Solution The frequency distribution for the above data set is:

In the frequency distribution, the class interval is taken as 10. In class 50-60, the number of forces used in breaking a certain gauge of wire is 3. The maximum forces used in breaking a certain gauge of wire lie in a class 20-30 which is 8.

Class-Limits	Tally Marks	Frequency
10 – 20	|||	3
20 – 30	卌 |||	8
30 – 40	卌	5
40 – 50	|||	3
50 — 60	|	1
		20

Activity

The 30 students of a school of Class-VII use various mode of transport for school. Find percentage and angles for each mode of travelling and then present it on a pie graph.

Method of Travelling	Number of Children
Walking	8
Car	9
Bus	4
Cycle	5
Train	1
Taxi	3

Exercise 13.1

1. The frequency distribution of the age of adults who listen to FM radio is

Serial Number	Age (years)	Frequency percentage of listeners
1	15–25	12
2	25–35	22
3	35–45	32
4	45–55	23
5	55–65	11

a. What is the percentage of listeners in the first class?
b. What is the minimum age limit of listeners in the frequency distribution?
c. What is the maximum age limit of listeners in the frequency distribution?
d. What is the class interval of the frequency distribution?

2. The number of units produced per day in a factory is:

Serial Number	Classes	Frequency (Units produced)
1	30 – 40	1
2	40 – 50	1
3	50 – 60	1
4	60 – 70	?
5	70 – 80	7
6	80 – 90	2
7	90 – 100	5
8	100 – 110	2
Total		25 days

a. How many days were studied in the frequency distribution?
b. What does 7 represent in frequency column?
c. What is the fifth class interval?
d. What is the unknown frequency of class 4?

13.3 Interpret and Draw Pie Graph

Pie Graph: The representation of a numerical data in the form of disjoint sectors of a circle is called a pie graph. A pie graph is generally used for the comparison of some numerical facts classified in different classes. In this graph, the central angle measures 360° which is subdivided into the ratio of the sizes of the groups to be shown through this graph. Following examples will help to understand the concept of a pie graph.

Example 3 Accidents at a potato chip plant are categorized according to the area injured

[illegible]	[illegible]
Fingers	17
Eyes	5
Arm	2
Leg	1

Draw a pie graph to show the percentage injuries in each category

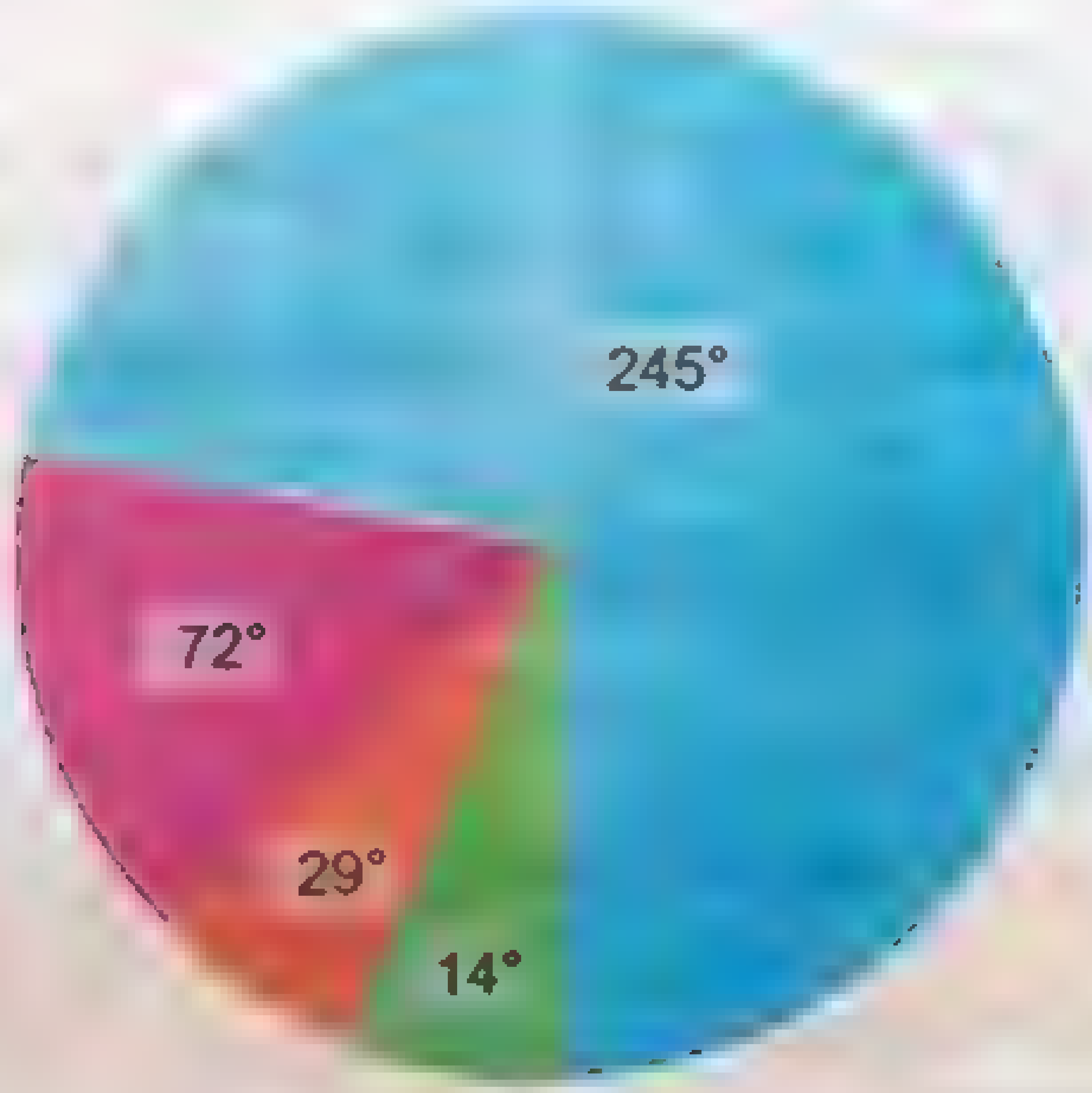
Solution

The frequency distribution in light of the above information is:

Area injured	Frequency	Percentage	Angle
Fingers	17	17/25×(100%)=68%	17/25×360°=245°
Eyes	5	5/25×(100%)=20%	5/25×360°=72°
Arm	2	2/25×(100%)=8%	2/25×360°=29°
Leg	1	1/25×(100%)=4%	1/25×360°=14°
Total	25	100%	36°

In constructing the pie graph 360° is multiplied by 68%, resulting in a part that takes up (17/25) 360° = 245° of the circle to plot the 68% fingers injuries, draw a line from 0 to the centre of the circle and then another line from the centre to 245° on the circle.

similarly, 360° is multiplied by 20% resulting in a part that takes up = (5/25) × 360° = 72° of the circle. To plot the 20% eyes injuries, add 245° to 72° that results 317°. Draw line from the centre of the circle to 317°, so thte area from 245° to 317° represents the percentage of the eye injuries.



Again 360° is multiplied by 80%, resulting in apart that takes up (2/25)360° = 29° of the circle To plot the 80% arm injuries, add 317° to 29° that results 346°. Draw a line from the centre of the circle to 346°, so the area from 317° to 346° represents the percentage of the arm injuries. The remaining area of a circle automatically holds for 4% of the leg injuries.

EXERCISE 13.2

1. Damage at a paper mill (millions of rupees) due to breakage can be divided according to the product:

Toilet paper	132,	Hand towels	85
Napkins	43	Other products	50

Draw a pie graph to indicate percentage damage in each category.

2. The number of units of electric power company consumed by consumers in different categories are the following:

Domestic 1950, Commercial 4000, Industrial 8000

Draw a pie graph to indicate the percentage unit's consumption in each category.

3. The number of students in a private sector university in each category are the following: BBA 61, BCS 40, MBA 28, B Eng 70.

Draw the frequency distribution and a pie graph to indicate the above data:

REVIEW EXERCISE 13

1. Choose the correct answer:

i. The numerical information is called:

a. data b. table c. knowledge d. calculation

ii. Data can be available in

a. grouped data b. ungrouped data

c. graph data d. ungraphed data

iii. The lower limit of a class 7 – 12 is:

a. 12 b. 7 c. 5 d. 19

iv. The upper limit of a class 5 – 15 is:

a. 5 b. 15 c. 10 d. 20

The class interval of of a class 14 – 18 is

a. 14 b. 18 c. 32 d. 4

If the central angle in a pie graph is 90°, then the proportion of that part is

a. 50% b. 25% c. 75% d. 90%

2. The time (in seconds) to run by 36 students a race of 500 m develops a data set:

45, 40, 44, 51, 40, 59, 44, 47, 42, 41, 54, 39, 50, 55, 61, 59, 47, 44, 49, 50, 52, 47, 44, 51, 59, 55, 59, 43, 44, 41, 41, 42, 46, 54, 51, 52

Organize the data in a frequency distribution with 5 as the class interval

3. The frequency distribution represents the annual temperature (°C) of a certain area:

[illegible]	Temperature °C	[illegible]
1	0 – 10	43
2	11–21	70
3	22–32	110
4	33–43	94
5	44–54	48
Total		365 days

How many days of the year are involved in a frequency distribution?

b. How many days of the year are found coldest?

c. What was the maximum temperature throughout the year?

4. A student of class 7th is getting rupees 25 as pocket money daily. He spent the pocket money in the following categories

Bus fare 14 rupees

Recess meal 8 rupees

He saved rupees 3 according to above expenditure Draw a pie graph to show the expenditure in each category

Glossary

- [illegible] Data are the condensed form of information
- [illegible] Class is one of the categories into which data can be classified
- [illegible] Class frequency is the number of observations in the data set falling in a particular class.
- [illegible] The relative percentage is the class relative frequency multiplied by 100%.
- [illegible] is collected in raw form and it provides us information about individuals such form of the data is called un-grouped data
- [illegible] The greatest value of a class interval is called the upper class limit.
- [illegible] The smallest value of a class interval is called the lower class limit.
- [illegible] The number of values that occurs in a class interval is called its frequency
- [illegible] The table which shows the frequency of class intervals is called frequency tables.
- [illegible] The representation of a numerical data in the form of disjoint sectors of a circle is called a pie graph

[illegible]

Find the population of all the districts of Khyber Pakhtunkhwa on internet and represent this on a pie-graph

Answers

Exercise 1.1

1.

(i) A is a set of first ten natural numbers.

(ii) B is a set of first six English alphabets.

(iii) C is a set of first five positive even numbers.

(iv) D is a set of prime numbers less than 20.

2.

(i) A={5,10,15,20,25}

(ii) {11,12,13,14,15,16,17,18,19}

(iii) {Monday,Tuesday,Wednesday,Thursday, Friday, Saturday, Sunday}

(iv) {2, 4, 6, 8}

3.

(i) A = { x/x is a natural number $\leq$ 20}

(ii) B = { x/x is a vowel}

(iii) C = { x/x is a capital of provinces of Pakistan}

(iv) D = { x/x is an odd number}

Exercise 1.2

1.

(i). {1, 2, 3, 4, 5}, {3, 4}

(ii). {–1, –2, –3, –4, –5}, {–2, –3}

(iii). {1, 2, 3, ..., 10}, {1, 3, 5, 7}

(iv). {1, 3, 5, 6, 7, 8, 9, 10, 11, 13}, {5, 7, 11}

(v). {1, 2, 3, ..., 10}, {2, 4, 6, 8, 10}

2.

(i) {0, 1, 2, 3, 4, 5}

(ii) {3, 5}

(iii) {1, 3, 4, 5}

(iv) {0, 1, 2, 3, 4, 5}

(v) {0, 1, 2, 3, 4, 5}

3. {1, 3, 5, 7, 9}, { }

4. {b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z}, { }

Exercise 1.3

2. (i) {1,3,5....19} (ii) {2,4,6....20}

   (iii) {1,2,4,5,7,8,10,11,13,14,16,17,19,20}

   (iv) {1,2,3,5,6,7,9,10,11,13,14,15,17,18,19}

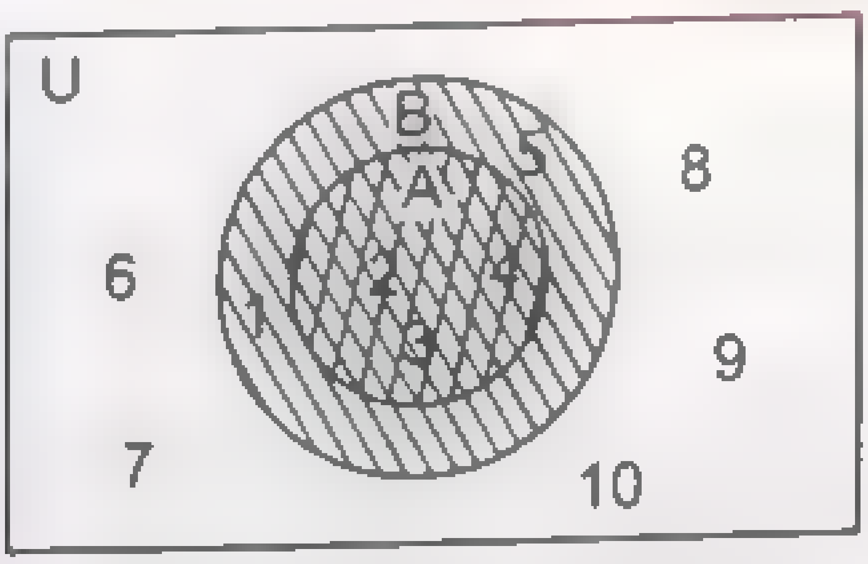
3. { }, U

5. (i) { } (ii) U (iii) { } (iv) U (v) A (vi) B

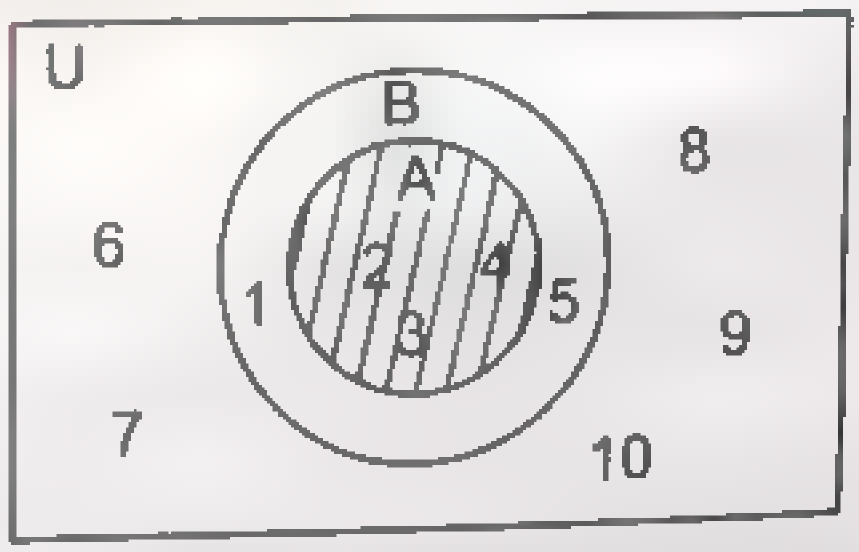
6. (i) Overlapping (ii) disjoint

Exercise 1.4

1. (i)

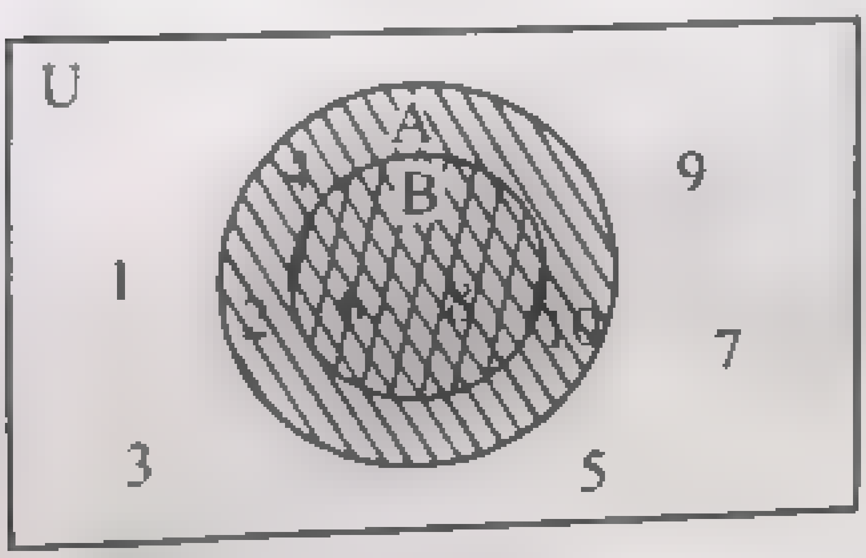


$A \cup B = \{1, 2, 3, 4, 5\}$

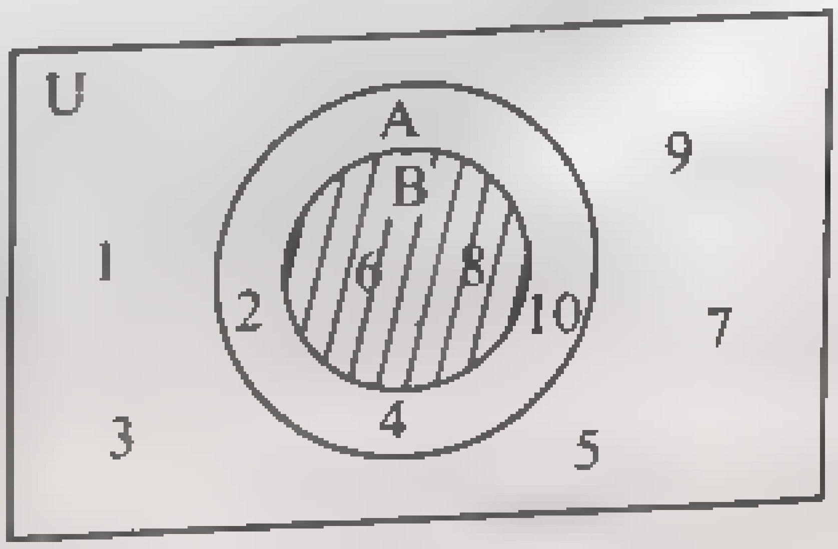


$A \cap B = \{2, 3, 4\}$

(ii)

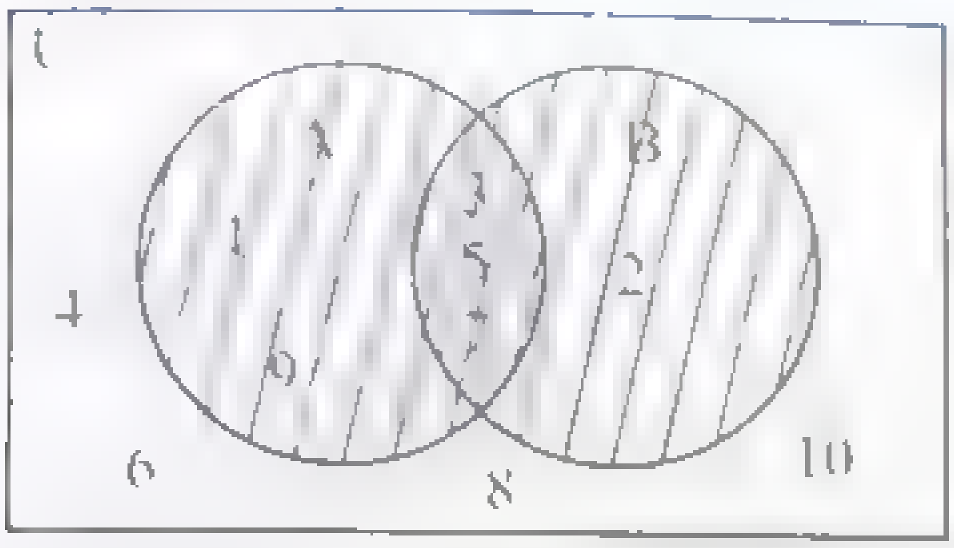


$A \cup B = \{2,4,6,8,10\}$

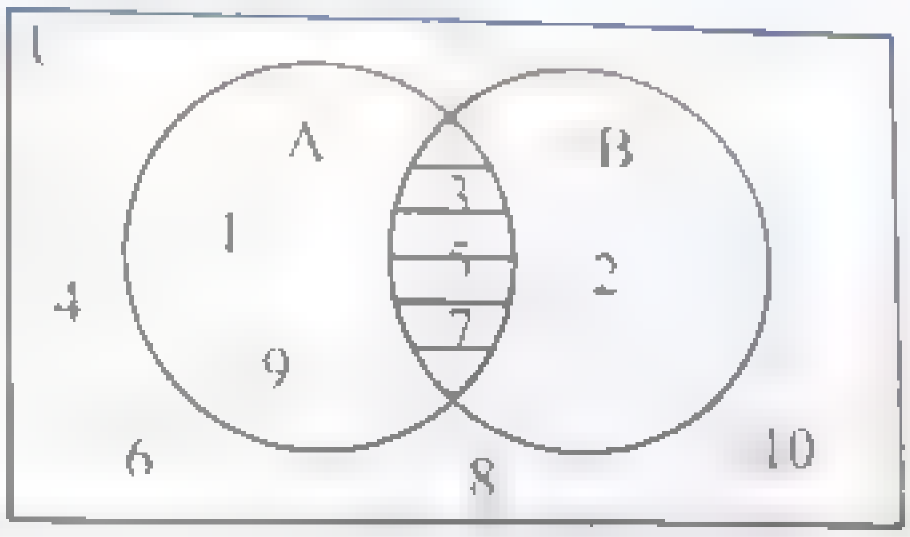


$A \cap B = \{6,8\}$

(iii)

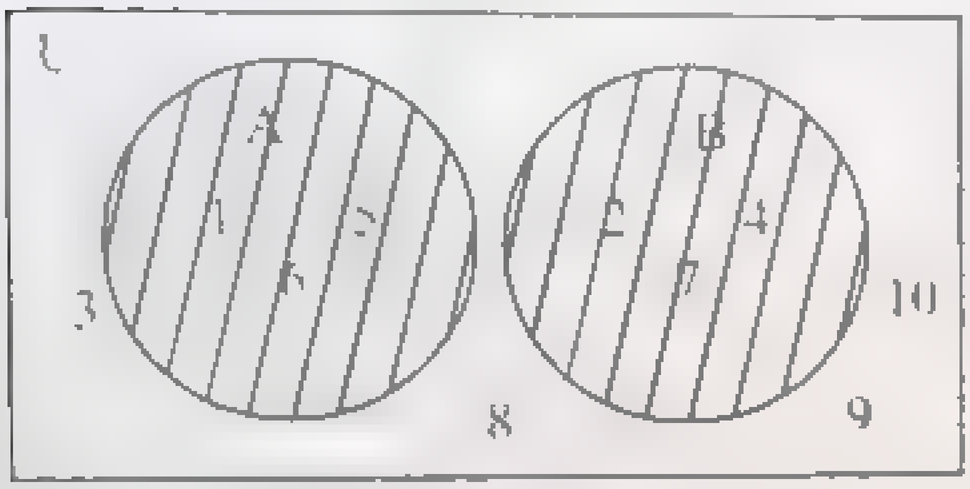


$A \cup B = \{1,2,3,5,7,9\}$

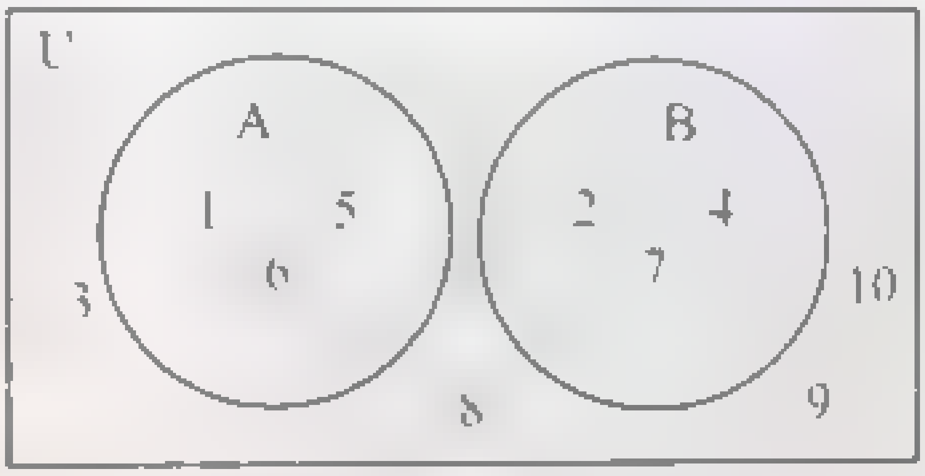


$A \cap B = \{3,5,7\}$

(iv)

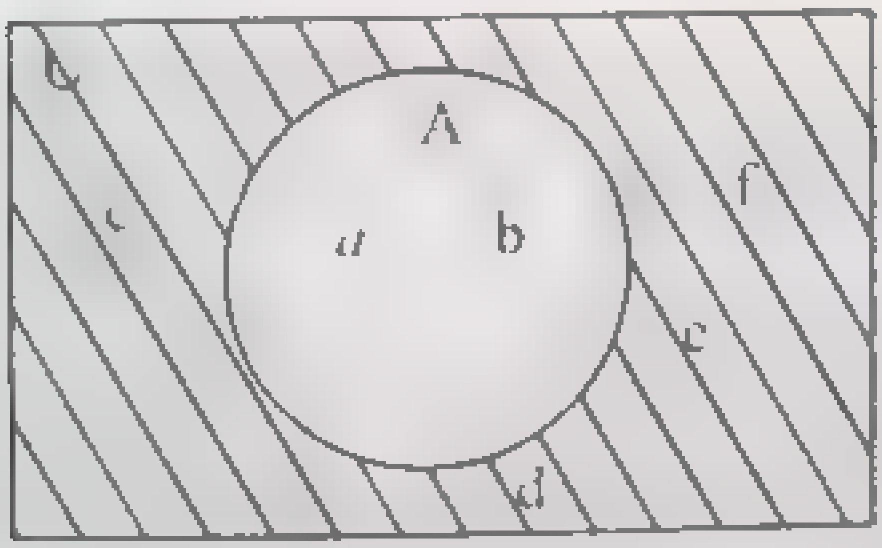


$A \cup B = \{1,2,4,5,6,7\}$

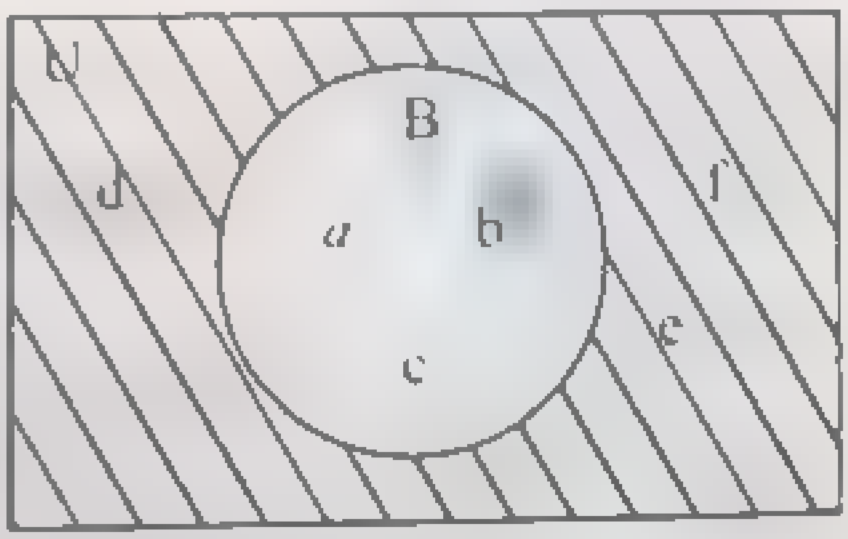


$A \cap B = \{\ \}$

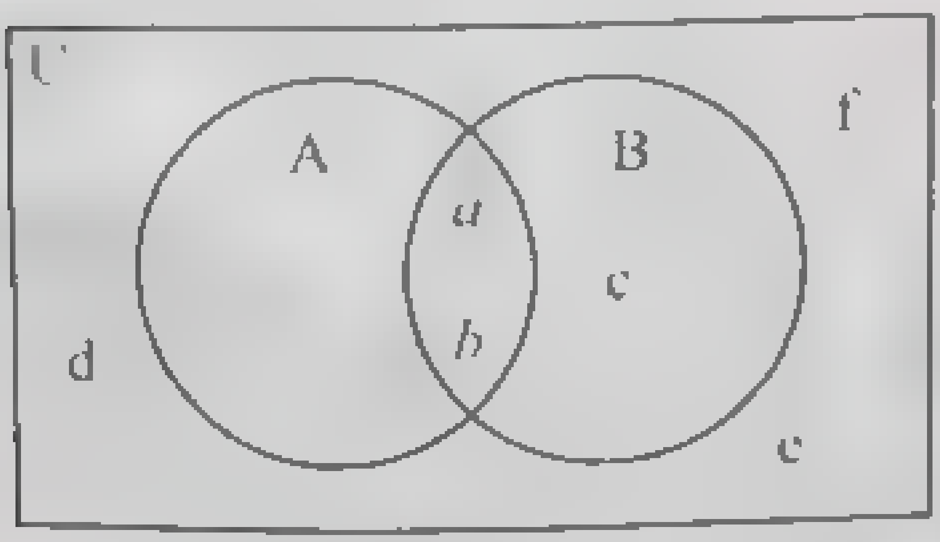
2 (i)



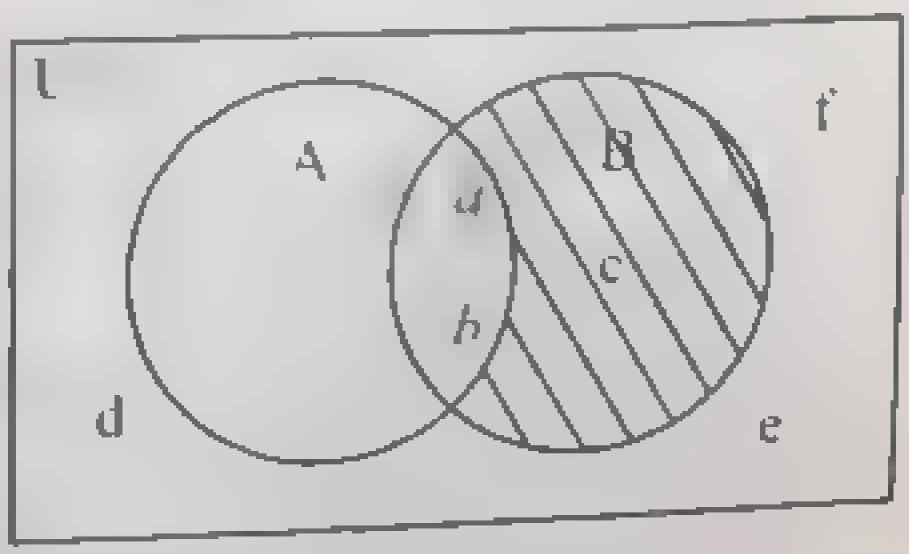
$A' = \{c, d, e, f\}$



$B' = \{d, e, f\}$

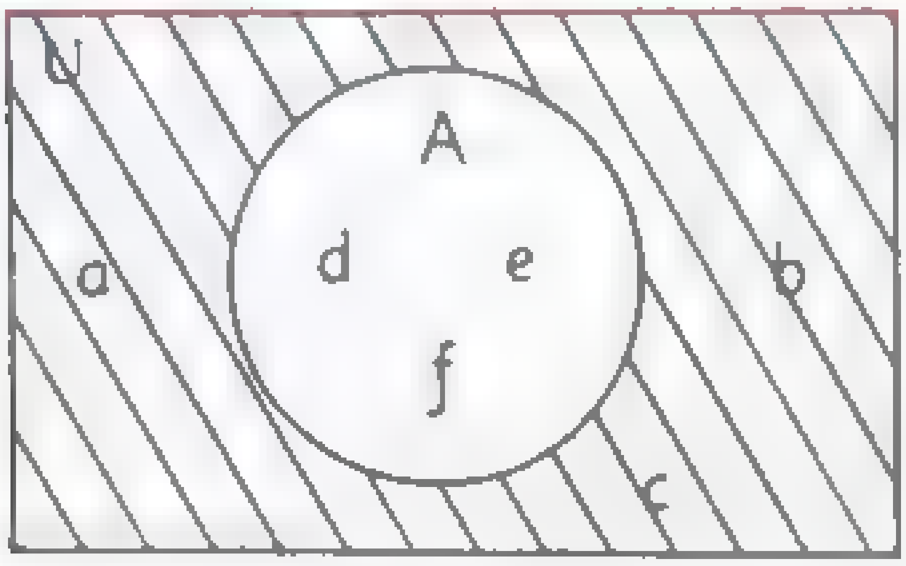


$A \backslash B = \{\ \}$

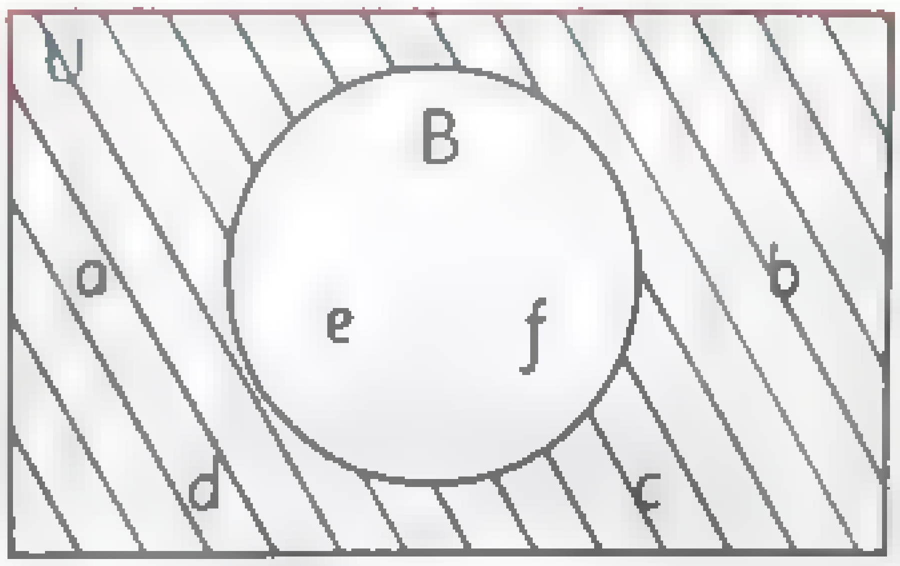


$B \backslash A = \{c\}$

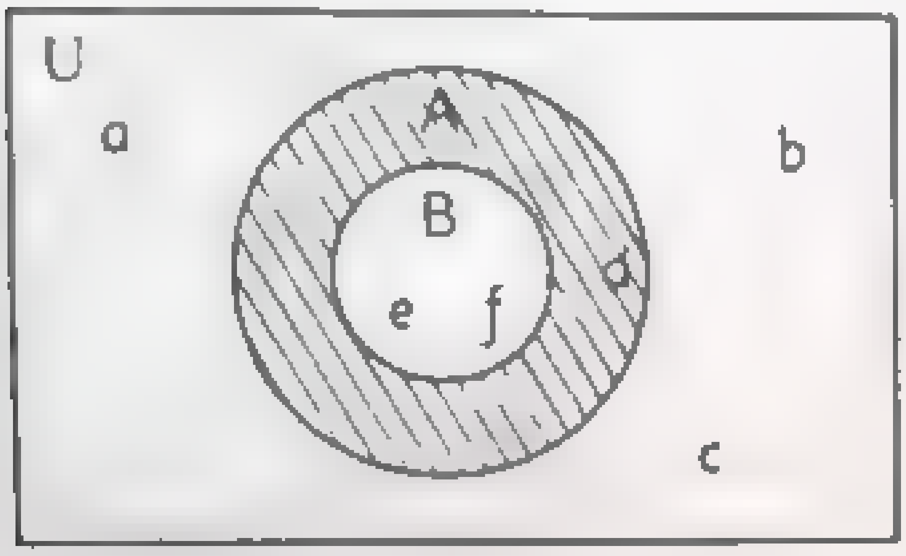
(ii).



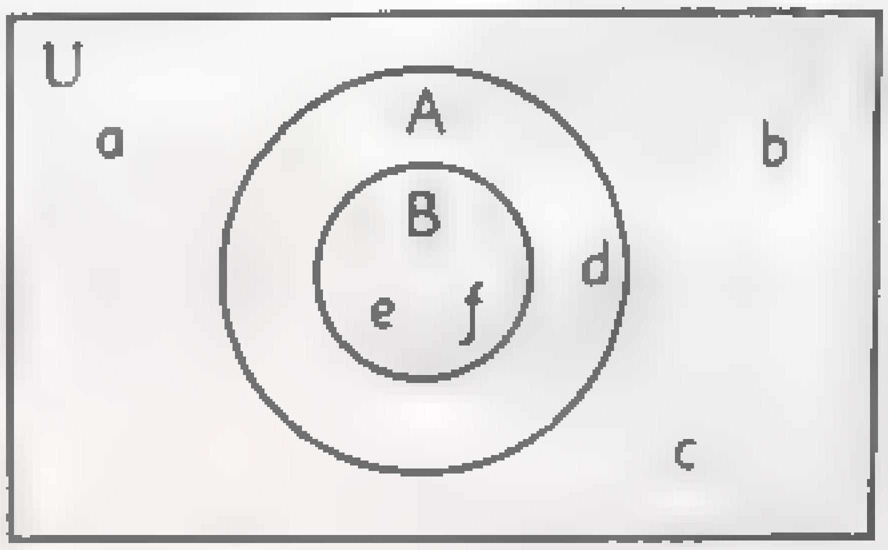
$A' = \{a, b, c\}$



$B' = \{a, b, c, d\}$

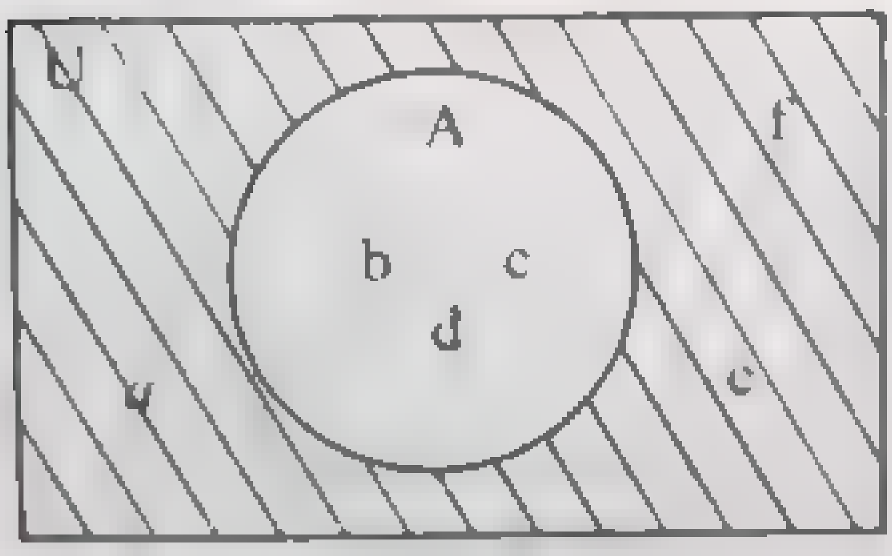


$A \setminus B = \{d\}$

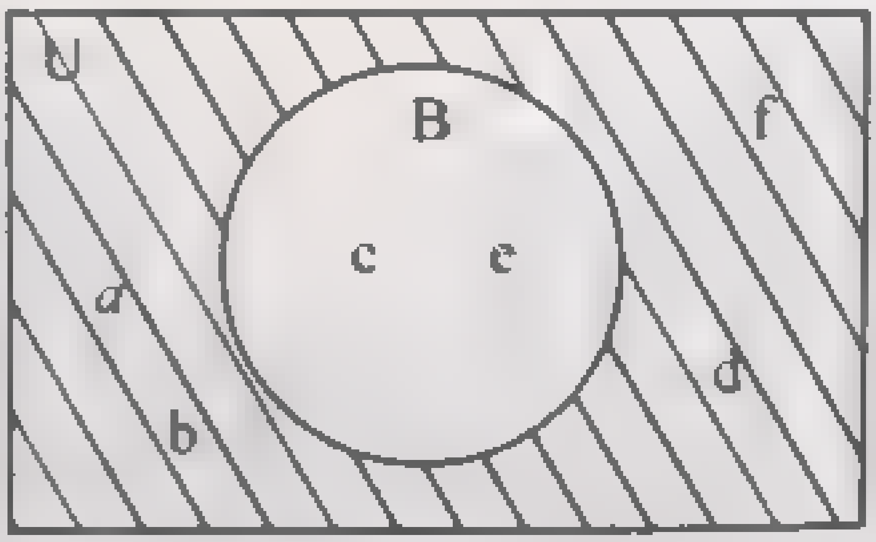


$B \setminus A = \{\ \}$

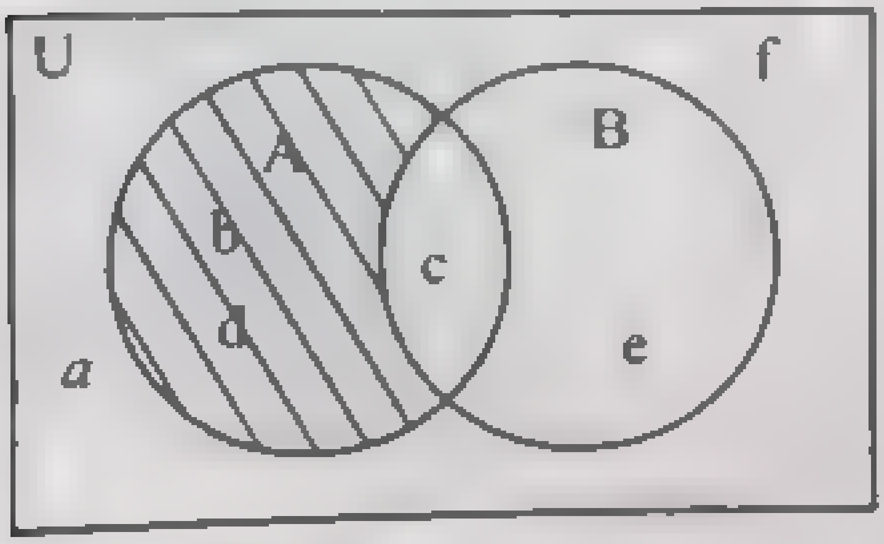
(iii)



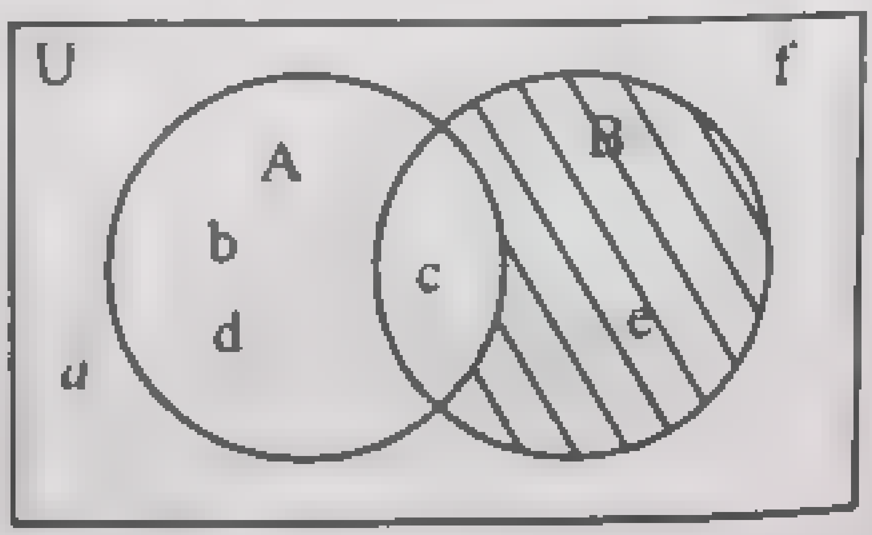
$A' = \{a, e, f\}$



$B' = \{a, b, d, f\}$

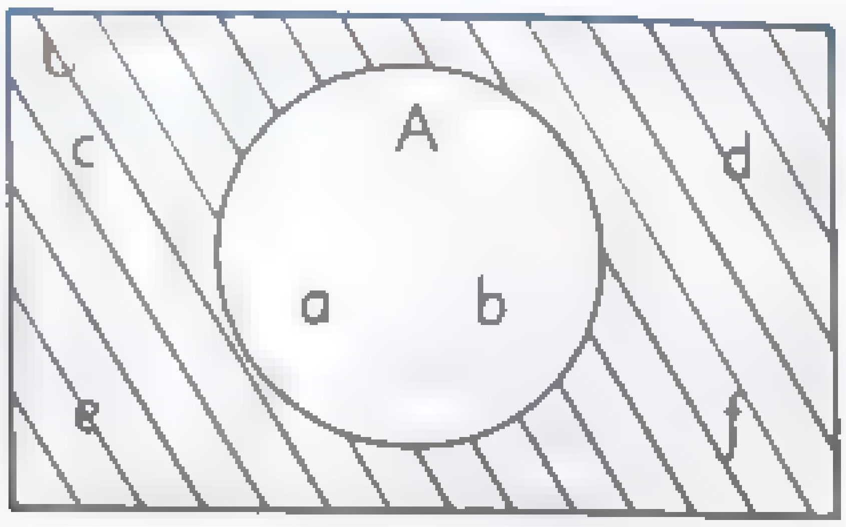


$A \setminus B = \{b, d\}$

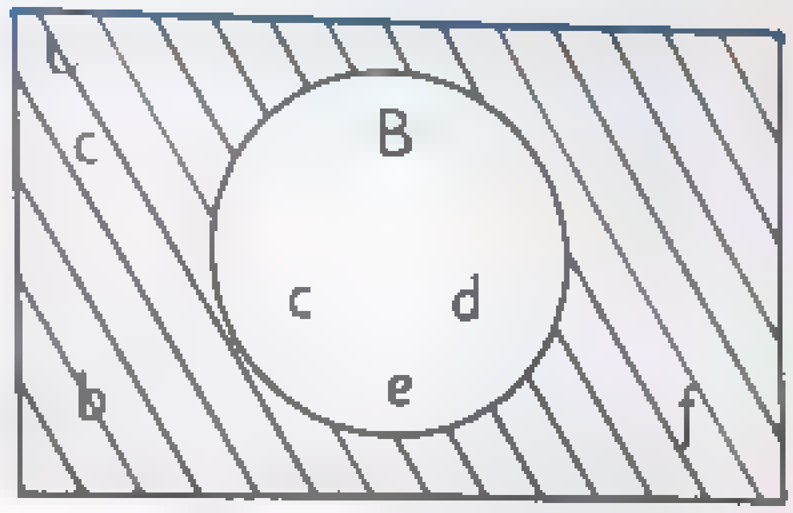


$B \setminus A = \{e\}$

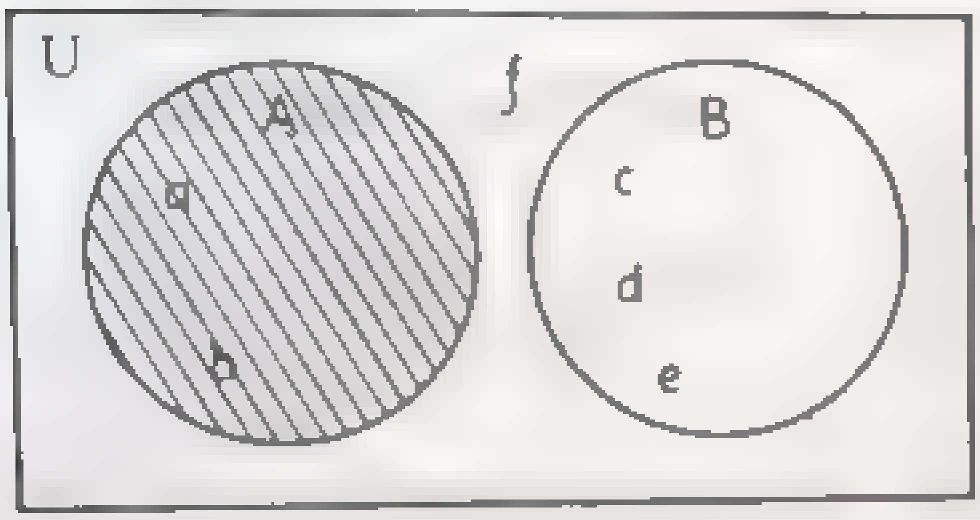
(v)



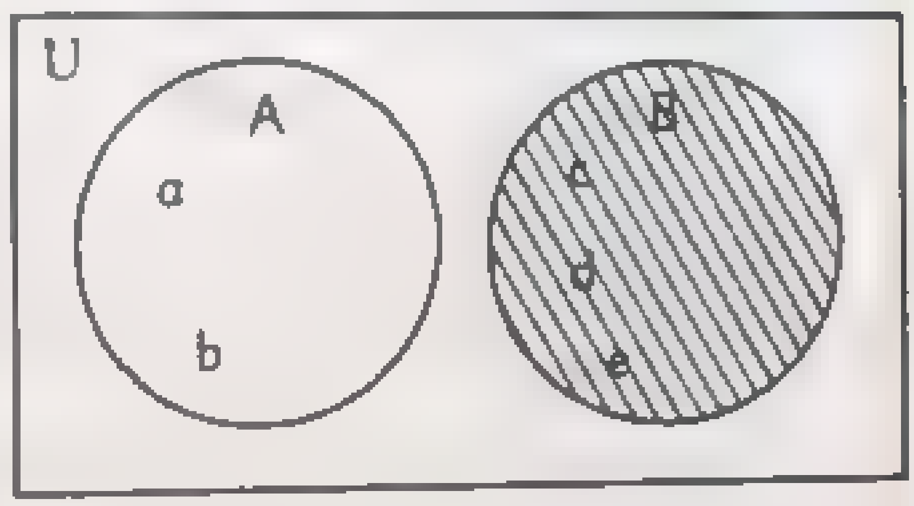
$A' = \{c, d, e, f\}$



$B' = \{a, b, f\}$



$A \setminus B = \{a, b\}$



$B \setminus A = \{c, d, e\}$

Review Exercise 1

1. True and false questions

   (i) T (ii) F (iii) F (iv) T (v) T

2. Fill in the blanks

   (i) U (ii) U (iii) φ (iv) A (v) φ

3. Multiple choices

   (i) b (ii) c (iii) d (iv) c (v) a

   (vi) b (vii) b (viii) a (ix) c (x) c

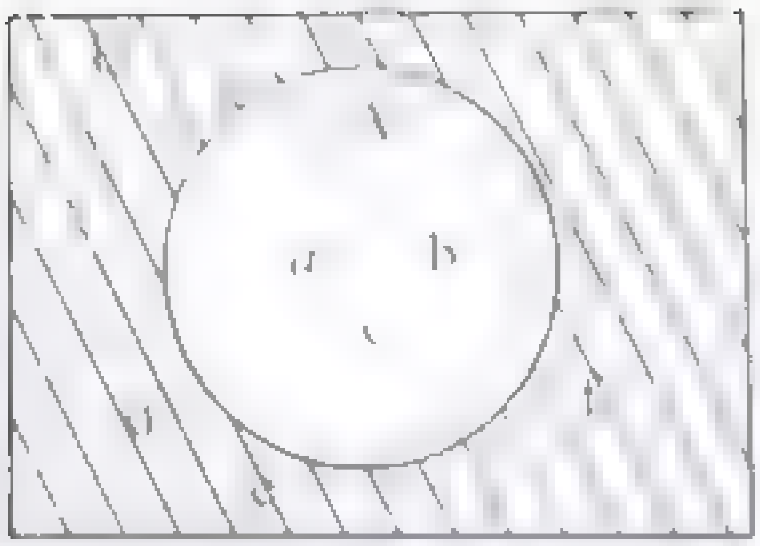
4. (i) {1, 2, 3, ... 10} (ii) {1, 2, 4, 6}
5. (i) {0, 1, 3} (ii) { 6 }
6. (i) {b, d, f} (ii) {a, c, e}

   (iii) φ (iv) U

   (v) {a, b, c, d, e, f} (vi) { }

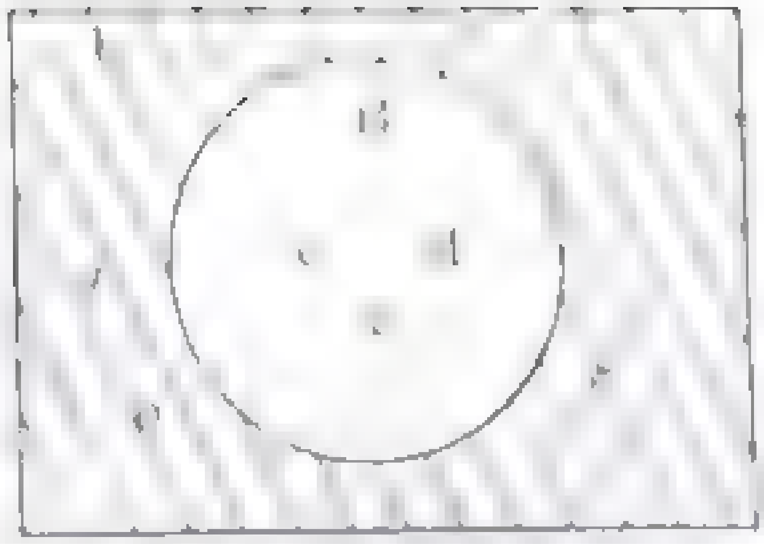
8. {1,2,3,4,5,6,7,8,15}

9 (i)



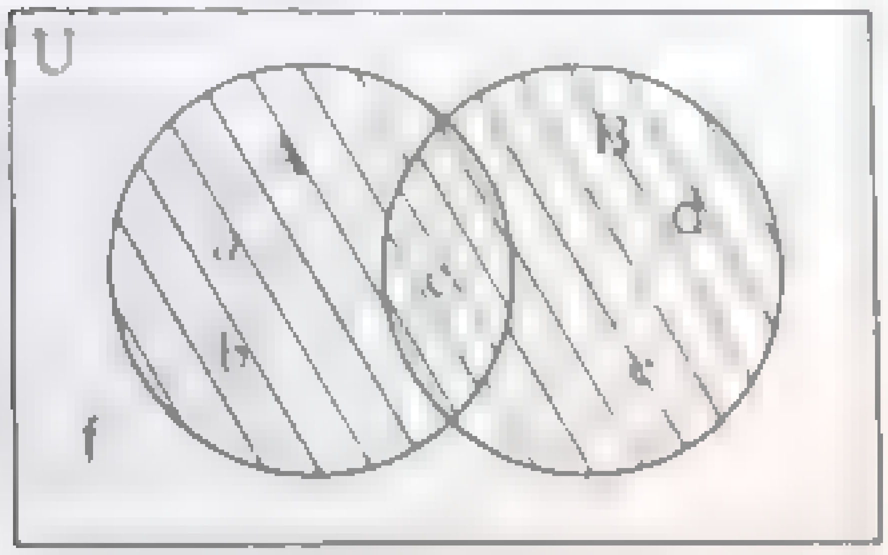
$A' = \{d, e, f\}$

(ii)



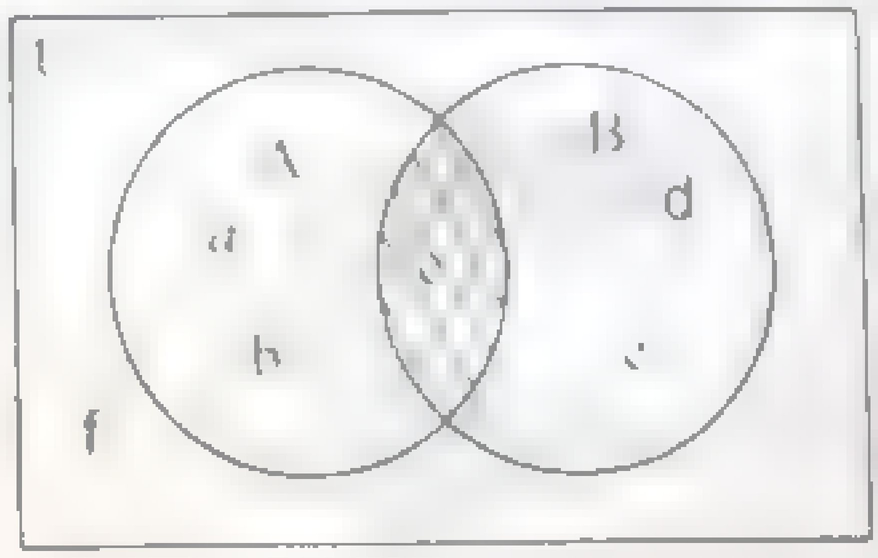
$B' = \{a, b, f\}$

(iii)



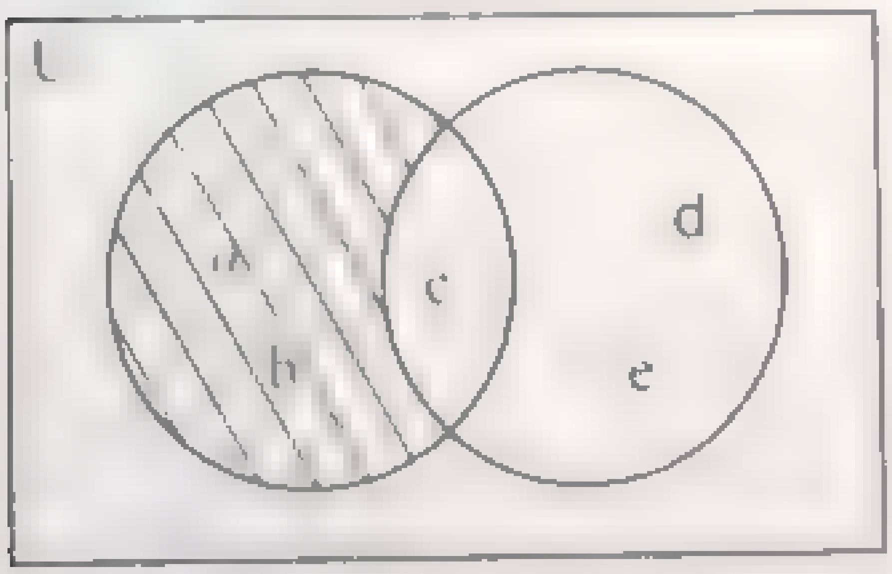
$A \cup B = \{a, b, c, d, e\}$

(iv)



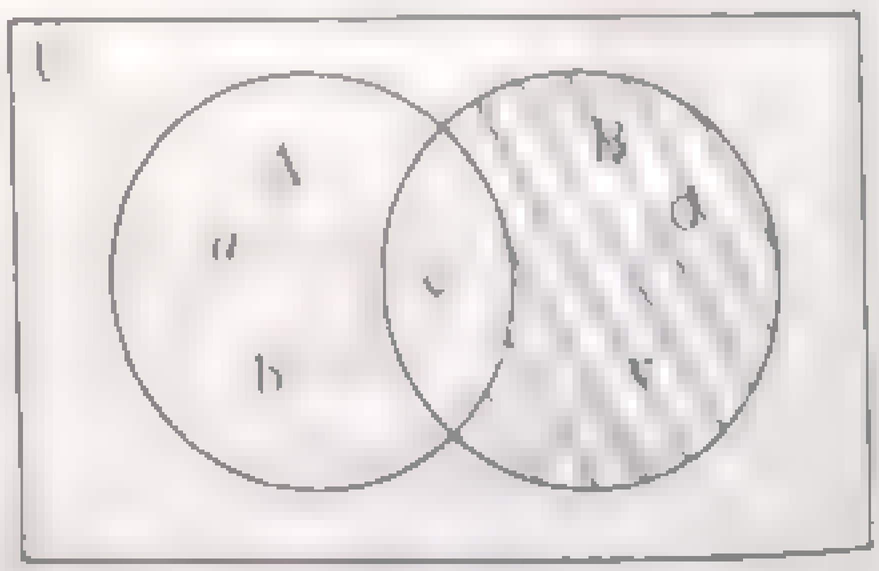
$A \cap B = \{c\}$

(v)



$A - B = \{a, b\}$

(vi)



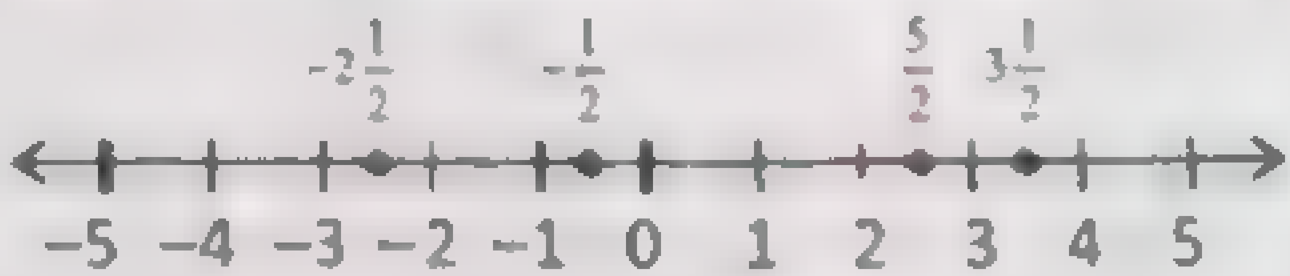
$B - A = \{d, e\}$

10 (i) Overlapping (ii) Disjoint

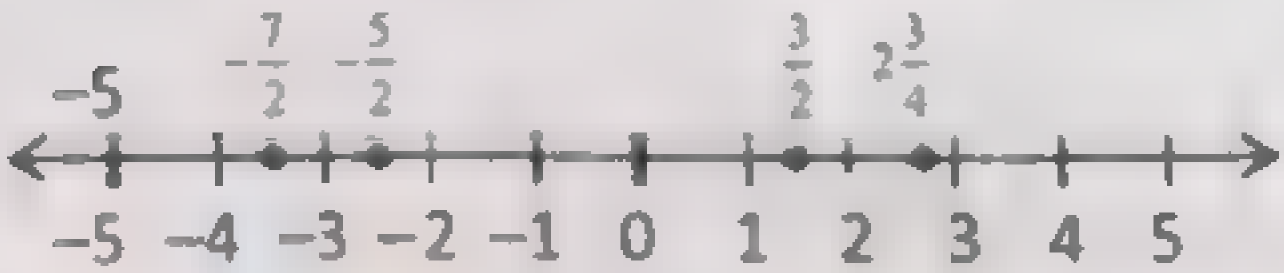
Exercise 2.1

1 (i) True (ii) False (iii) True (iv) True (v) False

2 (i)



(ii)



Exercise 2.2

(i) $1\frac{1}{5}$ (ii) $5\frac{1}{4}$ (iii) $2\frac{1}{14}$ (iv) $1\frac{5}{24}$ (v) $\frac{1}{7}$

(vi) 10 (vii) $5\frac{5}{8}$ (viii) $10\frac{4}{9}$ (ix) $1\frac{3}{4}$

Exercise 2.3

1 (i) $5, \frac{-1}{5}$ (ii) $\frac{23}{11}, \frac{-11}{23}$ (iii) $\frac{-4}{15}, \frac{15}{4}$

(iv) $\frac{-105}{200}, \frac{200}{105}$ (v) $\frac{-6}{7}, \frac{7}{6}$

2 (i) $\frac{7}{12}$ (ii) $\frac{14}{3}$ (iii) $9\frac{39}{40}$ (iv) $\frac{-9}{17}$ (v) 1

(vi) $-2\frac{2}{3}$ (vii) $4\frac{1}{2}$ (viii) 16 (ix) 10

Exercise 2.4

1 (i) Commutative property w.r.t addition

(ii) Associative property w.r.t addition

(iii) Commutative property w.r.t multiplication

(iv) Distributive property w.r.t multiplication over subtraction

(v) Associative property w.r.t multiplication

(vi) Distributive property w.r.t multiplication over addition

2. (i) $\frac{5x}{8}$ (ii) $6\frac{2}{3}y$ (iii) $\frac{3}{m}$

Exercise 2.5

(1) (i) $>$ (ii) $<$ (iii) $>$ (iv) $=$ (v) $>$

(2) $4\frac{2}{5}, 1\frac{1}{3}, \frac{3}{5}, -5\frac{7}{6}$

(3) $-5\frac{5}{3}, -5\frac{7}{12}, 3\frac{7}{25}, 3\frac{7}{8}$

Review Exercise 2

(1) (i) $\frac{1}{2}$

(ii) rational

(iii) no

(iv) 1

(2) (i) c (ii) a (iii) c (iv) b

(3) (i) $1\frac{8}{35}$ (ii) $1\frac{3}{55}$ (iii) $2\frac{1}{4}$ (iv) -3

(4) $\frac{-3}{10}, \frac{-1}{5}, \frac{4}{7}, \frac{6}{7}$ and $\frac{6}{7}, \frac{4}{7}, \frac{-1}{5}, \frac{-3}{10}$

(5) (i) $2\frac{3}{5}$ (ii) $2\frac{4}{5}$ (iii) $\frac{1}{5}$ (iv) $1\frac{8}{15}$

Exercise 3.1

(1) (i) $\frac{9}{20}$ (ii) $\frac{387}{500}$ (iii) $\frac{36}{5}$ (iv) $\frac{15771}{10000}$ (v) $\frac{7607}{50}$

(2) (i) Recurring (ii) Non-recurring (iii) Recurring

(iv) Non-recurring (v) Non-recurring

(3) (i) Non-terminating (ii) Terminating (iii) Non-terminating

(4) (i) Non-terminating (ii) Terminating (iii) Non-terminating

(iv) Terminating (v) Terminating

Exercise 3.2

1 (i) 5.28 (ii) 262.533 (iii) 1.4 (iv) 0.22 (v) 0.92 (vi) 72.169

(vii) 6.7 (viii) 53.6

Muhammad Ali 03101190027

Review Exercise 3

1 (i) b (ii) c (iii) b (iv) a (v) b

(2) (i) $\frac{63}{100}$ (ii) $\frac{213}{50}$ (iii) $\frac{14847}{100}$

(3) (i) Non-terminating (ii) Non-terminating (iii) Non-terminating
(iv) Terminating (v) Non-terminating

(4) (i) Terminating (ii) Non terminating (iii) Terminating
(iv) Non-terminating (v) Non-terminating

(5) (i) Non-terminating (ii) Non-terminating recurring
(iii) Non-terminating (iv) Non-terminating (v) Non-terminating

(6) (i) 5.7 (ii) 0.09 (iii) 4.8 (iv) 13.935

Exercise 4.1

1. (i) Base = 2, exponent = 5
   (ii) Base = –5, exponent = 7
   (iii) Base = $\frac{8}{5}$, exponent = 25
   (iv) Base = 100, exponent = 10
   (v) Base = $\frac{125}{32}$, exponent = –12
   (vi) Base = –115, exponent = 20
2. (i) 16 (ii) –243 (iii) 225
3. (i) 3^4 (ii) $(5)^2$ (iii) $\left(\frac{2}{7}\right)^4$ (iv) $\left(\frac{6}{7}\right)^3$ (v) 2^4 (vi) $(24)^2$
   (vii) $(20)^3$ (viii) 6^{11}
4. (i) $15f^5g^5$ (ii) a^4b^2 (iii) $154x^8$

5 (i) $64x^{6}$ (ii) [illegible] (iii) [illegible]

Exercise 4.2

1 [illegible]

2 $3xy$ 3 $10ab$

Exercise 4.3

1 i 1 ii 51 iii $\frac{8}{343}$

2 i 5^{4} ii $\frac{1}{5^{4}}$ iii $(-7)^{[illegible]}$ iv [illegible] v $\left(\frac{6}{5}\right)^{[illegible]}$

[illegible]

[illegible]

[illegible]

3. (i) (c) (ii) (b) (iii) (a) (iv) (b) (v) (d)
   (vi) (a) (vii) (d) (viii) (c)

4. (i) base = 2, exponent = 5, value = 32
   (ii) base = −3, exponent = 4, value = 81

5. (i) 3125 (ii) $\frac{1}{4}$ (iii) 1 (iv) 500 (v) 36
   (vi). 5^{20} (vii) 525 (viii) $-72v^{11}w^{18}$

Exercise 5.1

(1) 16, 25 are perfect square.
18, 33, 200 are not perfect square

(2) (i) 1225 (ii) 829921 (iii) 4708900 (iv) 15625

(3) (i) even (ii) odd (iii) even (iv) odd

Exercise 5.2

(i) 59 (ii) 46 (iii) 123 (iv) $10\frac{1}{13}$ (v) $6\frac{3}{29}$ (vi) $20\frac{1}{4}$
(vii) 7.089 (viii) 0.26 (ix) 12.36 (x) 1.13

Exercise 5.3

(i) 13 (ii) 42 (iii) 32 (iv) $1\frac{1}{5}$ (v) $3\frac{1}{4}$ (vi) [illegible]
(vii) 1.2 (viii) 4.4 (ix) 3.2 (x) 32.1

Exercise 5.4

(1) 5.6m (2). 68 km (3) 26 trees (4) 320m (5) 11yd

Review Exercise 5

(1) (i) an even (ii) less (iii) an odd (iv) 11 (v) 25

(2) (i) (b) (ii) (c) (iii) (a) (iv) (b) (v) (d) (vi) (b) (vii) (a)

(3) (i) 900 (ii) 4225

(4) (i) 89 (ii) $\frac{56}{65}$ (iii) 2 35

(5) (i) 42 (ii) $\frac{68}{38}$ (iii) 8 8

(6) 22m (7) 8 students (8) 160m

Exercise 6.1

1. (i) 20 (ii) 30 (iii) 21 (iv) 5
2. Yes
3. 32
4. 700 silver, 400 white and 200 black cars.
5. (i) 9:8 (ii) 1:9
6. 7
7. 30°, 60° and 90°
8. Photocopier is speedy

Exercise 6.2

1.	120 hours	2	6000 litres	3	2138 4 km/h
4.	1224 km/h	5	125 m/s	6	180 km/h
7.	44 44 m/s	8.	1250 metres		

Review Exercise 6

1. (i) (d) (ii) (a) (iii) (a) (iv) (b) (v) (b)
2. (i) 14 : 18 : 39 (ii) 1 2 : 3
3. 2 typists
4. 6 42 (6 months and 13 days)
5. 180 days
6. 7 7 cm and 12 6 cm
7. 5 Km/h

Exercise 7.1

1. 360,000 rupees
2. 158,666.7 rupees
3. 58.25 rupees
4. 8140.5 rupees
5. 3646.35 rupees
6. 25,050 rupees

Exercise 7.2

1. 100,000 rupees
2. 6.45 rupees
3. loss of 5000 rupees
4. 109.17 rupees per book
5. 22 22% profit
6. Rs 900
7. Rs. 13302.22

Exercise 7.3

1. 26700 rupees
2. 1,000,000 rupees
3. 8560 rupees
4. 51,000 rupees
5. 3000 kg wheat
6. 6025 rupees

Review Exercise 7

1. (i) (c) (ii) (a) (iii) (d) (iv) (b) (v) (c) (vi) (d) (vii) (a) (viii) (b) (ix) (d) (x) (b) (xi) (c)
2. 144000 rupees
3. 1800 sq yard
4. 58250 rupees
5. 1400 rupees, 18 42%

Exercise 8.1

1. (i) constant term = -3, variable = x
   (ii) constant term = 5, variable = y
   (iii) constant term = 4, variable = x, y
   (iv) constant term = -25, variable = x, y
   (v) constant term = 8, variable = z,
   (vi) constant term = 7, variable = t
2. (i), (iii), (iv)
3. Monomial = [illegible] Binomial = (ii), (iv), (vi),(ix)
   Trinomial = (v), (vii), (x)
4. (i) $bh - \frac{1}{2}bh$ (ii) $ab - 4x^2$

Exercise 8.2

1. (i) $4x^2 + 2x + 6$ (ii) $4x^3 - x^2 + x - 4$
   (iii) [illegible]$x^2 - 5x + 19$ (iv) $6y^3 - 6y^2 + 4y + 4$
   (v) $5p - q + r$
2. (i) $-3x^2 - 4x + 9$ (ii) $x^3 - 5x^2 - 5x - 13$
   (iii) $-2a + 7b - 3c$
3. (i) $y^3 - 6y^2 + 3y + 19$ (ii) $-7x^4 - 6x^3y + 9x^2 + 15$

Exercise 8.3

1. (i) [illegible] (ii) $-5x^5 - 10x^2$ (iii) [illegible] (iv) $x^3 + 2x^2 - 4x - 8$
   (v). $a^5 + a^4 + a^3$ (vi). $x^3 - xy^2 - yx^2 + y^3$ (vii). $a^2 + b^2 + 2ab + ac + bc$
   (viii). $6x^4 - 5x^3 - 50x^2 + 45x - 36$

Exercise 8.4

(i) $y^2 - xy$ (ii) $x^2 + y^2 - 2xy$ (iii). $-(a^3 + b^3)$
(iv) $5a^2 - 16b^2$ (v) $2x^4 - x^3 + 9x^2 + 17x - 11$
(vi) $3 - x$ (vii) $10y^5 - 35y^4 + 18y^3 + 26y^2 - 67y + 31$

Exercise 8.5

1. (i) $x^2+7x+10$ (ii) $x^2-4x-21$ (iii) $-8x-11$

2. (i) $4a^2+12ab+9b^2$ (ii) $\frac{1}{4}x^2+3xy+9y^2$ (iii) $x^2-4xy+4y^2$

   (iv) $\frac{9}{4}a^2-\frac{15}{4}ab+\frac{25}{16}b^2$ (v) $5a^2+44ab-9b^2$ (vi) $13x^2+4xy+41y^2$

3. (i) $(x+7)(x-7)$ (ii) $(xy+8)(xy-8)$ (iii) $(5a+7b)(5a-7b)$

   (iv) 3439

Exercise 8.6

(i) $(x+2)(x+2)$

(ii) $(3x-4)(3x-4)$

(iii) $(3x+5)(3x+5)$

(iv) $(4+x)(4-x)$

(v) $(2x+3y)(2x+3y)$

(vi) $(ab)(a+b)(a-b)$

(vii) $(y-3)(y-3)$

(viii) $(x^2+y^2)(x+y)(x-y)$

(ix) $-2(x-4)(x-4)$

(x) 729

Exercise 8.7

(i) $(x+1)(x+3)$

(ii) $(x+2)(x+4)$

(iii) $xy(x+9)(x-3)$

(iv) $(x+5)(x-3)$

(v) $(a-5)(a+3)$

(vi) $-2a^2(a-1)(a-4)$

(vii) $(y-3)(y-2)$

(viii) $(t-4)(t+3)$

(ix) $(x+8)(x-3)$

Review Exercise 8

1 (i) variable (ii) constant (iii) binomial

(iv) polynomial (v) $a^2-2ab+b^2$

2. (i) F (ii) F (iii) T (iv) F (v) T

3. (i) c (ii) b (iii) d (iv) c (v) a

(vi) d (vii) d (viii) a

4. $-2x^2+6x+4$

5. $-5x^3y-4x^2y+15$

6. x^3+1

7. (i). $2x+4$

(ii). $2y$

(iii). $xy+2y+1$

8. (i). $(x+8)(x+8)$

(ii). $(4x+5y)(4x-5y)$

(iii). $(x-7)(x+6)$

Exercise 9.1

(i) $\{-12\}$ (ii) $\{7\}$ (iii) $\{10\}$ (iv) $\{14\}$

(v) $\{2\}$ (vi) $\left\{\frac{29}{4}\right\}$ (vii) $\{5\}$ (viii) $\{5\}$

(ix) $\{7\}$ (x). $\left\{-\frac{1}{6}\right\}$ (xi). $\{25\}$

Exercise 9.2

1. 3
2. 21
3. 49
4. 6
5. 6m, 10m
6. Age of mother = 39 years
   Age of daughter = 13 years

Review Exercise 9

1. (i) T (ii) F (iii) T (iv) F (v) F
2. (i) Solution (ii) same (iii) {10} (iv) linear, one (v) 4
3. (i) (c) (ii) (a) (iii) (d) (iv) (d) (v) (d)
   (vi) (b) (vii) (d)
4. (i) $\{10\}$ (ii) $\left\{\frac{7}{3}\right\}$
5. 5 m
6. {7}
7. 16, 17, 18
8. Rectangle: length = 5,width = 1
   Square: side = 3

Exercise 10.1

1. 25°
2. 45°
3. 105°
4. 40°
5. 20°
6. 20°
7. 6°

Exercise 10.2

1. Yes
2. (i) 93 (ii) 5.4 (iii) 38 (iv) e (v) d

3 (a) 10, 11 and 12 (b) 4 (c) similar (d) congruent (e) 2,8 and 9

Exercise 10.3

3. (i) 10cm (ii) 17.2cm
4. (i) 5.5cm (ii) 7mm

Review Exercise 10

1. (i) a (ii) d (iii) a (iv) d (v) c
2. (i) F (ii) F
3. (i) $6x = 30$ (ii) $4 + 5x + x + 2 = 180$ (iii) $5x + 3x + 12 = 180$
   (iv) $6x + 4 + 32 = 90$
4. $30°$

Exercise 12.1

1. 220cm 2. 452.57cm 3 2640cm 4. 63360cm or 633.6 m

Exercise 12.2

1. 42.39 cm^2 2. 1131.4 mm^3 3. 3850 m^2 4. 2464 cm^2
5. 995.7 cm^3 6. 957. cm^2 7. 424.3 m^3
8. 7.2 cm 9. 3977.27 miles

Review Exercise 12

1. (i). b (ii). a (iii). c (iv). d (v). c (vi). c
   (vii). a (viii). a (ix). a (x). c
2. 88 cm
3. 3054.86 cm
4. 282.9cm^2 and 240.4 cm^3
5. 154 mm^2 6. 727 tiles

1 a 12% b 15 years c 65 years d 10

2. a. 25 days b. 7 days

c units pro duction from 70 units up to 80 units

d. 25-19= 6 days

Exercise 13.2

1 The frequency distribution for the above problem is:

Serial number	Classes/ Category	Frequency	Class relative Frequency
1	Toilet paper	132	132/310=.43(100%)=43%
2	Hand towels	85	85/310=.27(100%)=27%
3	Napkins	43	43/310=.14(1005)=14%
4	Other products	50	50/310= 16(100%)=16%
Total		310	100%

The proportion in each category is the following

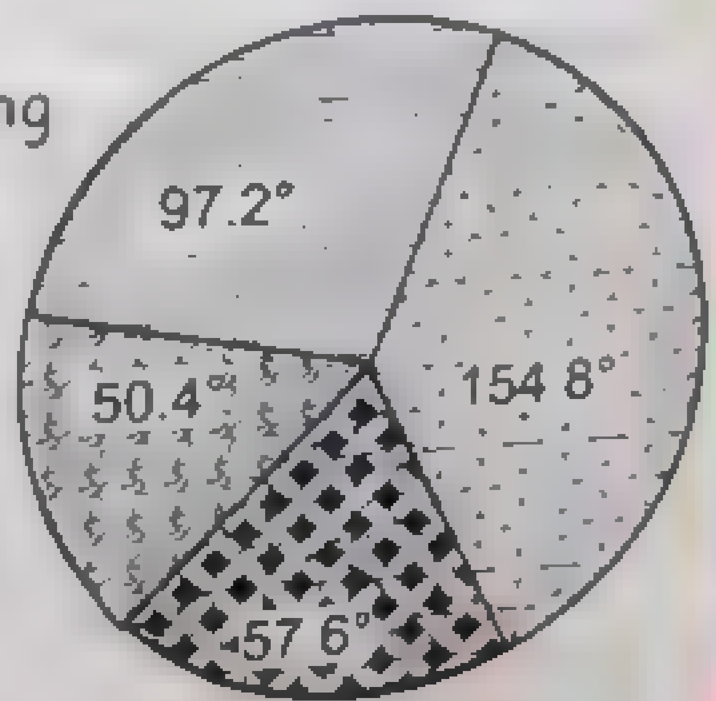
Toilet paper=360°(.43)=154.8°

Hand towels=360°(.27)=97.2°

Napkins=360°(.14)=50.4°

Other Products=360°(.16)=57.6°

The pie graph is as given:



2. The frequency distribution for the above problem is:

Serial number	Classes/ Category	Frequency	Class relative Frequency
1	Domestic	1950	1950/13950=.14(100%)=14%
2	Commercial	4000	4000/13950=.29(100%)=29%
3	Industrial	8000	8000/13950=.57(100%)=57%
Total		13950	100%

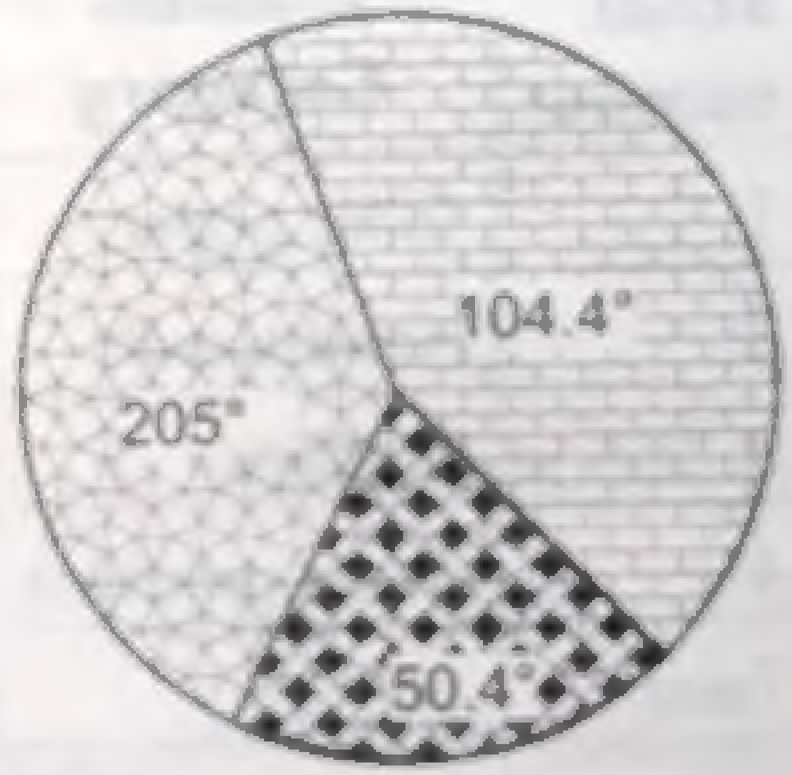
The proportion in each category is the following:

Domestic=360°(.14)=50.4°

Commercial=360°(.29)=104.4°

Industrial=360°(.57)=205.2°

The pie graph is as given:



3.The frequency distribution for the above problem is:

Serial number	Classes/ Category	Frequency	Class relative Frequency
1	BBA	62	62/200=.31(100%)=31%
2	BCS	40	40/200=.20(100%)=20%
3	MBA	28	28/200=.14(1005)=14%
4	B.Eng	70	70/200=.35(10050=35%
Total		200	100%

The proportion in each category is the following:

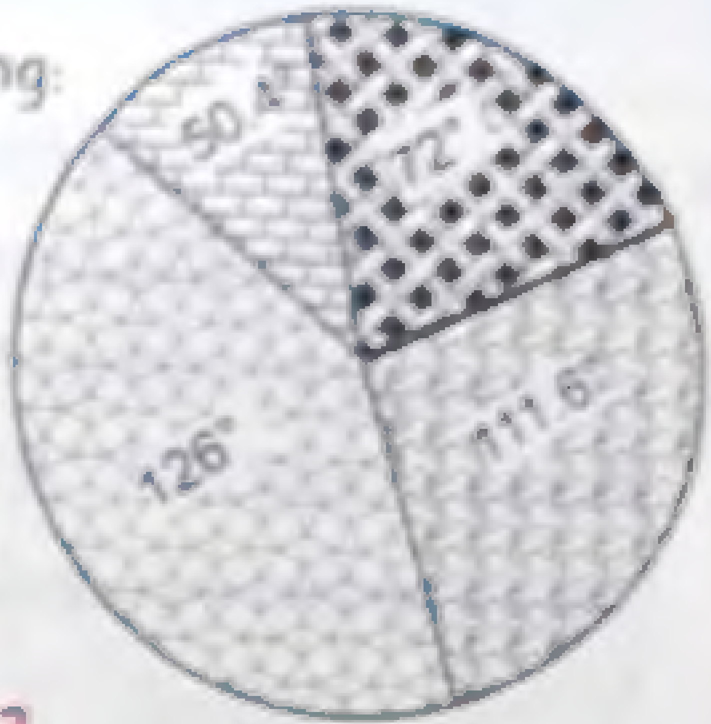
BBA=360°(.31)=111.6°

BCS=360°(.20)=72

MBA=360°(.14)=50.4°

B.Eng.=360°(.35)=126°

The pie graph is as given:



Review Exercise 13

1. (i) a (ii) . b (iii) b (iv). b (v) d (vi). b
2. The frequency distribution is as under:

Serial Number	Classes	Frequency
1	35-40	1
2	40-45	13
3	45-50	6
4	50-55	9
5	55-60	6
6	60-65	1
Total		36 Students

3. a. 365 days b. 43 days c. 54 C°
4. The frequency distribution is as under:

Serial Number	Classes	Frequency	Class relative frequency
1	Bus fare	14	14/25=.56(100%)=56%
2	Recess meal	8	8/25=.32(100%)=32%
3	Money Saved	3	3/25=.12(100%)=12%

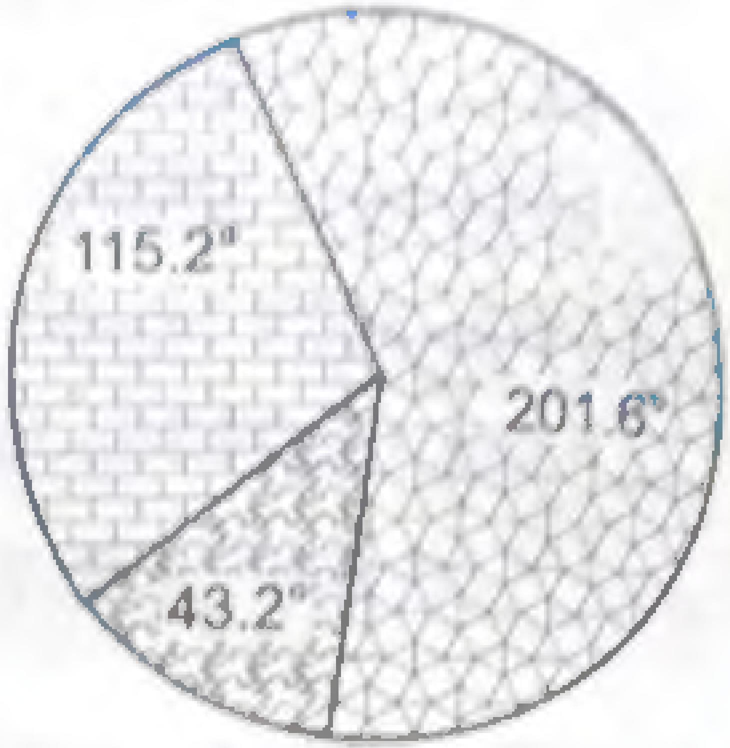
The proportion in each category is the following:

Bus fare= 360°(.56)=201.6°

Recess= 360°(.32)=115.2°

Money saved=360°(.12)=43.2°

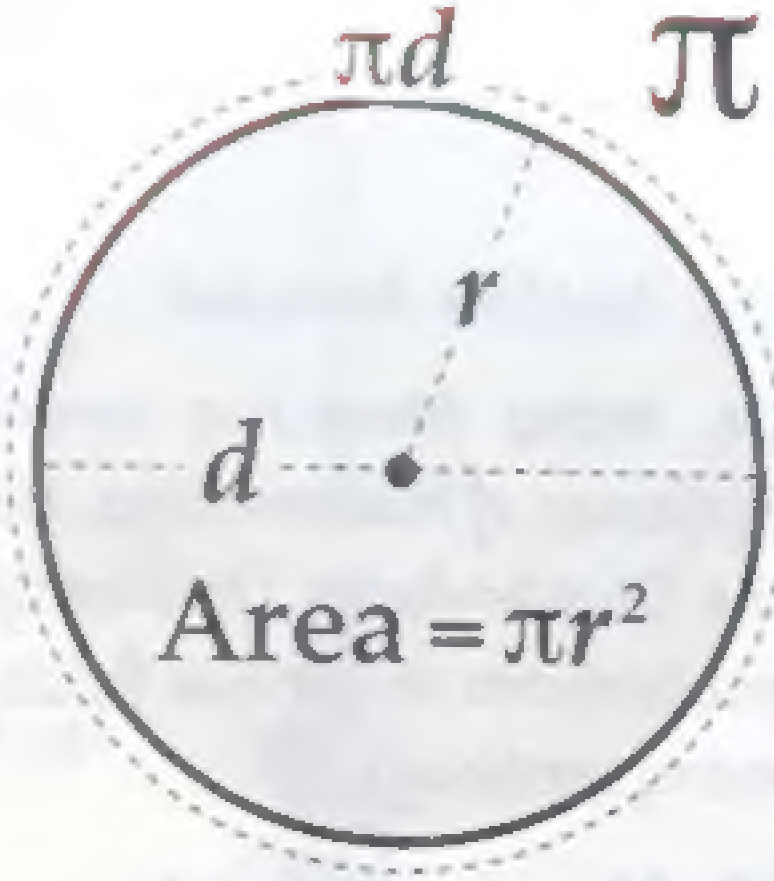
The pie graph is as under:



Did you know?

$$88 = 9 \times 9 + 7$$
$$888 = 98 \times 9 + 6$$
$$8888 = 987 \times 9 + 5$$
$$88888 = 9876 \times 9 + 4$$
$$888888 = 98765 \times 9 + 3$$
$$8888888 = 987654 \times 9 + 2$$
$$88888888 = 9876543 \times 9 + 1$$

π is the GREEK letter for p, but it is so much more than that. It is an irrational number with an infinite number of decimal points, but generally speaking five or six are enough to use it extremely accurately.



Al-Khwarizmi
The "Father of Algebra"

- The best known of the Islamic Mathematicians
- Considered one of the greatest Mathematicians of all times
- His books were studied long into the Renaissance
- To him we owe the words: Algebra and Algorithm